

DEPARTMENT OF THE INTERIOR

REPORT

OF THE

CHIEF ASTRONOMER

FOR THE

YEAR ENDING MARCH 31

1907

PRINTED BY ORDER OF PARLIAMENT



OTTAWA

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EXCELLENT MAJESTY

1908

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REPORT OF THE CHIEF ASTRONOMER AND INTERNATIONAL BOUNDARY COMMISSIONER.

DEPARTMENT OF THE INTERIOR,
DOMINION ASTRONOMICAL OBSERVATORY,
OTTAWA, CANADA, July 1, 1907.

W. W. CORY, Esq.,
Deputy Minister of the Interior,
Ottawa.

SIR,—I have the honour to report as follows upon the work of the Astronomical Branch of the Department of the Interior, and of the International Boundary Surveys for the nine months ending March 31, 1907.

The correspondence of the branch from July 1, 1906 to March 31, 1907 was:—

Letters received (exclusive of circulars)	964
Letters sent " "	2,008
Showing an increase over the previous fiscal year of $23\frac{1}{2}$ per cent.	
Accounts dealt with	743
Increase, $47\frac{1}{2}$ per cent.	

A statement of the work of the photographic division is appended. (Appendix No. 1.)

The library now contains 2,469 bound volumes, besides numerous pamphlets. The increase is rapid from the addition of scientific journals, reports of other observatories, &c. To meet the increase a large addition to the shelving is being made by the Department of Public Works.

The workshop has proved most useful. The appointment of a mechanic, last July, has enabled many improvements as well as repairs to be made to instruments, resulting in economy in both time and money. Repair work in the building obviates the necessity of sending an instrument away, which may involve the interruption of a series of observations, while the construction of apparatus to a required design, and under the direct supervision of the designer, is a most valuable feature. Construction has not been confined to minor apparatus; a spectrograph specially adapted to determination of radial velocities has been constructed. A description of this instrument, which was designed by Mr. Plaskett, will be found in his report appended hereto. A registering micrometer for attachment to one of the transit instruments is now in course of construction.

The Observatory has joined the 'astronomical exchange.' At the Observatory of Harvard University is a central bureau for the receipt from observers all over the continent of reports of any discoveries or notable observations which they may make. These reports are telegraphed to the observatories which are members of the exchange, and are of service in keeping the members of the staff informed on current astronomical matters. A well devised cipher enables a great deal of astronomical information to be conveyed in a short telegram.

In December, a section of the Royal Astronomical Society of Canada, comprising now over one hundred members, was formed. Fortnightly meetings were held in the Observatory during the winter, at which papers on astronomical subjects were read

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and discussed. These meetings have been of great service to the members of the staff, by the interchange of ideas, and they have also evoked much public interest.

The number registering in the Visitors' Book, has been 2,688, during the nine months ending on March 31, last. Many of these have called during the day, to see the 15-inch telescope and other instruments, including the, to many, more interesting apparatus, that by which the time system is operated.

On Saturday nights, the public is admitted to view the heavens through the large telescope. Members of the Astronomical Society also have this privilege on the nights of meeting.

It has been necessary to refuse the applications which occasionally are made to look through the telescope on other nights. The instrument is in use on every clear night, with the spectroscope or other auxiliary instrument attached, and to grant a request to see through the telescope would necessitate replacing the attachment by the visual eye-piece, and an adjustment of the counterpoises, with a resulting loss of time which would be fatal to regularity of observations.

The transit instrument is still housed in the temporary shed to the east of the main building, the western wing, built to accommodate this instrument as well as the meridian circle, not having yet been completed. Work, however, is now progressing upon the piers for the instruments, and on the roof of the wing. The meridian circle has not yet been received from the makers.

It is expected that work will soon be commenced on the coelostat house, and the house for standardizing measures of length. The plans and specifications have been completed by the Public Works Department and it is expected that tenders will be called for in the near future. Both buildings are much needed.

The astrophysical work has been continued under the direction of Mr. Plaskett. It has comprised, mainly, observation of velocities of stars in the line of sight for determination of the orbits of spectroscopic binaries; also, solar photographs for record of sun-spot areas. Micrometric work on double stars has been begun. Mr. Plaskett has undertaken investigations of the errors entering into spectrographic work. He has also prepared drawings of mechanism for coelostat telescope, of house for the same, and of various instruments, an account of which will be found in his report hereto appended. In the summer of 1906 he visited a number of observatories at which spectrographic work is carried on, with a view to familiarizing himself with the processes employed.

Daily records are obtained from the seismograph of earth movements. The large scale of the record (90 c.m. per hour) is of advantage in the accurate determination of the time of disturbances. A discussion by Dr. Klotz of these observations, and their scientific bearings, will be found in his report.

Arrangements have been made for commencing during the present summer, systematic observations of the magnetic elements at various points, as well as observations for gravity with the half-seconds pendulum, in continuation of the observations with this apparatus made by Dr. Klotz some years ago at Ottawa, Toronto and Washington, and at points on the route of the transpacific cable.

The time service has worked satisfactorily. There are now 215 dials operated under the system of control from the Observatory, described in previous reports, with one tower clock (at the Observatory). A system of twenty dials and a tower clock will shortly be in operation in the Post Office, and provision is being made for 29 dials in the Printing Bureau, 29 in the Mint and 7 in the Archives Building. The Ottawa Electric Company have offered to place, at their own expense, a large dial in front of their office on Sparks Street, to be operated from the post office circuit. Some improvements have been made in the mechanism for sending the noon signals.

The time-keeping of the standard sidereal clock at the Observatory has been brought to a high degree of perfection by means of an automatic temperature regulation through a Callendar recording (and controlling) thermometer. A description

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of this instrument will be found in the appended report by Mr. Stewart on the time system.

My last report, dated October 9, 1906, brought the account of the field astronomical work for the determination of latitudes and longitudes up to the close of last summer's operations. Necessarily there is nothing to report as to the occupation of new stations, since this work cannot be done in winter. In April, last, Mr. F. A. McDiarmid, who is our principal field astronomer, was detailed to accompany an officer of the United States Coast and Geodetic Survey to the 141st meridian, at the Yukon river, to observe an initial azimuth for the survey of that meridian. As a better determination of latitudes and longitudes of points on the Yukon river has long been a desideratum, it was thought advantageous that Mr. McDiarmid's services, while he was in that region should be utilized, after he had completed the azimuth work, in the determination of the geographical positions of various points between the boundary at the 141st meridian and the boundary at White Pass, including Dawson. Mr. W. C. Jaques was detailed as the second observer.

Arrangements have also been made for the observation of the geographical co-ordinates of several points in Ontario, Quebec and the maritime provinces, for cartographical purposes.

The trigonometrical survey of Canada is being continued. Owing to the unusually late spring this year, operations were much delayed, although reconnaissance was made by Mr. Bigger, during the winter, eastward as far as the boundary of New Hampshire, and at this date, the selection of angular points has been completed between this point and a meridian about 30 miles west of Ottawa. The building of observing scaffoldings where necessary at the angular points has been almost completed over this whole extent, and observing is now proceeding.

Lines of level are being run over the principal railway lines in the eastern townships to connect with the levels which are being carried along the international boundary line (45th parallel). Two parties are engaged on this work.

At the request of the Militia Department a connection is being made with the United States Lake Survey primary stations on the Niagara peninsula, with a view to triangulating across the lake to the neighbourhood of Toronto. This connection will afford a basis for the topographic work of that department around Toronto.

It is my painful duty to record the death of Mr. J. D. McLennan, D.L.S., who was employed on the triangulation for two years, 1905 and 1906. His health failing, he was compelled to apply for sick leave last winter. He died at his home, at Port Hope, on April 19, 1907.

In pursuance of an order in council, dated November 13, last, a committee was formed of representatives of the departments which conduct surveys, to consider what steps, if any, should be taken towards the systematizing of surveys for topographical purposes.

The committee was composed of seven departmental representatives, together with representatives of the Universities of Toronto, McGill and Laval. The undersigned was elected chairman of the committee, which held frequent meetings in December, January and February, and completed its report on February 15.

In my last report an account was given of the observations for longitude preparatory to the survey of the 141st meridian. As already stated, before the opening of navigation on the Yukon, Mr. McDiarmid was sent to the point where the meridian crosses the Yukon river to locate, in co-operation with Mr. Baldwin, of the United States Coast and Geodetic Survey, the initial point of the meridian by measurement from his observing station of last year, and to lay down the meridian of that point, to be produced southward by the line surveyors.

Mr. A. J. Brabazon, in charge of the Canadian section of the line surveying party, followed at the opening of navigation. It is intended to produce the line south as rapidly as possible in order to reach the mining region near the White river before

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winter, if possible. A topographical survey will be made extending two miles on each side of the line, based upon a triangulation.

The demarcation of the boundary of the Alaska Coast strip is proceeding as usual. Mr. J. D. Craig, D.L.S., is working on the line back of Bradfield canal, and Mr. W. F. Ratz, D.L.S., is marking the boundary line at Taku and Whiting rivers, and making an exploratory survey of the unmapped region east of Stephens passage, in order that the commissioners may be able to decide (in accordance with the agreement of March, 1905) which peaks should determine the boundary line. Each of these surveyors is accompanied by a representative of the United States commissioner.

Mr. D. H. Nelles, D.L.S., accompanies, as my representative, an American party under Mr. Fremont Morse. This party is engaged in making a triangulation up Glacier bay, in order to determine the geographical positions of the mountains on the boundary line westward from the termination of the survey made two years ago by Mr. Ratz, south of the Salmon river. One peak is especially important, the first peak east of the Alsek river, for between it and the next peak west, as determined by the Tribunal, there is a stretch of 50 miles. Hence the proper identification of this peak, and the determination of its geographical position is most important. An attempt was made last year to reach it from the other side, by way of the Alsek, but it was not identified with certainty.

Another United States party is working to the east of Lynn canal.

The survey of the 49th parallel is being continued by Mr. J. J. McArthur, D.L.S. Of the part of this line west of the summit of the Rocky mountains, there remains to be completed but a few miles in the foothills of the Cascade range. This will be completed this year.

I have arranged for a tour of inspection of this line in company with the United States Commissioners, Mr. Tittmann and Dr. Walcott. We start in a few days.

The survey of the Eastern section of the boundary line (from the Richelieu river to the St. Croix river) is proceeding under Mr. G. C. Rainboth and Mr. J. B. Baylor, the Canadian and United States engineers, respectively. The work, which consists of a resurvey of the line and the placing of new monuments, was begun last August at Hall's stream, at the northeastern corner of the State of Vermont. It is expected to reach Richelieu river by the close of the present season.

A survey of the international boundary line was made at Portal, on the C.P.R. at the southern boundary of the province of Saskatchewan.

The boundary line (the 49th parallel) had been surveyed in this locality by the Joint Commission of 1872-75, but the nearest original monuments were at some distance on each side of the railway station, where a closer definition of the line was desired on account of a question of jurisdiction which had arisen.

In October, 1906, I gave Mr. C. A. Bigger instructions to perform the demarcation in co-operation with Mr. O. B. French of the United States Coast and Geodetic survey, who was detailed by Mr. Tittmann, the American Commissioner, for the work. The survey was accordingly made, and the line marked with iron bolts driven into the ground, and by nails in the station platform. The demarcation was approved by Mr. Tittmann and myself in a joint report dated November 23, 1906, which has been accepted by the two governments.

Appended hereto will be found the following statements and reports:—

Appendix 1.—Report of work done in the photographic division.

Appendix 2.—Report by Otto Klotz, LL.D., on gravity, seismology and magnetics.

Appendix 3.—Report by J. S. Plaskett, B.A., on astronomical and astrophysical work.

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Appendix 4.—Report by R. M. Stewart, M.A., on time service and transit observations.

Appendix 5.—Observations for latitude and longitude.

I have the honour to be, sir,
Your obedient servant,

W. F. KING,
Chief Astronomer
and International Boundary Commissioner.

APPENDIX 1

REPORT OF THE CHIEF ASTRONOMER, 1907.

STATEMENT OF WORK PERFORMED IN THE
PHOTOGRAPHIC DIVISION

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APPENDIX 1.

STATEMENT of work done in the Photographic Division between the 1st July, 1906, and the 31st March, 1907.

	Size of Prints, Negatives, &c.														Total.
	In. 3 1/2 x 5 1/2	In. 4 x 5	In. 4 1/2 x 6 1/2	In. 5 x 7	In. 8 x 10	In. 10 x 14	In. 14 x 17	In. 16 x 20	In. 20 x 24	In. 8 x 36	In. 28 x 36	In. 30 x 40	In. 40 x 60	In. 3 x 14	
Survey plates developed			914	132											1,046
Films developed	200	220													420
Copies, maps and plans					46		7	29							82
Lantern slides and transparencies		202													202
Black and white and blue prints							24	17	10		30	25	10		116
Bromide prints						1,149	11	78	8	48					1,294
Argo paper prints (contact)		405	130	2,306	292										3,133
Seismograms developed										274					274
Star spectrum plates enlarged														12	12
" print enlargements														18	18
Sun observation plates developed					145										145
Platinum prints				75											75
Total	200	827	1,044	2,513	483	1,149	42	124	18	322	30	25	10	30	6,817

J. D. WALLIS,
Photographer.

APPENDIX 2.

REPORT OF THE CHIEF ASTRONOMER, 1907

GRAVITY, SEISMOLOGY AND MAGNETIC WORK

BY

OTTO KLOTZ, LL.D.

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APPENDIX 2.

GRAVITY, SEISMOLOGY AND MAGNETIC WORK, BY OTTO KLOTZ, LL.D.

W. F. KING, Esq., B.A., LL.D.,
Chief Astronomer,
Department of the Interior,
Ottawa.

OTTAWA, ONT., July 1, 1907.

SIR,—I have the honour to make the following report of the work carried out under my charge, which may be classified under three different headings:—Gravity, Seismology and Magnetism.

GRAVITY.

The pendulum observations made by me with our half-seconds apparatus at McGill University, where previously Commandant Defforges of Paris had observed, and at the School of Practical Science, Toronto, are given in abstract; by comparison with my observations taken at the international base station of the United States Coast and Geodetic survey, Washington, I was enabled to give satisfactory absolute values for gravity.

The most interesting and valuable gravity observations taken by me are those in the South Seas at Suva, Fiji, and those at Doubtless bay, at the northern extremity of New Zealand. The importance is two-fold; in the first place the vast Pacific is more or less a virgin field for gravity work, and in the second place the number of gravity stations in the vastly greater water-area of the globe is very limited, so that the anomalies of gravity deduced from theoretical considerations based on latitude and an assumed ellipsoid of revolution from geodetic and pendulum measures, are known only for a relatively small part of the earth's surface. The results obtained therefore from ocean stations are at present of far greater value than for those on land.

Advantage was taken of the Eclipse expedition to Northwest river in August, 1905, under your charge to obtain gravity observations there. These were carried out by Professor L. B. Stewart, and are given in abstract together with the reduction to absolute value based on my Washington observations with the same pendulums.

The observations for gravity at the five stations, Montreal, Toronto, Suva (Fiji), Doubtless Bay (New Zealand), and Northwest river were made with the half-seconds pendulum apparatus described in my report for 1905.

Each of the three pendulums was swung for about eight hours in one position and then for a similar time in the reversed position. Time observations, the interval between which serves as a scale expressed in sidereal seconds for determining the period of a pendulum, were obtained at the beginning, at the end, and during the swings of the pendulums. Two sidereal chronometers were always used by me, except for Montreal, where the standard sidereal observatory clock was used, for noting coincidences, and one of them was used for the time determinations. The latter were always obtained from two positions of the transit, circle east and circle west, and the transits recorded on the chronograph. A comparison of the two chronometers was generally made three times a day and on the chronograph so that a good differential rate between the two time pieces was obtained. This differential rate was interpolated for the middle time of each swing for obtaining the rate correction for the

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period. In order to make this part of the report complete in itself, it will be well to repeat briefly the method of reduction of the observations. The lengths of the pendulums, upon which the period depends, are supposed to be invariable except as affected by temperature. As a test of invariability they are generally swung again at a base station after a pendulum campaign. Gravity observations with a half-seconds pendulum apparatus give only differential values for the acceleration of gravity, that is, a comparison is made of the period of a pendulum, or of the 'mean pendulums,' which is the mean of the several pendulums swung, at a given station with that at a base station for which the absolute value is known. Washington, where the pendulums were first swung, is the base station for the following observations.

In order to make the periods comparable with those obtained with the same pendulums at other times and stations, it is necessary to reduce them to certain standard conditions. These conditions, which are arbitrarily adopted are: an infinitely small arc; temperature 15° C; pressure 60^{mm} of mercury at 0° C; true sidereal time; and inflexible support.

Arc Correction.—Were the pendulums swinging for a very brief time only, the reduction might be made from the observed arc or amplitude. We would then have

$$t = t_0 \left(1 + \left(\frac{1}{2}\right)^2 \sin^2 \frac{a}{2} + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 \sin^4 \frac{a}{2} + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 \sin^6 \frac{a}{2} + \dots \right)$$

where t_0 = reduced time or time for infinitely small arc, t = observed time, and a = the amplitude or half of total arc.—

The above expression reduces to

$$t = t_0 \left(1 + \frac{a^2}{16} + \frac{11a^4}{3072} \dots \right) \quad a \text{ being in radians,}$$

from which

$$t_0 = t \left(1 - \frac{a^2}{16} + \frac{a^4}{3072} \dots \right)$$

As a is generally less than 3°, the term containing the fourth power may be omitted and we obtain

$$t_0 = t \left(1 - \frac{a^2}{16} \right) \quad (1)$$

This reduction pertains to a uniform arc, or when the swing is of short duration, and for such may be put in the form, $t_0 = t \left(1 - \frac{a' a''}{16} \right)$, where a' and a'' are respectively the arcs at the beginning and end of a swing.

This latter form is the one used in the reduction of the oscillations of a magnet.

When, however, the swing continues for a considerable time, formula (1) is no longer applicable.

On the assumption that the amplitude decreases in geometrical ratio, the relation between amplitudes is $a = a_1 e^{-kt}$, where k is the logarithmic decrement, t the time interval and e the base of the natural logarithms. For reduction, the adaptation of Borda's formula is,

$$\text{arc correction} = - \frac{P M}{32} \frac{\sin(\phi + \phi') \sin(\phi - \phi')}{\log \sin \phi - \log \sin \phi'}$$

where P is the period of the pendulum in seconds, M the modulus of the common logarithmic system, ϕ and ϕ' the initial and final semi-arcs respectively.

Temperature Correction.—The coefficient necessary for this correction was determined experimentally at Washington with pendulums of the same material and construction as those of our apparatus, by swinging the pendulums at temperatures differing about 20° C, and obtaining the periods for the different temperatures.

From these experiments the formula for correction for temperature was derived

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Temperature correction = $+ \cdot 00000418 (15^\circ - T^\circ)$, where T° is the temperature of the 'dummy' pendulum within the air chamber in degrees Centigrade.

Pressure Correction.—The air chamber during a swing is exhausted to about 60^{mm} pressure, and the swings are reduced to this pressure. From observations by G. R. Putnam, at Washington, in 1894, the

$$\text{pressure correction} = + \cdot 000000101 \left[60 - \frac{\text{Pr}}{1 + \cdot 00367 T^\circ} \right]$$

where Pr is the mean of the observed pressures at the beginning and end of the swing, and T° the mean temperature of the pendulum during the swing. The expression $\frac{\text{Pr}}{1 + \cdot 00367 T^\circ}$ is simply a reduction of the air pressure to a temperature of 0° Centigrade.

Rate Correction.—In field observations very good time determinations are obtained with the astronomic transit, chronometer and chronograph; the comparisons between the chronometers, three times daily, on the chronograph for differential rate are very satisfactory, yet for absolute rate we are dependent upon one or other of the chronometers keeping a uniform daily rate as deduced from the time observations. The fluctuations of the rate during the 24 hours must be sought mainly in the change of temperature, which it is very difficult to maintain uniform in an observing hut, ten feet square.

As there are 86,400 sidereal seconds in a day, if R is the daily rate then the correction per second = $\frac{R}{86400} = \cdot 000011574 R$, and for a period P

$$\text{Rate correction} = \cdot 000011574 RP$$

For a chronometer gaining the correction is subtractive, and for losing additive.

Flexure Correction.—As all the observations to date with one exception have been made on solid stone piers detached from the immediate floor, the flexure correction is small and practically constant. From observations made statically by means of a weight, 1.5 kilogrammes, the following formula is derived:—

Flexure correction = $\cdot 00000065 D$, where D is the displacement of the knife-edge in microns.

Applying the above four corrections, the periods of the pendulums are obtained and expressed in sidereal seconds. The acceleration of gravity, or g , is expressed, however, in terms of a mean time second. For differential gravity observations it is quite immaterial which time, sidereal or mean, we employ, as the ratio between them would remain the same in the deduction of the unknown g from the relation $P_o^2 : P^2 = g : g_o$ where P_o , g_o pertain to the base station.

A word about the theoretical value of g .

In 1743 was published Clairaut's celebrated work 'Theorie de la Figure de la Terre,' in which is given his famous theorem: $\frac{g' - g}{g} = \frac{5}{2} m - e$ (2)

where g' and g are respectively the values of gravity at the pole and equator, m the ratio of the centrifugal force at the equator to gravity, and e the ellipticity of the meridian or flattening.

Furthermore for any latitude $g_\phi = g \{ 1 + (\frac{5}{2} m - e) \sin^2 \phi \}$ for sea level. (3)

Todhunter* says: 'The assumptions on which Clairaut's demonstration of his famous theorem rests should be carefully noticed. The strata are supposed to be ellipsoidal, and of revolution round a common axis, and nearly spherical. Each stratum is homogeneous, but there is no limitation on the law by which the density varies from stratum to stratum: the density may change discontinuously if we please.

* History of the Theories of Attraction and the figure of the earth. Vol. 1, p. 221.

It is not assumed that the strata were originally fluid; but it is assumed that the *superficial* stratum has the same form as if it were fluid and in relative equilibrium when rotating with uniform angular velocity. There is no limitation on the law by which the ellipticity varies from stratum to stratum, except that the ellipticity must be continuous, and at the surface must be such as would correspond to the relative equilibrium of a film of rotating fluid.'

Fundamentally Clairaut's theorem is used to-day, it is simply a matter of substituting values for g , m and c , and the different values adopted for these latter constitute practically the differences between different formulas that we have for the theoretical determination of the value of g_ϕ . As data accumulate slight modifications in the constants are made.

Another form of expressing (3) is $g_\phi = g_{45} (1 - B \cos 2\phi)$, in which form Harkness gives $g_\phi = 980.60 (1 - .002662 \cos 2\phi)^*$ (4)

The most recent accurate general formula for sea-level is that of Helmert

$$g_\phi = 978.046 (1 + .005302 \sin^2 \phi - .000007 \sin^2 2\phi)^\dagger \quad (5)$$

This formula differs from Helmert's 1884 formula principally in the new value 978.046 for equatorial gravity, instead of 978.000.

It has been found that the actual figure of the earth as determined by the surface of the oceans differs but very little from a figure of revolution, the differences that are found, however, from Clairaut's assumption are the deviations from homogeneity in the distribution of the matter of the earth. The difference between the observed value of g at or near sea-level and the computed one (5), dependent upon the latitude gives the 'anomaly.'

In the reduction to sea-level, that is, merely for height of stations, we have directly, from theoretical consideration that gravity varies inversely as the square of the distance, the change in g due to an elevation h , where h is small compared with the radius r of the earth, equal to $\frac{2h}{r}$ hence formulae (3), (4), (5) obtain the additional factor $\left(1 - \frac{2h}{r}\right)$. Putting $r = 6.378 \times 10^6$ metres, we find that 33 metres elevation decreases gravity by $^{\text{cm}}.01$, which Helmert expresses as $^{\text{cm}}.0003086 H$, where H is in metres. This reduction is simply for elevation and disregards the matter lying between the station and sea-level.

If we take into consideration the matter lying between the station and sea-level, and on the assumption that it extends indefinitely in a horizontal plain or is a shell of depth H , then gravity is increased by $\frac{3H}{r} \frac{\delta}{\Delta}$, where δ is the density of the matter above sea-level, and Δ the mean density of the earth. Any deviation from the latter condition by mountain masses above the station, or valleys beneath the same will decrease the gravity at the station and hence a correction, 'the topographical' correction, making a third term to the above two is introduced, it has the positive sign. When this term is introduced topographical sheets are necessary and some value for density must be assumed.

It must be admitted that with reference to these latter reductions considerable uncertainty is involved dependent upon the assumptions involved and the constants adopted.

Professor Everett says: 'The reduction for elevation is largely a matter of guesswork; and in records of observations the actual values are much more important than the so-called 'values reduced to sea-level.'

For deducing the absolute value of g for a station, we have the relation with reference to the base station at Washington, $P_w^2 : P^2 = g : g_w$ (6)

* Smithsonian Geographical Tables 1897—Appendix.—

† 'Der normale Teil der Schwerkraft im Meeresniveau.' K. Preuss. Akad. der Wissenschaften zu Berlin. 1901 S. 336.

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In this reduction, as formerly, g_w is taken at 980.098 dynes, this value when similarly applied to Potsdam, for observations with the same three half-seconds pendulums, gives for the latter $g_p = 981.261$, while the absolute determination by an elaborate series of reversible seconds pendulums gives* $g_p = 981.274$, a difference of .013. For any of the following deduced values of g by adding .013 we obtain a value based on Potsdam.

Corrections to thermometers, as tested at Washington Bureau of Standards. Test No. 229, December 27, 1902.

SCALE.	CORRECTIONS.	
C	Green 116.	Green 121.
0.0	— .10	— .20
5.0	— .10	— .10
10.0	— .10	— .15
12.5	— .15	— .15
15.0	— .15	— .15
17.5	— .15	— .20
20.0	— .15	— .20
22.5	— .15	— .15
25.0	— .20	— .20
27.5	— .20	— .20
30.0	— .20	— .20
32.0	— .20	— .20
35.0	— .20	— .20
40.0	— .20	— .20

That is, the Green thermometers read too high.

Montreal.—Pendulum apparatus was mounted on and cemented to a solid brick pier 2' 5" x 3' 5" and 3' above the floor, slate top. The pier was 27 feet west of the pier on which Commandant Defforges observed in 1893, being in the north basement of the Physics building, McGill University. Latitude of station $45^{\circ} 30' 22''$ (McLeod), longitude $73^{\circ} 34'$; elevation 131 ft. (40^m).

A telegraph line was strung between the nearby McGill College observatory, connecting the sidereal observatory clock with the flash apparatus. Professor McLeod, director of the observatory, kindly supplied the daily rate of the clock.

It may be noted that in the report† of Commandant Defforges for his Montreal observations he gives the height of the station as 100 metres instead of 40 metres, the actual height. Hence his reduced value for g requires a correction for the erroneous height.

Toronto.—Here the apparatus was set up in the southeast basement of the School of Practical Science. It was mounted on and cemented to the comparator stand, a solid steel structure on firm foundation and free from the floor. Latitude of station $43^{\circ} 39' 35''$, longitude $79^{\circ} 24'$; elevation 349 ft. (106^m). Professor L. B. Stewart observed for time and the daily rates of the chronometers were deduced therefrom.

Suva.—This station was primarily occupied as a longitude station in the series 'Transpacific Longitudes,' given in the report of the Chief Astronomer for 1905.

Adjoining the transit hut was erected another one 7 feet square and in it a concrete pier was built 2 feet square, 5 feet high, and rising two feet above the floor. The circulation of the air was facilitated by an opening 3 inches wide between the

* "Bestimmung der absoluten Grösze der Schwerkraft für Potsdam mit Reversionspendeln"—Berlin, 1906.

† Translation in United States Coast and Geodetic Report, 1894.

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roof and the walls. The flash apparatus was in the transit hut and a small opening in the board partition permitted coincidences to be noted. The two chronometers were kept in the 'artificial line' cabinets within the contiguous cable building, in order to assure as uniform a temperature as possible, and was satisfactory under the circumstances, the range for the twenty-four hours being confined to about two degrees Fahrenheit. In the huts where the range of temperature is greater, it is (in the tropics) far less than in the temperate zone, where in our observing (transit) huts the temperature in the summer during the day-time readily runs up to 100° F. and more. Latitude of station—18° 08' 45", longitude 178° 25' 36" E, elevation 2^m+

Time observations were made with the astronomic transit as in longitude work, observing clamp east and clamp west, recording on the chronograph, on which also the chronometers were compared.

From the transit observations, the daily rate of Bond 516 is found to be uniform, -1^s.60 (gaining), which is adopted, and from the three daily comparisons of the two chronometers, the varying rate of Dent 48,419 is deduced. Although the rate of Dent varies considerably during the 24 hours, yet by the frequent comparisons of the two chronometers, a good differential rate is obtained and from it the rate is interpolated for the middle time of swing; as the periods by the two chronometers show, a satisfactory result is obtained by this interpolation.

DATE.		Dent 48419			Bond 516			D—B	Difference corrected for Bond rate.	DENT.			Middle time of pend. swing.	Dent daily rate during swing.	No. Swing	
										At time.	Daily rate					
1903		h.	m.	s.	h.	m.	s.	s.	s.	h.	m.	s.	h.	m.	s.	
July	10	7	29	00	7	29	08.69	— 8.69								
"	12	17	04	00	17	04	10.52	—10.52	—12.53							
										22	00	—1.92	22	00	—1.92	1
"	13	2	56	00	2	56	10.39	—10.39	—11.18	7	07	—1.14	6	08	—1.23	2
"	13	11	19	00	11	19	10.55	—10.55	—10.95	14	46	—1.53	14	06	—1.50	3
"	13	18	13	00	18	13	10.57	—10.57	—11.01	22	27	— .85	21	48	— .90	4
"	14	2	41	00	2	41	10.83	—10.83	—11.13	6	41	— .36	6	06	— .40	5
"	14	10	41	00	10	41	11.24	—11.24	—11.36	14	59	—1.09	14	16	—1.03	6
													22	32	— .48	7
"	14	19	17	00	19	17	11.42	—11.42	11.81	3	22	— .09	6	11	— .70	8
"	15	11	27	00	11	27	12.44	—12.44	—12.50	15	35	2.73	14	38	—2.52	9
"	15	19	43	00	19	43	12.05	—12.05	—12.99	23	56	—2.02	23	09	—2.09	10
"	16	4	09	00	4	09	11.90	—11.90	—12.61	7	59	—1.53	7	24	1.57	11
"	16	11	49	00	11	49	11.92	—11.92	—12.41	15	46	—3.12	15	15	—3.02	12
"	16	19	44	00	19	44	11.42	11.42	—12.45							

Doubtless Bay, N.Z.—Here, as in Suva the pendulum observations were made after completing the longitude determinations.

In the store-house of the Pacific Cable Company, was built a solid concrete-brick pier of the same dimensions as the one at Suva, and on it the apparatus was mounted, the footplates of the air-chamber being as usual cemented with plaster of paris to the top of the pier.

The observations with pendulum No. 2 were unsatisfactory and were discarded.

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Latitude $-34^{\circ} 59' 20''$, longitude $173^{\circ} 29' E$, elevation 7^m. The time observations were made similar to the ones at Suva; and the chronometers were also kept in the artificial-line cabinet and the comparisons made thrice daily on the chronograph.

Although in the interval of five months and transport from Fiji, the Bond chronometer had materially changed its daily rate, from $-1^s.60$ to $-10^s.42$, yet the daily rate at either place was fairly uniform, so that the rate of the Dent is referred to the Bond by means of the differential rates from chronograph comparisons.

Date.	Dent. 48419.			Bond. 516.			D.—B.	Diff. corrected for Bond rate.	Dent.		Middle time of pend. swing.	Dent daily rate during swing.	Swing No.
	h.	m.	s.	h.	m.	s.			At time.	Daily rate.			
1903.	h.	m.	s.	h.	m.	s.	s.	s.	h.	m.	s.		
Dec. 16	2	56	00	2	56	04.00					
" 21.	1	09	00	1	09	57.24	—57.24	13	05	—04	23 49 —3.48	1
" 22.	1	01	00	1	02	07.56	—67.56	—67.60	3	05	—4.53		
" 22.	5	09	00	5	10	08.57	—68.57	—69.35			8 07 —2.83		2
" 23.	21	44	00	21	45	15.06	—75.06	—75.77	13	26	—1.03	16 37 —1.59	3
" 23.	1	15	00	1	16	16.18	—76.18	76.59	23	30	—2.80		
" 24.	15	36	00	15	37	21.64	—81.64	—82.41			0 29 —2.63		4
" 24.	15	36	00	15	37	21.64	—81.64	—82.41	8	26	—1.29	8 36 —1.22	5
" 24.	23	57	00	23	58	25.92	—85.92	—85.26	19	46	+1.90	16 42 +1.04	6

Northwest River.—This station was primarily occupied to observe the total eclipse of the sun on August 30, 1905. Professor L. B. Stewart, of the School of Practical Science, Toronto, was entrusted with the selection of the site and the determination of time and geographical positions, besides making gravity observations with the half-seconds pendulum apparatus used at the preceding stations. Professor Stewart writes: ‘A small wooden structure 8 feet by 10 feet was speedily erected to serve as an observatory, having a transit slit in the roof closed by a trap door. Two concrete piers were also built, one in the middle of the building to serve as a support for the (10-inch) theodolite, and the other, a lower one, in the northeast corner for the pendulum receiver. * * * * Latitude was determined by observing the meridian altitude of stars’ north and south. Moon culminations were observed for longitude. Time was determined by observing the transit of stars over the five threads of the theodolite, 12^s.7 intervals, clamp east and clamp west, recording the same on a chronograph. Dent sidereal chronometer No. 49,950 was used, as also for the pendulum observations. There were besides in the outfit sidereal chronometers Bond 516 and Dent 2,071, also mean time Bond 511.

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The following are the results of the time observations for Dent 49,950 as given by Prof. Stewart.

Date.	Time.	Correction.	Interval.	Rate.
1905.	h. m. s.	s.	d.	s.
August 18.....	20 00 00	+25.74		
" 21.....	19 00 00	+29.60	2.9583	+1.30
" 26.....	18 10 10	+39.35	4.9653	+1.96
" 31.....	3 40 00	+53.59	4.3958	+3.24

The pendulum observations were reduced as the preceding, to infinitely small arc; temperature 15° C; and pressure 60^{mm}. There was no observation for flexure, and hence no correction. The apparatus having been mounted on a solid concrete pier the flexure correction would be confined to the units of the seventh decimal of a second in the period.

For each pendulum, direct and reversed, twelve coincidences were noted, giving thereby between the 1st and 11th, 2nd and 12th the interval for ten coincidences, from which the uncorrected period is obtained.

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Station—Montreal, Physics Building, McGill University.
PENDULUM OBSERVATIONS AND REDUCTIONS.
Observer—OTTO KLOTZ.

Date.	Swing Number.	Pendulum	Position.	Knife-edge.	COINCIDENCE INTERVAL.		ARC.	PERIOD UNCORRECTED.				CORRECTIONS (7TH DECIMAL PLACE).				PERIOD CORRECTED.					
					CHRONOMETER.	Standard Observatory sidereal clock.		Initial.	Final.	Temperature.	Pressure.	CHRONOMETER.	Observatory clock.	Arc.	Temperature.	Pressure.	RATE	Observatory clock.	Flexure.	CHRONOMETER.	Observatory clock.
1902.																					
Sept. 10..	1	1	D	I	s.	188.282	56'	19'	17°.73	47.5	s.	.5013313	—9	—114	+13	—230		—5	s.	.5012968	
" 11..	2	1	R	I	188.125	64'	24'	24'	17°.49	49.5	.5013324		—12	—104	+11	—229		—5	.5012985		
" 11..	3	2	D	II	169.828	67'	24'	24'	17°.63	49.2	.5014764		—13	—110	+11	—228		—5	.5014419		
" 11..	4	2	R	II	170.208	58'	20'	20'	17°.63	51.8	.5014731		—9	—110	+8	—228		—5	.5014387		
" 12..	5	3	D	II	173.145	58'	21'	21'	17°.63	48.8	.5014481		—10	—110	+11	—227		—5	.5014140		
" 12..	6	3	R	II	173.120	55'	19'	19'	17°.70	48.1	.5014483		—8	—113	+12	—226		—5	.5014143		

Mean pendulum..... .5013840
Mean pendulum at Washington..... .5015221

PENDULUM OBSERVATIONS AND REDUCTIONS.

St. Louis University School of Practical Science.

However, (The Körz,

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PENDULUM OBSERVATIONS AND REDUCTIONS.

Station Sinaloa, F.M.

Observer OTTO KLOTZ.

Date.	Swing Number.	Pendulum.	Position.	Knife-edge.	COINCIDENCE INTERVAL.		Arc.	PERIOD UNCORRECTED.		CORRECTIONS (7TH DECIMAL PLACE).				PERIOD CORRECTED.		Mean.		
					CHRONOMETER.			Time. (Green No. 116).	CHRONOMETER.		RATE.		CHRONOMETER.					
					No. 48419 Dent.	No. 516 Bond.			No. 48419 Dent.	No. 516 Bond.	Temperature.	Pressure.	Dent.	Bond.	Elevation.		No. 48419 Dent.	No. 516 Bond.
1903.					s.	s.	(mm.	s.	s.					s.	s.		
July 12	1	D	I	I	135.855	135.988	20.6	43.25	5018470	5018452	-15	234	+20	-111	-93	-6	5018124	5018124
" 13	9	D	I	I	134.85	135.29	22.275	52.6	5018608	5018547	-11	306	+11	-146	93	6	5018150	5018142
" 13	2	R	I	I	135.86	135.643	21.65	47.75	5018469	5018499	-11	279	+16	-71	-93	-6	5018118	5018126
" 13	10	R	I	I	135.63	135.811	21.15	51.0	501854	5018476	-15	258	+13	121	-93	-6	5018114	5018117
" 13	3	D	I	I	125.283	125.216	23.75	43.0	5020035	5020046	-15	368	+21	-87	93	6	5019580	5019585
" 14	7	D	I	I	126.520	126.110	21.625	48.25	5019838	5019903	-11	278	+15	-28	-93	-6	5019530	5019530
" 13	4	R	I	I	126.067	125.865	21.975	44.75	5019910	5019941	-11	293	+19	52	93	6	5019567	5019557
" 15	8	R	I	I	126.624	126.010	21.7	48.25	5019822	5019919	-13	282	+15	-41	-93	-6	5019495	5019540
" 14	5	D	I	I	128.08	127.60	22.05	51.75	5019596	5019670	-9	296	+12	23	-93	-6	5019274	5019276
" 16	11	P	I	I	127.513	127.50	21.3	50.5	5019683	5019685	-15	265	+13	-91	93	6	5019319	5019319
" 14	6	R	I	I	127.137	126.938	22.95	51.0	5019741	5019772	-12	334	+10	-60	-93	-6	5019339	5019338
" 16	12	R	I	I	126.96	127.486	21.3	49.5	5019769	5019687	-16	265	+14	-175	93	-6	5019321	5019321

Mean pendulum 5018996
Mean pendulum at Washington 5015221

PENDULUM OBSERVATIONS AND REDUCTIONS.

Station—Doubtless Bay, N.Z.

Observer OTTO KLOTZ.

Date.	Swing Number.	Pendulum.	Position.	Knife-edge.	COINCIDENCE INTERVAL.		ARC.	Thermometer temperature. Green No. 118.	Pressure.	CHRONOMETER.		PERIOD UNCORRECTED.		CORRECTIONS (7TH DECIMAL PLACE).				PERIOD CORRECTED.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
					No. 516 Bond.	No. 48419 Dent.				CHRONOMETER.	Initial.	Final.	Temperature.	Pressure.	RATE.		CHRONOMETER.	Mean.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																
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1903.	1	1	D	I	s.	s.																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																												

Mean pendulum, I, III..... 5015631
Mean pendulum, I, III, at Washington..... 5014933

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PENDULUM OBSERVATIONS AND REDUCTIONS.

Station Northwest River.

Observer -L. B. STEWART.

Date.	Swing Number.	Pendulum. Position.	Knife-edge.	COINCIDENCE INTERVAL.		ARC.		PERIOD UNCORRECTED.			CORRECTIONS (7th DECIMAL PLACE).			PERIOD CORRECTED.	
				Chronometer.	No. 49950 Dent.	Initial.	Final.	Temperature.	Pressure.	Chronometer.	No. 49950 Dent.	Rate.	Dent.	Chronometer.	No. 49950 Dent.
1905.															
Aug. 29.	1	D	...	s.	231.0	93'.8	77'.6	9'.3	92.5	s.	5010846	-48	+238	-30	+174
" 29.	1	R	...	s.	231.8	81'.0	70'.6	9'.6	99.0	s.	5010808	-38	+226	-36	+174
" 29.	3	D	...	s.	203.2	86'.8	75'.2	9'.4	97.0	s.	5012333	-43	+234	-34	+174
" 30.	4	R	...	s.	203.5	89'.1	78'.7	9'.7	98.0	s.	5012315	-47	+222	-35	+177
" 30.	5	D	...	s.	205.4	85'.7	77'.6	14'.23	99.2	s.	5012201	-44	+32	-31	+180
" 30.	6	R	...	s.	205.3	86'.8	77'.6	14'.90	98.2	s.	5012207	-45	+4	-33	+180

Mean pendulum at Washington 5012043
Mean pendulum at Washington 5015221

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In the following abstract the computed g_c has been obtained from Helmert's formula (5).

$$g_c = 978.046 (1 + .005302 \sin^2 \phi - .000007 \sin^2 2 \phi)$$

The observed g_o is obtained from the observed periods compared with that at Washington, $g_o = \frac{P_w^2 g_w}{P_o^2}$, g_w being taken at 980.098 dynes.

Station.	Difference from Mean Pendulum in 7th decimal.		
	No. 1.	No. 2.	No. 3.
Washington, 1.....	+880	—570	—309
2.....	+888	—578	—308
Ottawa, 1.....	+879	—578	—321
2.....	+883	—576	—289
Montreal, 1.....	+864	—563	—301
Toronto, 1.....	+872	—578	—292
Suva, 1.....	+873	—575	—311
2.....	+865	—528	—324
Northwest River, 1.....	+886	—605	—281
Mean.....	+877	—572	—304

Station.	Latitude.	Longitude.	Elevation.	Computed g_c .	Observed g_o .	$g_o - g_c$
Washington.....	38° 53' 13"	77° 01'	10 ^m	980.683	980.098	+ .015
Ottawa.....	45 25 23	75 42	73	980.670	980.593	— .077
Montreal.....	45 39 22	73 34	40	980.678	980.658	— .040
Toronto.....	43 39 35	79 24	106	980.511	980.433	— .078
Suva.....	—18 08 45	178 26 E	2	978.547	978.624	+ .077
Doubtless Bay.....	—34 59 20	173 29 E	7	979.745	979.825	+ .080
Northwest River.....	53 31 31	60 10	2	981.395	981.341	— .052

SEISMOLOGY.

The last quarter of a century stands out pre-eminently as the most marked in seismic disturbances of which we have any historic record. It began with that cataclysmic explosion of Krakatoa in 1883, noted for the red sun-sets that followed for the next two years, due to the suspended dust in the upper regions of the atmosphere. Of the more important disturbances we may mention those of Ischia near Naples; Charleston, South Carolina; Tarawera, New Zealand; the calamitous Mino-Owari earthquake in 1891, in Japan, when more than 20,000 lives were lost; Saint Pierre in the West Indies; Formosa; Vesuvius; the Alaska upheavel in 1899; the great Indian earthquake at Kangra in 1905 which cost close on 20,000 lives; and the recent destructive quakes at San Francisco, Valparaiso, Kingston (Jamaica) and Chilpancingo (Mexico). It is estimated that the total loss of life from these disturbances is at least 150,000.

The great Mino-Owari earthquake was the immediate reason for the birth of the Earthquake Investigation Committee, which has since then contributed so much by its 'Publications' to the study of earthquakes. Five years later the British Association for the Advancement of Science, through the indefatigable labours of the eminent seismologist, Professor John Milne, formed a Seismological committee; and now through the destructive earthquake in San Francisco the American Association for

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the Advancement of Science has formed a committee on Seismology of 15 members, of whom the writer is one.

Some of the objects in view in forming the Committee on Seismology in America are as follows:—

1. To be available for, and to initiate counsel in connection with legislation which provides for investigations of earthquakes or the means of investigating their dangers.

2. To bring into harmony all American and Canadian institutions doing seismological work, and to guard against unnecessary duplication of studies.

3. To organize, if thought best, a correlated system of earthquake stations, which should include the outlying possessions and protectorates.

4. To advise regarding the best type or types of seismometers for the correlated stations.

5. To disseminate information regarding construction suited to earthquake districts.

6. To collect data regarding the light as well as the heavy shocks, and to put the results upon record.

7. To start investigations upon large problems of seismology.

8. To advise with some weight of authority where catastrophic earthquakes have wrought national calamity.

Since its appointment the Committee has held one meeting and amongst the important resolutions may be mentioned, 'that the time has come to ask the support of the federal government for seismological work.'

The scope of the committee will in course of time broaden and many questions not only of scientific but of immediate practical importance will be taken up. It will undoubtedly fall to the lot of this newer science of seismology to answer unsolved questions in astronomy, geodesy, geology and meteorology pertaining to the physics of the earth. The field is large, but most promising for cultivation.

The most important organization for the study of seismology is the 'International Seismological Association,' which will hold its second conference next September at the Hague.

Practically every civilized country in the world has joined the association and appointed a representative for the quadrennial meetings, showing that the study of seismology is not confined to countries that are notably subject to more or less destructive earthquakes, but the subject is recognized as one of grave importance and for its full development and evaluation, co-operation is necessary. The more widely stations are distributed over the world and records obtained, the more readily will the true nature of earthquake waves with the accompanying phenomena producing them, be determined. Although the study is that of earthquakes, yet it involves and embraces much besides, which is of the greatest interest to all countries, irrespective of their susceptibility to quakes or not.

In the pursuit of knowledge for the amelioration of mankind, science knows neither political nor geographical boundaries.

The seismograph at Ottawa has been in continuous operation, with slight interruptions due to repair of the clock work of the registering apparatus, and the study of the instrument and seismograms have received considerable attention.

An investigation has been begun with reference to the behaviour of the horizontal pendulum, when compared with the fluctuation of pressure on the earth's surface due to barometric changes. This is a very involved problem, for results obtained for a given change of atmospheric pressure at one place does not necessarily produce the same effect upon a similarly sensitive pendulum at another place, for the reason that the coefficient of elasticity of the immediate earth's crust

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may be not the same for the two places. The series is not as yet sufficiently extended to enable definite conclusions to be drawn.

The method of investigation may be briefly outlined. Every morning during the interval of taking off and putting on a fresh sheet the position of the images of the two pendulum mirrors is taken by means of a millimetre scale, read to tenths, and referred to the south solid brick wall of the seismograph room, and immediately opposite the slit of the registering apparatus. That is, the position and change of the zero point are daily measured. This is in the first place, on the supposition that the change is due to the tilting of the pier. The angular movement of the wall from which the measurements are made is subject of course to the same movement as the pier from bending of the earth's crust, and therefore shows no relative displacement, whatever the bending. However as the image is magnified 120 times, and its movement is confined to individual millimetres, the movement of the wall in comparison with it is evanescent.

These movements in linear measure are then converted into arc by the formula

$$\beta = \frac{\pi^2 D}{2 n g t^2 \sin 1''}$$

where t = period of pendulum (one-half complete vibration).

g = gravity.

D = linear displacement.

$2 n$ = magnification.

Assigning values in the above we find $\beta = ".2122 D$.

Estimating measurements to tenths of a millimetre, it will be seen then an inclination of about one-fiftieth of a second of arc can be read, which is equivalent to a grade of one foot in 2,000 miles.

We obtain therefore the apparent tilting in the planes normal respectively to the two pendulums, i.e., in the meridian and in the prime vertical, and by composition the resultant of the two, that is, the actual direction as well as the magnitude of tilting.

From the daily weather maps issued by the Meteorological service, Toronto, are obtained the isobars, or lines of equal barometric pressure, drawn at intervals of one-tenth inch difference of pressure. For our immediate purpose the two isobars between which Ottawa falls, are examined in order to obtain the pressure gradient and direction, the latter being normal to the two isobars. The closer the isobars are together, the steeper will be the gradient and the greater will be the apparent tilting of the pier, being in the direction of the normal to the two isobars which passes through Ottawa. The pier will tend to incline from an area of 'low' barometer to the area of 'high' barometer. It may be remarked that a difference of pressure of one millimetre is equivalent to a load of 13,600,000 kg. on a square kilometre, and that a change of 15 millimetres within the 24 hours has on more than one occasion been experienced here. This latter change of pressure is equivalent to 600,000 tons per square mile. By the investigations of Professor (now Sir) G. H. Darwin* 'On variations in the vertical due to elasticity of the earth's surface,' some very interesting results were obtained with reference to the amount of distortion to which the upper strata of the earth's mass are subjected, when a wave of barometric depression or elevation passes over the surface. He showed a very remarkable relation to exist between the slope of the surface of an elastic horizontal plane and the deflection of the plumb-line caused by the direct attraction of the weight producing the slope, and this relation is expressed by the ratio $\frac{v}{g}$ to $\frac{1}{3} a \delta$ in which g is gravity, v is modulus of rigidity, a is the earth's radius, and δ is the earth's mean density. 'This ratio is

* Philosophical Magazine, vol. 14, Fifth series, p. 409.

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independent of the wave-length of the undulating surface, of the position of the origin, and of the azimuth in the plane of the line normal to the ridges and valleys. Therefore the relation is true of any combination whatever of harmonic undulations; and as any inequalities may be built up of harmonic undulations, it is generally true of inequalities of any shape whatever.'

If we take the barometric range at five centimetres, about two inches, we have a difference of pressure on every square yard of nearly 1,300 pounds, or over half a ton. Darwin in order to obtain a numerical value for his deduced equations for slope and attraction assumed the rocks as one-quarter as stiff again as the stiffest glass, that is, the rigidity of glass in gravitation units as 3×10^8 , for the range of pressure 2.5 centimetres on each side of the mean. With this assumption he finds the minimum apparent deflection of the plumb-line, consequent on the elastic compression of the earth to amount to ".0117, and this is augmented to ".0146 when the true deflection due to the attraction of the air is added. So that the whole range of the deflection between high pressure and low pressure would be ".0292; and furthermore, that the ground is nine centimetres higher under the barometric depression than under the elevation, that is when the barometer is very high we are at least three inches nearer the earth's centre than when it is very low.

Darwin concludes his article, 'If barometric pressure, tidal pressure, and the direct action of the sun and moon combined together to make apparent slope in one direction, then, at an observatory remote from the sea-shore, that slope might perhaps amount to a quarter of a second of arc. . . . I venture to predict that at some future time practical astronomers will no longer be content to eliminate variations of level merely by taking means of results, but will regard corrections derived from a special instrument as necessary to each astronomical observation.'

It has been shown that with the present adjustment of the instrument, and it is near its limit of sensitiveness the smallest reading that can be made with any degree of certainty is ".0212 which is but slightly less than Darwin's maximum value.

As Darwin's assumed values can suffer but little change, it will be seen that the investigation presents great difficulty for showing clearly the relationship between barometric gradients and oscillations of the pendulum zero, for other factors are involved too. However, the series of observations when sufficiently extended will undoubtedly disclose the cause of the fluctuation or deviation from the vertical.

In tabulating the occurrence of earthquakes during the various months of the year it is found that there is a marked predominance for the colder season of the year. Long observations and investigations have established a relationship between earthquakes and atmospheric conditions, not, however, in the sense in which it was held by the ancients, that is, that earthquakes produced meteorological phenomena, such as storm, hail, rain, cold, heat—in fact the whole gamut of weather. The relationship is, however, just the reverse of that attributed by the old philosophers. The predominance that has been found for the colder season is not attributable, however, to atmospheric temperature, this is a mere co-incidence, but during that period of the year the barometric gradients are in general greater than at other times, and thereby in a secondary manner act as 'the last straw to break the camel's back.' That is, as stresses are ever existant in the crust of the earth, increasing towards the limit of elasticity or resistance, the differential loading of the surface of the earth into those areas subject to quakes and which are intersected by regional geological faults or planes of weakness, may set the fuse to bring about the downfall, or adjustment to temporary equilibrium,—an earthquake takes place.

Although the stress produced by atmospheric loading is very small compared with the tectonic stresses set up within the crust by other causes, it is evident that if a steep barometric gradient passes over a line or plane of weakness in a part of the crust under great tension or stress approaching rupture, that the rupture will be accelerated by such atmospheric conditions.

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In a great earthquake if the surface shows no marked manifestations of rifts, of torn ground, it seems to indicate that the hypocentre was very deep beneath the surface. The various influences that may be considered as affecting the vertical or the slow movement of the pendulum are atmospheric pressure; atmospheric attraction; deformation of the surface by lunar and solar attractions; direct effect of lunar and solar attractions; deformation of the surface by solar radiation and by constant tectonic movements or strains; change of temperature of the apparatus; and irregular strains in the pier itself. As will be seen, some of these influences are periodic. The maximum horizontal force due to the moon deflects the vertical $''\cdot0174$, and that due to the sun $''\cdot0074$. The effective force varies with the zenith distance and azimuth of the attracting body, and as the two pendulums are mounted respectively in the meridian and prime vertical each requires its own reduction for eliminating the effect of the disturbing influences. Ehlert records* a change of level for the horizontal pendulum of $27''$ during three and one-half days, this being an abnormal fluctuation. For the month May 10—June 10, 1907, the horizontal pendulum here showed a mean daily change of the vertical of $0''\cdot36$ and $0''\cdot34$, the former for the E.—W. pendulum and the latter for the N.—S. one. Ehlert found from observations at Strassburg that the mean diurnal fluctuation of the ground due to the heat of the sun expanding the surface of the earth, the hemisphere turned towards the sun suffering ellipsoidal distortion amounted to $''\cdot112$, being of course greater in summer ($''\cdot208$) with clearer sky and longer sunshine than in winter ($''\cdot016$). As the pendulum pier at Strassburg is 5 metres beneath the surface, A. Schmidt, quoted by Sieberg,† has raised doubts about the above values, maintaining that the daily variation of temperature is confined to about 1 metre depth of soil, and certainly does not extend to a depth of 5 metres. The pendulum pier here is at a depth of 3 metres beneath the surface of the soil. E. V. Reubur-Paschwitz noted at Wilhelmshaven‡ a marked movement of the pendulum synchronizing with the varying height of the barometer. It corresponded to a deviation of the vertical of $''\cdot29$ for a change of 1 millimetre in the height of the barometer. This extraordinary fluctuation was attributed to the marshy spongy nature of the environment.

The later and extended observations of v. Paschwitz at Strassburg make the deflection of the effect of barometric pressure on the pendulums there uncertain. It undoubtedly exists, it is a question of measurement and subsequent elimination of the other disturbing influences.

It is found that about 1 p.m. throughout the year the periodic deviation of the pendulum is practically zero,§ so that readings at this time are best adapted for the determination of the irregular movements. During a period of ten and a half months the Strassburg || pendulum slowly moved from $+12''$ to $-130''$ or a total of $142''$. During the same period the change of nadir was only $25''$. The curve for the latter followed closely that of the temperature of the cellar while the curve for the pendulum does not synchronize with that for temperature. The relative humidity of the basement does not appear to influence the pendulum.

In the accompanying table I, is shown the daily movement of each pendulum and the deduced apparent tilting of the pier in magnitude and direction since the beginning of the investigation. Plates 1 and 2 show graphically this movement. Plate 3 shows the relative humidity, which is markedly higher during that part of the year when the basement and building are not artificially heated.

* Beiträge Zur Geophysik IV. p. 70.

† Beiträge Zur Geophysik II., p. 334.

‡ Handbuch der Erdbebenkunde p. 197.

§ Beiträge Zur Geophysik II. p. 329.

|| Ibid p. 318.

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TABLE I.

DAILY CHANGE OF PENDULUM ZERO.

S=+ N=—
W=+ E=—

CHANGE 3					CHANGE 3								
DATE.		N—S component.		E—W component.	Apparent tilting of pier	DATE.		N—S component.		E—W component.	Apparent. tilting of pier		
1907.					1907.								
Mar.	7-8..	-	·21	-	·21	SW	·30	May	3-4	-	·30	NE	·47
"	8-9..	+	·53	+	·32	SW	·62	"	4-5	-	·11	NE	·15
"	9-11..	+	·53	-	·11	SW	·54	"	5-6..	-	·32	NW	·60
"	11-13..	-	1·06	+	·21	NW	1·08	"	6-7..	-	·30	NE	·35
"	13-14..	-	·74	-	·64	NW	·98	"	7-8..	-	·49	SE	·54
"	14-15..	-	·95	+	·85	NW	1·27	"	8-9..	-	·57	NE	·58
"	15-16..	-	·21	-	·74	NW	·77	"	9-10..	-	·81	NE	·89
"	16-18..	-	3·92	-	·00	N	3·92	"	10-11..	-	·85	SW	·98
"	18-19..	-	·39	+	1·70	NW	1·74	"	11-12	-	·25	NW	·28
"	19-20..	-	·04	+	·48	NW	·48	"	12-13..	-	·51	NW	·75
"	20-21..	-	·53	-	1·59	NE	1·67	"	13-14..	-	·46	SE	1·07
"	21-22..	-	·53	-	2·44	NW	2·49	"	14-15..	-	·06	SE	·36
"	22-23	-	·48	+	·74	NW	·88	"	15-16..	-	·34	SE	·69
"	23-25..	-	1·64	-	1·59	NW	2·28	"	16-17..	-	·51	NE	·63
"	25-26..	-	1·33	+	·57	NW	1·45	"	17-18..	-	·68	NE	·71
"	26-27..	-	1·35	+	·80	NW	1·57	"	18-19..	-	·04	NE	·09
"	27-28..	-	·23	-	·57	NE	·61	"	19-20..	-	·15	NE	·16
"	28-29..	-	·42	+	1·00	NW	1·08	"	20-21..	-	1·21	NE	1·25
"	29-30..	-	·87	-	·17	NW	·88	"	21-22..	-	·91	SE	·91
"	30-31..	-	·08	+	1·19	NW	1·19	"	22-23..	+	·11	SE	·22
"	31-1..	-	·21	+	·55	NW	·59	"	23-24..	-	·06	NE	·29
April	1-2..	-	·99	+	·66	NW	1·19	"	24-25	-	·57	N	·57
"	2-3..	-	·32	+	·57	NW	·65	"	25-26	-	·28	N	·28
"	3-4..	-	·21	+	·13	NW	·25	"	26-27..	-	·30	NE	·50
"	4-5..	+	·34	+	·34	SW	·48	"	27-28..	-	·25	SE	1·78
"	5-6..	-	·64	+	·68	SW	·93	"	28-29..	-	·44	NE	·45
"	6-7..	+	·21	+	·95	SW	·97	"	29-30..	+	·62	SE	·64
"	7-8..	-	·89	-	·15	NE	·90	"	30-31..	-	·34	SE	·49
"	8-9..	+	·29	+	·34	SW	·45	"	31-1	-	·15	NE	·43
"	9-10..	-	1·14	-	1·04	NE	1·54	June	1-2..	-	·32	SE	·44
"	10-11..	-	·00	+	·19	W	·19	"	2-3..	-	·40	N	·40
"	11-12..	+	1·02	+	1·76	SW	2·03	"	3-4..	-	·36	NE	·41
"	12-13	-	·02	+	·44	NW	·44	"	4-5..	-	·08	NE	·22
"	13-14..	-	·68	+	·06	NW	·68	"	5-6..	-	·57	NE	1·05
"	14-15..	-	·60	+	·42	NW	·73	"	6-7..	-	·62	NE	·62
"	15-16..	-	·87	+	·25	NW	·90	"	7-8..	-	·06	NE	·20
"	16-17..	-	·25	+	·17	NW	·30	"	8-9..	-	·08	NE	·14
"	17-18..	-	·51	+	·28	NW	·58	"	9-10..	-	·49	NE	·72
"	18-19..	-	·44	+	·83	NW	·94	"	10-11..	+	·23	SE	·23
"	19-20	-	·60	+	·64	NW	·88	"	11-12	-	·17	NE	·53
"	20-21							"	12-13..	+	·13	SW	·33
"	21-22							"	13-14..	-	·19	SE	·73
"	22-23							"	14-15..	+	·23	SE	·45
"	23-24..	-	·04	-	·11	NE	·12	"	15-16..	-	·02	NE	·62
"	24-25	-	·15	-	·23	NE	·27	"	16-17	-	·55	NE	·57
"	25-26..	-	·15	-	·36	NE	·39	"	17-18..	-	·55	NE	1·75
"	26-27..	-	·62	+	·06	NW	·62	"	18-19..	-	·17	SW	1·39
"	27-28..	-	·28	-	·34	NE	·44	"	19-20..	-	·25	NW	·39
"	28-29..	+	·34	-	·36	SE	·49	"	20-21..	-	·34	SE	·34
"	29-30..	-	·19	-	·53	NE	·56	"	21-22..	-	·06	SW	·10
"	30-2..	-	·19	+	·15	SW	·24	"	22-23	-	·32	NE	·77
May	2-3..	-	·08	-	·25	NW	·26	"	23-24..	-	·55	NE	·56

The atmospheric pressure over a portion of the surface of the earth may be represented as a meniscus resting on the earth and whose dimensions are determined by the isobars. This meniscus or cap with ever varying form slides over the earth like a heavy weight, deforming the surface. If we imagine the surface studded with vertical rods, these, as the meniscus moves along, will always be inclined towards the summit of the cap. For our geographical position the path of the meniscus is from

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the northwest and thence down the valley of the St. Lawrence, so that we must expect to find the prepondering swaying at Ottawa to be in our east-west direction, and manifested more by the north-south pendulum than by the one in the prime vertical.

Beside these slow movements of the pendulums that have been considered there are others, spoken of as microseismic to distinguish from the macroseismic or those from actual earthquakes. It appears that pulsations are set up in the earth, just how is not yet clearly established, and these are communicated to the pendulum. Were it possible to disentangle the movement of the pendulum itself, the pulsations of the earth would be much simplified. The pendulum for no length of time remains a steady point, but instead is set oscillating by the pulsations. The period of the pulsations may be very long, 200^s, but generally is very much less. The pendulum is the more readily set in vibration when the period of the oscillation corresponds to or is a multiple of the period of the pendulum. Professor Milne concludes from his observations that the oscillations may be and are in certain cases produced by steep barometric gradients and by winds. A very satisfactory proof of this is our barograph record and the seismogram of June 18, last. At 3.50 p.m. on that day the barograph rose abruptly from 755^{mm} to 756.2^{mm} only to drop again to its former reading, repeating this zigzag movement during fully four hours, recording five maxima, and five minima with a difference of about a millimetre in that interval. It may be remarked that a high 'gusty' wind was noted that afternoon before the records of the barograph or seismograph were known. Apparently at the same moment, as closely as one can read the time scale on the barograms, both pendulums—the E—W one particularly—began recording earth tremors in pulsations. The displacement of the pendulum as shown in the seismogram was fully a millimetre (magnification 120), but the line traced was very irregular, and was in keeping with the 'puffy,' 'gusty' wind and the jerky behaviour of the barograph. The bottom of the pier, resting on boulder clay and supporting the seismograph, is very nearly 3 metres beneath the outside surface, so that the wind was very effective at this depth. Dependent upon the nature of the soil, it has been found for sandy ground that even at a depth of 25^m, the wind-effect was reduced only one-half of that for the immediate surface. In the case cited above for June 18, the seismogram was undoubtedly a direct and almost instantaneous response to the fitful barometric gradient and its concomitant wind, and the pendulums were not set swinging, responding to vibrations set up in the earth's crust by friction of a steady wind as is maintained by some as necessary. Oscillations produced by this latter means would undoubtedly show rhythm which is absent in our case.

Then we have oscillations set up by internal stresses. They come and go quite independent of atmospheric conditions or cosmic influences. The pulsations and records are generally the most uniform seismograms obtained. They may last for a short time, or for hours or days. Earth tremors or oscillatory pulsations as interpreted on our seismograms appear to be fitful impulses, which set the pendulums swinging *uniformly* for a minute or so, with an amplitude of less than a millimetre (120 magnification), then the oscillation dies down, interference is noticed changing the uniformly serrated record, to be followed again by half a dozen or more uniform swings. It will be hence noted that in these earth pulsations the pendulum does not act as a steady point at all. When we come to look at the record for a macroseism or earthquake, the apparent behaviour of the pendulum is different. Beginning with the 'preliminary tremor' the pendulum bob apparently does not respond to the movements of the earth but acts as a 'steady point' or fairly so; when, however, we come to the principal portion with the more or less violent oscillations we find practically the period to be that of the pendulum with occasional or frequent interference phenomena, by which the pendulum is momentarily stopped and the direction of swing at the time reversed.

It must be borne in mind that the period of the pendulum is about 5.7^s, and as that value is near that of predominating periods of earthquake waves, there is difficulty in separating or distinguishing the two on the seismograms. With the

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earth pulsations or microseisms we have a more or less continuous adjustment towards equilibrium, until the earth settles down for a time to a quiescent state. Earth tremors may be looked upon, however, as a manifestation of the normal condition of the earth's crust, which must necessarily be subject to strains and stresses. It is not to be expected that an instrument can be devised that will faithfully record only the movements of the earth particles and not the movements of the apparatus also.

On account of the small movement in the first preliminary tremors, the Bosch photographic seismograph, with light steady mass and high magnification, is well adapted to record them.

The hieroglyphics drawn by the seismograph have not yet been fully read; they await the finding of their Rosetta stone.

The earth tremors are frequently the precursors for hours of an earthquake, *e.g.*, that of San Francisco and of Mexico (April 15, 1907), at other times no disturbance is noticed until the earthquake shock arrives.

From a seismogram we may (not always) obtain the movement of the earth-particle, that is, its amplitude a , (half-range) and period t , then on the assumption of simple harmonic motion, we have for the maximum velocity $v = \frac{2\pi a}{t}$, and if f = the maximum acceleration per second per second $f = \frac{v^2}{a} = \frac{4\pi^2 a}{t^2}$. The maximum acceleration is proportional directly to the amplitude and inversely to the square of the period. The amplitude is measured in millimetres and the time in seconds. For earth tremors a is a very small fraction of a millimetre and t generally more than 5 seconds, so that the value of v is less than a hundredth of a millimetre per second, for $a = .01^{\text{mm}}$ $t = 10^{\text{s}}$ and the maximum acceleration is correspondingly small, a thoroughly evanescent quantity as far as the effect on objects is concerned. Even the records of severe distant earthquakes show a very small acceleration, say a millimetre per second per second.

If V is the speed of propagation, and λ the length of the wave, then $\lambda = Vt$, t being the time of period of oscillation of the earth particle, which is the time interval also from crest to crest of the wave.

V the velocity of propagation is dependent upon the medium transmitting the oscillations—its co-efficient of elasticity and of rigidity.

The expression for V is, $V = \sqrt{\frac{E}{\rho}}$, where E is the elasticity of the medium and ρ its density. This applies to a longitudinal wave in a homogeneous medium.

To give an example for the latter, Adams* gives the value of E for plate-glass as 7.24×10^{11} , and for diabase 9.49×10^{11} in C. G. S. units. Taking this latter value and the mean density as 2.57 we find $V = 6.08^{\text{km}}$ per second or 365^{km} per minute which is about two-thirds of the velocity of the first preliminary tremors of the San Francisco earthquake of last year (April 18).

The elasticity required to give the observed velocity would hence be 22×10^{11} , which is the modulus for steel. If the density were increased we would obtain a still greater value for E .

Professor E. Oddone has deduced from some large quakes (Balkans, San Francisco, Valpariso) that the time of diametral passage through the earth of the first preliminary tremors, longitudinal waves, is 16.5^{m} . Then taking the mean density of the earth as 5.6 he obtains for E 85×10^{11} which indicates a rigidity 'quatre fois plus rigide que le fer, et sept fois circa plus rigide, que les plus rigide roches archaïques.'† We see therefore that the resistance to compression of the matter of the earth is far in excess of that of any known material of the surface of the earth. Although the direction of the path of the various vibrations is not accu-

* Elastic Constants of Rocks, p. 69.

† Quelques constantes sismiques trouvées par les macrosismes, p. 25

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rately known, *i.e.*, whether the pulsations travel along chords, circular arcs or other curves, data necessary for getting accurate velocity determinations, nevertheless seismology has definitely settled the question of the nature of the interior of the earth as to solidity or otherwise. Not only has solidity been established, but also a fair measure of its rigidity.

Oddone draws attention to a curious coincidence between the time of propagation of a longitudinal wave along a diameter of the earth and the time of a light wave to cross the earth's orbit, each being approximately 16.7^{min} .

Oddone rounds off the figure to $17^{\text{m}} \pm 1^{\text{m}}$, and takes it as the constant for longitudinal waves through the earth. He designates it by the letter *P*, in honour of the President (Prof. Palazzo) of the International Seismological Association.

In a recent paper* by Professor A. E. H. Love on 'The Gravitational Stability of the Earth' he says: 'The elastic constants of the earth, in its present state, can be estimated from the observed velocities of propagation of the three types of waves which are transmitted when a great earthquake takes place. There are two sets of preliminary tremors propagated directly through the earth with nearly constant velocities of about 10 kilometres per second and 5 kilometres per second, and a main shock propagated over the surface with a velocity of about 3 kilometres per second. The two sets of tremors have been identified with waves of dilatation and distortion, and the main shock with superficial waves of the type first investigated by Lord Rayleigh. In the present paper reason is given for thinking that the manner of propagation is not much affected by gravitation and initial stress, and thus the observed values of the velocities of propagation of earthquake tremors and shocks would yield (1) for the seismic effective modulus of compression of the earth as a whole the value 36.9×10^{11} dynes per square centimetre; (2) for the seismic effective rigidity of the earth as a whole the value 13.8×10^{11} dynes per square centimetre; (3) for the seismic effective rigidity of surface rocks a value approximately equal to 6×10^{11} dynes per square centimetre.'

It is evident that seismograms contain the story, however involved, of the earth-wave; whence it came, its vicissitudes *en route*, passing through different media of different densities, being reflected and refracted, encountering in the upper part of the crust geological dislocations, faults and dikes, each having its effect upon the velocity, acceleration and destructive force. Professor A. E. H. Love in his presidential address recently before section A, British Association for the Advancement of Science, says: 'If we knew the distribution of density of the matter within the earth it would be a mathematical problem to determine the form of the geoid, *i.e.*, of the equipotential surface of the earth.' There is no doubt but that the seismogram bears a message on this subject. With the accumulation of seismograms from many and widely distributed stations and their collective study and analysis, there can be no doubt that the interior of the earth must yield up its secrets, and the seismologist will be able to furnish data of the greatest value for which scientists have hitherto groped in vain.

In the accompanying diagram, Plate IV., is shown the effect of air-damping on the amplitude of the oscillations. The record was obtained by giving each pendulum a slight tap with a lead pencil. It may be stated that a fresh electric candle (32 c.p.) was put into the lamp in order to obtain a good impression for the rapid motion of the light spot. The curve is exponential, and the ordinate is a function of the time.

The relationship between any two complete amplitudes is $\frac{y_1}{y_n} = f^n$, where f is the factor of diminution in amplitude for successive oscillations, *i.e.*, for equal increments of time, or the period of the pendulum comprising the two oscillations. f may be expressed as equal to e^k , where $e = 2.71828$, the base of the Napierian or natural logarithms, and k is designated the logarithmic decrement.

* Proceedings Royal Society, vol. 79, p. 194

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Taking linear measures from the (enlarged) diagram for values of y , we find f for the E-W pendulum to be 1.31, and hence $k = .27$; while for the N-S pendulum the corresponding values are 1.22 and .20. The damping in the small air-chambers, with the moveable sides, of the two pendulums was not quite the same.

If we put T^1 for observed period of the pendulum, and τ for the time in which the amplitude has decreased the e^{th} part, then evidently $f^2 = e^{\frac{T^1}{\tau}}$ or $\frac{T^1}{2\tau} = \text{nat. log } f$.

Taking linear measures again, and taking the mean value for independent determinations of τ , we obtain its value for the E-W pendulum to be 10.6s, and for the N-S pendulum 14.5s.

This is the damping when the instrument was set up, but it is intended by some means to increase the same.

The following are the principal earthquakes recorded since the beginning of the year. Unfortunately the first one, that in January at Kingston, Jamaica, has no time scale, from failure of the relay which works the shutter. This was only noticed when the sheet was developed. The two following days, January 15 and 16, showed much unrest of the earth on the seismogram, especially on the latter day.

The next earthquake of consequence was that on the Pacific coast of Mexico, by which several towns were destroyed on the morning of April 15. The pulsations as shown on the seismogram were more intense than those for the destructive earthquake at Kingston.

The following is the record:—

	N-S Component.			E-W. Component.		
	h.	m.	s.	h.	m.	s.
First preliminary tremors began...	1	15	04	1	15	03
Second preliminary tremors began...	1	19	26	1	19	30
Principal portion began...	1	27	20	1*		
Principal portion ended...	1	43		1	43	
End of earthquake...	3	01		3	09	

(The amplitudes gradually decreased into irregular wavy lines and merged into 'sawtooth' tremors, which lasted for some hours.)

Maximum amplitude...	14 ^{mm} .	14 ^{mm} .
Period of pendulum...	5.7 ^s	5.7
Magnification...	120	120

The time scale is direct from the standard mean time clock, 75° meridian and requires no correction.

The following is the record of the earthquake in Ecuador on the morning of June 1, 1907:—

For hours before the quake was registered, the earth was in a state of unrest and the seismogram for both pendulums shows the characteristic regular 'sawtooth' pulsations, amplitude .2^{mm} to .3^{mm} (magnif. 120).

These tremors were replaced on the N-S component (E-W pendulum) by more or less irregular movements, amplitude 1^{mm}, beginning at 3^h 48^m 44^s, while for the other pendulum the tremors continued, showing absolutely no movement corresponding to the preceding time, until 3^h 55^m 14^s, when an abrupt movement, amplitude 2.5^{mm}, took place, the other pendulum showing a marked increase in amplitude, 1.5^{mm}, at 3^h 55^m 20^s. The amplitudes thereafter for both decreased irregularly until at 3^h 58^m 56^s, the N-S pendulum increased to 2.5^{mm} and the other to 1.5^{mm}, which neither exceeded thereafter. The last large oscillations occurred for the E-W pendulum between 4^h 11^m and 4^h 12^m, and for the N-S pendulum between 4^h 10^m and 4^h 11^m. The quake continued till 4^h 30^m showing as an irregular wavy line, when the tremors similar to those preceding the quake manifested themselves for many hours.

* Photo record too faint.

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As far as can be gathered from the press reports of the time, it appears that the epicentre was about due south of Ottawa. If the horizontal movement of the pier is in the plane of the great circle between the epicentre or hypocentre and here, then the preliminary tremors coming through the earth would not disturb the pier relative to the N-S pendulum, which records essentially an E-W motion, the pendulum would be hit end-on. On the other hand the E-W pendulum, giving N-S component would have its maximum effect.

Although it is considered that the seismogram reveals but little of the direction of the epicentre from the place of observation, giving instead the direction of the end or last part of the impulse, yet the above record would seem to show clearly that the first preliminary tremors, longitudinal waves, arrived in the direction of the epicentre from here. Furthermore it is to be noted that when the N-S pendulum did move, it did so abruptly and with an amplitude as large as was recorded during the whole quake. Such an abrupt movement required essentially an east and west oscillation of the earth particles, transverse waves in this case. This movement agrees too with the record of the E-W pendulum, whose amplitude for that time is only about one-half that of the other pendulum. The E-W pendulum recorded the quake for about 10 minutes longer than the N-S pendulum did (which had lapsed into the tremor stage) indicating that as the first movements of the quake were north-south, so the last movements were.

Professor Marvin* noted a similar condition for the Washington seismogram of the Kingston, Jamaica, quake of January 14, 1907.

Knowing the position of the epicentre, that is, its distance from Ottawa, Laska's empirical expression for the relation between the various phases and distance does not appear to apply in this case very satisfactorily, at least not for the 'principal portion.'

We may take the distance as 3,400 miles, say 5440^{km}, this would require an interval between the first and second preliminary tremors of 6.45^m. If we recognize the first movement of the N-S pendulum as the beginning of the second preliminary tremors, in which case we must admit that the second preliminary tremors are transverse and not longitudinal waves like those of the first preliminary tremors, then the interval between the first and second as found on the seismogram is 6^m 30^s or 6.5^m, which is in close accord with the above empirical or theoretical 6.45^m. Correspondingly we should find the 'principal portion' to begin 16.3^m after the first preliminary, that is, at 4^h 05^m 02^s. Examining the diagram there is nothing thereabout to show the beginning of the 'principal portion' with a rapidly increasing amplitude.

The next earthquake to be recorded was on the morning of June 13, 1907. On the following day the press reported an earthquake of the preceding day at Kingston, Jamaica, at 1.20 a.m., which 'was especially severe at Port Royal, destroying the walls of the temporary buildings under construction,' also 'a severe earthquake was experienced yesterday at Valdivia, Chile. Several buildings and the railroad bridges were destroyed, and five persons were killed.'

If the above time for Kingston is correct we have evidently no record for that earthquake.

The seismogram of the 13th June shows somewhat the characteristics of the one just described for Ecuador.

Both Kingston and Valdivia are nearly due south of Ottawa, the former being about a degree west and the latter a little over two degrees east of our meridian. In distance, however, there is a great difference while the former is about 1,900 miles (3,000 km.), the latter is nearly 5,900 miles (9,400 km.) distant. Hence the phases of the quake should determine which earthquake was recorded. The earth was comparatively quiet preceding the quake, there were very few 'sawtooth' tremors recorded in the preceding hours, and those that were, were very minute, scarcely readable.

* Monthly Weather Review, Jan. 1907.

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The E-W pendulum showed the first preliminary tremor at 4^h 30^m 46^s, amplitude .7^{mm}. At that same moment the N-S pendulum showed a minute disturbance, like an earth tremor, and this continued till at 4^h 41^m 04^s a decided movement, amplitude 1.5^{mm}, was recorded. The E-W pendulum after the first preliminary tremor showed little movement until at 4^h 41^m 00^s it too recorded a decided deflection, amplitude 3^{mm}. The amplitudes of both slowly decreased till 5^h 19^m when the traces were practically straight lines. Recognizing then the second disturbance as that of the 'second preliminary' we have for interval from the first 10^m 18^s, this would indicate a distance from the epicentre of 9,300 km., whereas our assumed distance is 9,400 km., a close and satisfactory agreement. The beginning of the 'principal portion' should occur 27.9^m after the first preliminary, but in this case too, there are no marked oscillations corresponding to this time or near it. By a coincidence the interval of 10^m 18^s is approximately that between the first preliminary tremors and the beginning of the 'principal portion' for the distance of Kingston, and at the same time that between the first and second preliminary tremors for the distance of Valdivia.

TERRESTRIAL MAGNETISM.

Declination was generally obtained by means of a magnetic needle supported on a pivot and the whole attached to a transit by means of which the direction of true north was obtained. Both inclination or dip and intensity or total force were obtained with the dip circle. The latter was determined statically, Lloyd's method, where a dipping needle is loaded with a small fixed and constant weight acting in opposition to magnetism, that is, the earth's magnetic attraction is weighed, so to speak, against the force, assumed to be invariable, of gravitation. This gives the comparison of the forces in the plane of the magnetic meridian. Then the loaded needle whose magnetic constant is determined at a base station, is used as a deflector to a dipping needle whose polarity is not reversed as is done with the dipping needle used for the determination of inclination. The data are then sufficient to deduce the value of the force.

The method is simple, it is well adapted in connection with exploratory work, requiring less skill and manipulation and less patience than is necessary in the use of the fibre magnetometer. Lefroy in his extensive survey in Canada employed Lloyd's method. It is only applicable to a limited portion of the globe, being especially useful in the higher magnetic latitudes.

The instrument that is used at present is a Tesdorpf magnetometer, similar to those supplied to Drygalski of the 'Gauss' on her Antarctic expedition.

Of the three elements, the inclination is obtained in the usual way, common to all magnetic instruments, a symmetrical magnetic needle with cylindrical axis is supported on two horizontal agate edges. The plane of oscillation is in the magnetic meridian. If all the conditions were perfect then the inclination to the horizon shown by the needle would be the desired magnetic dip. To eliminate imperfections of figure, inequality of pivots or axis, unequal distribution of magnetism, observations are made in duplicate and in all possible positions of the needle together with reversal of its polarity, which latter is affected by means of two bar magnets.

The declination, however, is obtained not by a magnetic needle, which suffers somewhat in sensitiveness on account of pivotal friction, but by the direction of a suspended cylindrical hollow magnet. Formerly the suspending fibre was of silk, but now a metallic fibre, extremely fine, is used.

The horizontal force (total force a derivative) is obtained from deflection and oscillation observations. The principles involved in the two observations are simple. In the first—that of deflection—we obtain the ratio of the magnetic moment of a deflecting magnet whose constants are known to the earth's horizontal magnetic force, *i.e.*, there are two forces pulling at the suspended magnet, one that of the deflecting magnet, which is placed at a given distance from the suspended magnet,

and in a horizontal line from the centre of the latter and perpendicular to its direction, and the other that of the earth.

In the second, that of oscillations or vibrations, by noting the duration or time of an oscillation, the product of the same two quantities is obtained. As the lines of force lie in the magnetic meridian, the total force could be obtained were it practicable to observe the oscillations in that plane. Hence the oscillations are observed in a horizontal plane. There is an analogy between these magnetic observations and those of gravity by means of a pendulum.

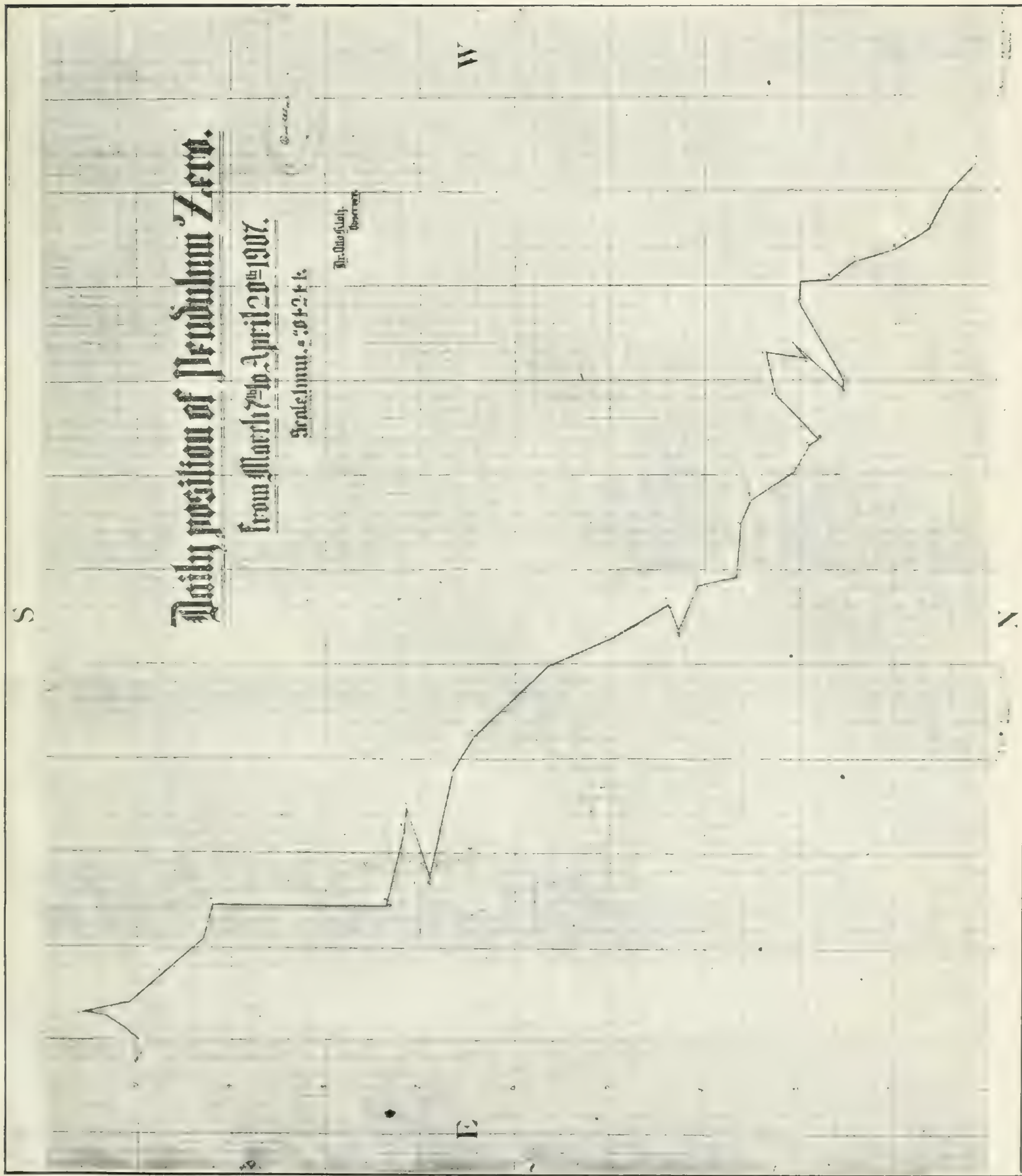
By noting the period of a pendulum at two stations we obtain a ratio of the force of gravity at the two places; similarly, when oscillations are noted with a permanent magnet at two points of the earth's surface, the ratio of the magnetic force is obtained. In each case, for gravity and terrestrial magnetism, the intensity of the force varies inversely as the square of the period of oscillation. The oscillating magnet is the one used as a deflecting magnet in the preceding case.

The constants of the magnet have been determined by Mr. R. F. Stupart, director of the Magnetic Observatory at Agincourt, the magnetic base station for Canada.

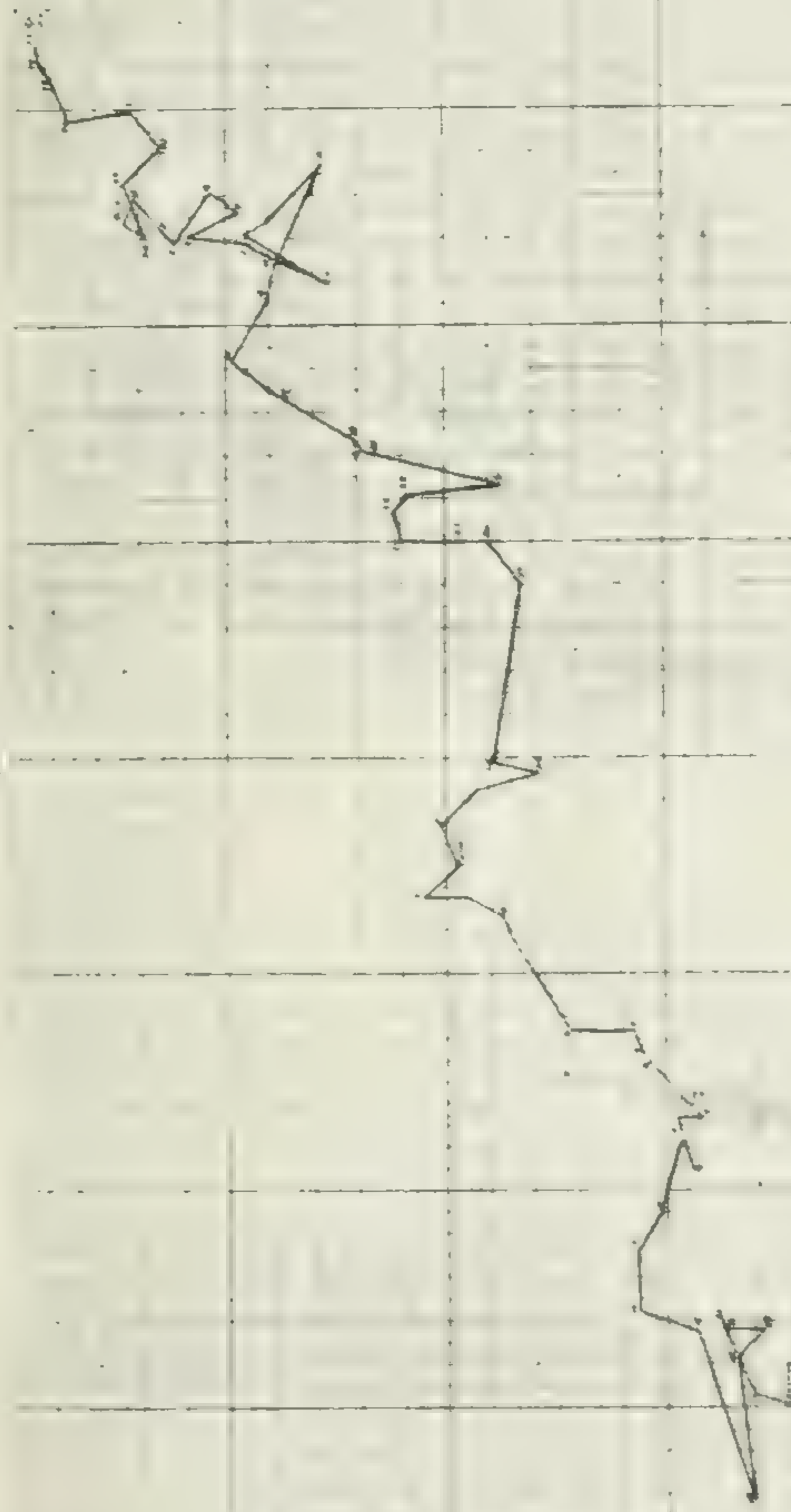
I have the honour to be, sir,

Your obedient servant,

OTTO KLOTZ.



111



Daily position of Pendulum Zero.

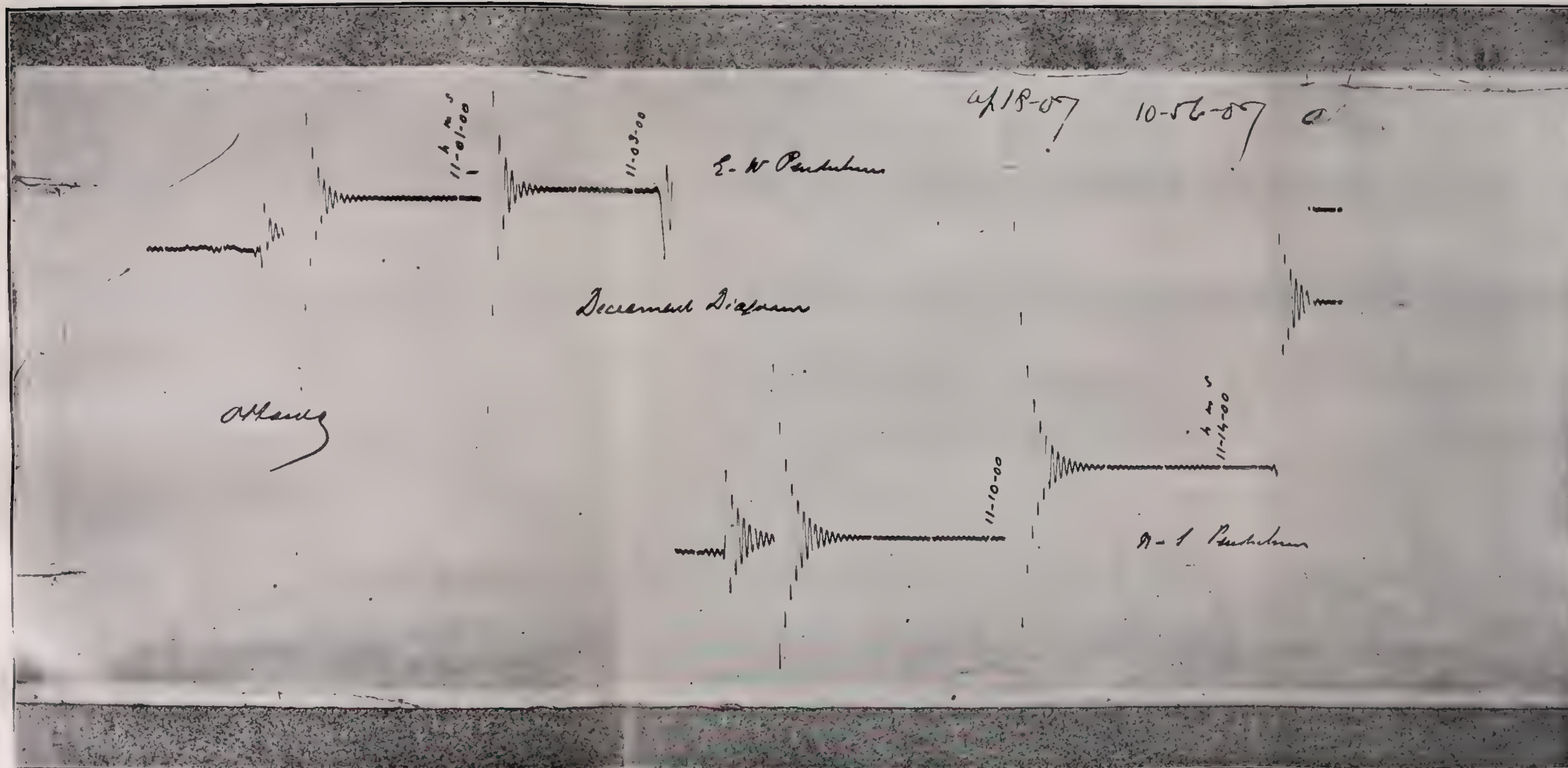
from April 23 to June 21, 1907.

Scale 1 m. 1021.

Dr. No. 5111.

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APPENDIX 3.

REPORT OF THE CHIEF ASTRONOMER, 1907.

ASTRONOMICAL AND ASTROPHYSICAL WORK.

BY

J. S. Plaskett, B. A.

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APPENDIX 3.

ASTRONOMICAL AND ASTROPHYSICAL WORK, BY J. S. PLASKETT, B.A.

OTTAWA, ONT., July 10, 1907.

W. F. KING, Esq., B.A., LL.D.,
Chief Astronomer,
Department of the Interior,
Ottawa.

SIR,—I have the honour to present the following report of the work carried on by me and under my direction during the past year.

Since presenting my last report, considerable progress has been made towards getting the work arranged and systematized, but considerable remains to be done before it is completely organized. A certain amount of experimental and tentative work is unavoidable when an observatory is being started and new work initiated; indeed, it is only by intelligent experiment that any real progress can be made, and it has been my aim from the beginning by suitable experimenting to place our instruments and methods of work in as efficient a form as possible. Before entering into any details of the work, however, I wish to express my appreciation and gratitude for the readiness you have always shown in meeting any needs that have arisen either in the way of assistance or apparatus, for the very effective help you have so readily given me in many of the problems connected with my work, and for the advice and encouragement so willingly afforded me. To these must also be added the privilege of visiting the principal observatories of the United States, which you arranged for me. This visit proved of great value, as many ideas and methods were thereby obtained which have been and will still be of much service in the work here, and which could have been obtained in no other way than by a personal visit. A full report of this visit will be found below.

The greater part of my time, as in the previous year, has been occupied with spectrographic work and I have to report satisfactory progress in that line. As outlined last year, the work has been confined almost wholly to determining the velocity curves and orbits of some spectrographic binaries, and, although I can only report one binary, α Draconis, as completed, the preliminary elements of ι Orionis have been obtained and work on η Piscium, ν Geminorum, η Virginis, η Bootis, α Corona Borealis, and σ Andromedae is well under way. Besides these stars a number of other binaries as well as some early type stars are under observation, but, up to the present, none of the plates have been measured and reduced. The same difficulty occurs here as is felt elsewhere, the practical impossibility of having the measurement and reduction keep pace with the observing. This is especially necessary in the case of binary stars to determine at what part of the period observations are most needed. With the additional assistance you have provided, however, and with the spectro-comparator, whose purchase you have authorized, together with a simplification in the method of reduction, there seems to be good ground for believing that this difficulty will be to a great extent obviated in the future. I wish to report in this connection the very satisfactory work of W. E. Harper, who has during the year he has been with me, measured and reduced a large number of spectrograms in a most careful and efficient manner.

The designing and the making of the detail drawings for the new combined three-prism and single-prism spectrograph, which is now completed and in regular use,

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occupied considerable time. Besides the above, the working drawings of the mechanism for the coelostat telescope have been completed, and the work, which is being done at the Victoria Foundry, Ottawa, is well under way. A new form of polarizing photometer has been designed and the drawings made, while the instrument itself has been constructed in our workshop. These will be described in more detail below.

The workshop has proved itself an invaluable adjunct to the observatory, and I think we are to be congratulated on obtaining so able a mechanician as Mr. Mackey to preside over it. During the year he has been employed, all the mechanical parts of the new spectrograph, (the manufacturers price for a similar but less complete instrument being \$2,000) have been constructed, including an outside temperature case of aluminum. The photometer above referred to has been completed and a travelling wire micrometer for Cooke Transit No. 1, is nearly finished. Besides these larger pieces of work, many small accessories have been constructed and the numerous repairs, so frequently required wherever instruments are used, attended to.

The supervision of this work and the testing and adjusting of the new spectrograph have occupied considerable time, as has also the arrangement for the automatic heating of the temperature case. Owing to the great difference between day and night temperature in the equatorial room, the spectrograph has frequently to be maintained from 5° to 10° C higher than the surrounding air, and, under such conditions, it is difficult to prevent the radiation and conduction then going on from making itself felt in a lowering of the temperature within the prism box, this taking place at the rate of about 0.1° C per hour, if the outside temperature continues to fall. However, most of the difficulty has, by suitably arranging the heating coils, been successfully overcome.

Three special investigations on spectrographic work have been undertaken, two of them completed, the third only partially so. The first dealt with the form of image given by the system of objective and correcting lens. I had long felt that there was need for some work in this line, especially after my trouble of the previous year with the correcting lens as detailed in the last report. This investigation attacked the problem in various ways and showed that the image given was at least twice as large in diameter as it should be. The paper was published in the *Astrophysical Journal* for April, 1907, and is given as Appendix A to this report. It is hoped as a result of this work to obtain a correcting lens giving a normal-sized image. Such a lens has been computed by Dr. C. S. Hastings, and is now being made by the Brashear Company. It should very materially reduce the required exposure time and moreover, by the more uniform illumination of the collimator objective thereby obtained, ensure greater freedom from chance of systematic error.

The second investigation referred to was on the spectrum of α Ceti, the well known variable star, which reached an unusually high maximum last December and was well within the range of our equipment. Several facts of interest were elicited from the measures of the spectra obtained, which are detailed more particularly in the copy of the paper below (Appendix B), which was published in the *Journal of the Royal Astronomical Society of Canada*, vol. No. 1.

The third incompleeted investigation was on the influence of slit widths on the accuracy of velocity determinations and, although not completed, the results show that the slit can be widened considerably in early type stars with few lines without much increasing the probable error of the determination of the velocity. The whole question will be investigated in greater fullness in the near future, with especial reference to the new spectrograph and the different dispersions obtainable by it.

Since the last report the solar camera has been in regular use and a photograph of the sun's surface, recording the spots, has been made on every clear day. This work has been attended to either by Mr. Harper or myself, but now that Dr. DeLury has been appointed he has taken charge of it. No other solar research has yet been commenced owing to the delay in the erection of the coelostat house. As stated before

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the heavy mechanical parts of the telescope are being made at the Victoria Foundry. The secondary plane 20" mirror and the concave of 18" aperture and 80 feet focus have been ground and figured by Brashear, and are now stored in the instrument room. A combined collimator and camera objective of 6 inches aperture and 23 feet focus, also figured by Brashear, has been obtained, and I am glad to report that, through the kindness of Dr. Brashear, we are now in possession of a 6 inch plane grating of 12,500 lines to the inch to be used in connection with the above objective. The optical parts of the installation are all ready and, as soon as the building is completed, work can be commenced at once.

It was found difficult, indeed practically impossible, to obtain satisfactory results with the registering wedge photometer, and work with it has been abandoned. I designed a photometer which depends on the comparison of the star with an artificial star, the latter being varied in intensity by the rotation of a Nicol prism in a polarized beam from it, this variation being a function of the angle of rotation. This design was completed, and the necessary optical parts for it obtained about the New Year, but the press of work in the workshop has prevented its completion until now. It is hoped that, by its use, accurate results in the measures of variable stars may be obtained.

A beginning has been made in micrometer work with the equatorial by Mr. Motherwell, recently appointed. It is proposed to obtain measures of those double stars within the range of the telescope of which the measures are few in number, and of those binaries which are in sufficiently rapid motion to require frequent measurements. It is also proposed to, as far as possible, measure the positions of any comets visible. Mr. Motherwell is also including in his work, at your suggestion, the observation of the time of occultations of the brighter stars by the moon. For this purpose and for photometric work on the brighter stars, the 4½" Cooke equatorial telescope is to be mounted on the roof of the observatory and can be used when the 15-inch is otherwise engaged.

The popularity of the Saturday open nights for visitors has continued unabated, the average attendance being over 50 and on several nights having been over 100.

That there is a real interest in astronomy in the Capital is shown not only by these figures, but by the splendid membership obtained for our Astronomical Society, by the large attendance at the meetings and the interest in the proceedings.

The care of the instruments has taken up considerable time, and one is frequently interrupted in important work to give out a minor instrument. I propose, with your permission, to entrust the bookkeeping part of it to Mr. Motherwell, reserving only the general supervision over the instruments and attention to the necessary repairs.

It is hoped, now that the staff under my direction has been increased by three, and that the instruments and method of work are gradually getting into satisfactory condition, to much increase the output of useful work in the near future. I feel, however, that considering the circumstances, a good beginning has been made and that, as the work becomes more and more systematized, both the amount and quality will increase.

VISIT TO OBSERVATORIES.

Lick Observatory.

When, by your kindness, I had the privilege of visiting the principal observatories in the United States, the first one to be reached was the Lick on Mt. Hamilton in California. The journey was made from Ottawa by way of the Canadian Pacific Railway to Vancouver and thence from Seattle through Portland to San Francisco, reaching there on Sunday, August 26. Mt. Hamilton is reached from San José, 50 miles from San Francisco. From San José a daily stage makes the 25 miles of mountain climb in about 5 hours. The observatory is commandingly situated on the summit of Mt. Hamilton, at an altitude of about 5,000 feet, and, owing to its com-

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parative inaccessibility is surrounded at a somewhat lower level by a number of other buildings serving as dwelling houses for the astronomers, as offices and workshops, &c. In fact the summit is like a small town, has its own electric light plant and water works system, its school, its boarding houses, &c., all presided over in a paternal way by the director of the observatory, Dr. W. W. Campbell. I had not known, until I reached San José, the inaccessibility of the observatory nor of the fact of there being no hotel accommodation at the summit, and I was in a quandary as to arrangements for accommodation during my visit. I had written Dr. Campbell of my proposed visit, but, owing to his absence, had received no reply. However, I was heartily welcomed at the summit and kindly and hospitably entertained by Dr. and Mrs. Campbell, who did everything in their power to make my stay a most pleasant one. I can safely say that they succeeded fully, and I wish to record here my appreciation of their kindness to an entire stranger.

I wish also to mention that every member of the staff seemed to take pleasure in showing me every detail of their work, and as a consequence, I had both a pleasant and profitable time.

The summit of the mountain was blasted off to form a level place for the observatory building, which faces the west, overlooking the Santa Clara valley and San José. The 75-foot dome containing the 36-inch telescope is at the south end, and the dome for the 12-inch telescope at the north end of the building. The offices in the central part of the building between the domes are entered from a hall at the rear running the whole length of the building and entering the large dome at one end and the small one at the other. In this hall, arranged as transparencies, in stands for daylight or artificial illumination, are examples in the form of photographs of the different kinds of astronomical work carried on at the observatory. They comprise photographs of the sun and sun spots, of the solar corona and prominences, of the spectrum of the reversing layer and chromosphere, photographs of the moon at different phases, of some of the planets, and of star clusters and nebulae. There are many fine examples of the latter taken with the Crossley reflector by the late Director, Professor J. E. Keeler. There are also enlargements of different types of stellar spectra made by the present Director, Dr. Campbell, who has made a specialty of stellar spectroscopy and radial velocity work, and practically revolutionized the methods of determining the velocities of stars in the line of sight.

These transparencies, as well as numerous photographic prints on the walls, serve to render the hall very interesting to visitors of an astronomical turn of mind and, during the summer, there is hardly a day that does not bring its quota. As at Ottawa, visitors are, on Saturday evening until ten o'clock, allowed the privilege of a look through the telescope, and this I learned is frequently taken advantage of by large numbers who make the 25 mile trip from San José for that purpose.

On the afternoon of my arrival, Dr. Campbell showed me over the observatory, introducing me to the various members of his staff and explaining the nature of the work under way at the observatory. The amount of work accomplished is marvellous, and as to the quality, there is no need to speak of that as its reputation is world wide. The climate of Mt. Hamilton is probably better suited for astronomical work than that of any other observatory on the globe. For eight months in the year practically every night is fine, and I can testify that the seeing is much better there than at any other observatory I visited. Dr. Campbell told me that the average number of working nights was 250 in a year, and the work is so arranged and divided that every night and all night long is utilized. The custom there is to divide the nights into two portions, one person using the telescope from dark till midnight and then being relieved by a second observer, who works until dawn.

The work undertaken is quite wide in its range, embracing nearly every class of astronomical activity. Spectrographic work, the radial velocity of stars, has perhaps the greatest attention paid to it, but micrometric work is a good second, and besides,

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considerable attention is devoted to observations with the meridian circle. Moreover the Crossley reflector, under charge of Dr. Perrine, is doing splendid work in the photography of stars and nebulae; while photometric work also receives attention.

The determination of the radial velocities of the brighter stars is the especial work of the director, and he has not only revolutionized the methods of obtaining accurate values, but has now probably more data concerning radial velocities than all other observatories engaged in this work combined. Several thousand star spectra have been photographed, and the work of measurement is making good progress. When his catalogue of the radial velocities of the brighter stars has been completed, our knowledge of the universe will be very considerably increased. The Mills spectrograph attached to the 36-inch equatorial is the instrument with which this work has been done, and this instrument with the method of measurement and reduction employed are fully described by Campbell in the *Astrophysical Journal*, VIII., p. 123. The optical parts of this spectrograph have, however, recently been remounted in an ingenious and original manner after designs by Dr. Campbell who very kindly explained to me the whole mechanism. It consists briefly of a steel box, oblong shape, made of two $\frac{1}{8}$ -inch thick plates on the sides joined together about 3 inches apart by braces and cross braces and by plates, which form the channels or tubes along which passes the light from slit to collimator lens and from camera lens to plate, the prisms being mounted on one of these plates in the proper position. These plates are further stiffened by being screwed to castings which act as the ends of these tubes and the points of support of the instrument. The whole spectrograph is thus complete in one part, homogeneous and self contained. An axis, passing through the prism box and near the centre of gravity of the spectrograph, is supported in bearings on a truss framework rigidly attached to eye end of the equatorial. This axis can rotate, if necessary, in these bearings which are moveable in slides parallel to the optical axis of the telescope. The other point of support is on the tubular casting projecting from the box to which the slit is attached and which thus forms one end of the collimator. This slides in a ring pivoted between two points attached to the same truss. Thus it is not possible for any stresses to be introduced in the spectrograph proper by any flexure of the truss, and this is a decided improvement over the usual form, where truss and spectrograph are so connected together that flexure of the former is likely to introduce a similar trouble in the latter. Again, a change in the star focus is compensated by moving the spectrograph as a whole and not the collimator tube only.

Dr. Campbell informed me that flexure is entirely eliminated by this form of construction, and that I can readily believe. He further stated that if he were building another instrument he would construct it of brass which is amply strong to prevent flexure, which is easier to work, and which, with the usual type of collimator and camera lenses forms a combination which requires only very slight changes of focus for changes of temperature. The old Mills spectrograph had $H\gamma$ central, the new $\lambda 4,500$. I was told, however, that not much advantage is thereby gained in exposure time over $H\gamma$ central, this being probably due to two causes:—first, the focal length of camera requires increasing to obtain the same linear dispersion in the two cases, second, a denser spectrum is required around $\lambda 4,500$ to get the same accuracy. He advises about $\lambda 4,400$ as the most useful compromise.

The compactness of the instrument allows the temperature case to be small and neat. The case is supplied with coils for electric heating, the temperature being maintained automatically constant by an electric contact thermometer working in conjunction with a relay for controlling the heating current. As this responds to a change of temperature 0.2°C , the temperature in the prism box will vary only very slightly during a night's work.

The outside temperature case is placed on the spectrograph and the automatic heating arrangement switched on after the dome has cooled down and only shortly before starting the evening's work as the difference between day and night temperature

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on Mt. Hamilton is small, about 3° C. The automatic control works admirably, as the reading of the inner thermometer did not change appreciably during 4 hours working.

I had the privilege of staying with Dr. Campbell while he made several exposures on the spectrograph, and observed the remarkable ease with which the large equatorial is handled and the convenience of all the auxiliary apparatus required. The guiding is now performed by the reflecting slit method, and this method seems to me decidedly preferable to that in which one guides by the light passing through the slit jaws and reflected from the front surface of the first prism. I had an opportunity of comparing the guiding and following with that of our own telescope and, considering the great focal length of the instrument, practically three times as great as our 15-inch, the image remains remarkably steady and is very clear and distinct. This is undoubtedly due in great part to the exceedingly steady and almost perfect seeing which Mt. Hamilton possesses, but the quality of the image is also due to the magnificent objective of the 36-inch.

Dr. Campbell was most kind in giving me every possible assistance that might be of service in our spectrographic work and in many points and details his experience and advice is invaluable. I can only mention here a few of the points that occur to me as of sufficient moment to record.

The exposures actually required for stars of different magnitudes are found to vary directly as their intensities, *i.e.*, the exposure is increased 2.5 times for every magnitude fainter. This has not been found to be the case at Ottawa, the faint stars requiring relatively longer exposures than the bright stars. A well known property of the photographic plate by which, when the product of light intensity and time is constant, equal densities in the resulting negatives do not result when there are considerable differences in the light intensities probably accounts for this difference in the case of Ottawa, but why it should be different at the Lick is not evident. The comparison spectrum is introduced beside the star spectrum by a separate attachment at each side of the slit, reflecting the spark light to the collimator lens by a special diagonal prism. Hence when the spark spectrum is exposing there is no interference with the star spectrum which is being exposed at the same time. The exposure on the spark is divided into 4 parts at $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{7}{8}$ of the time of star exposure. The titanium comparison is now used with the new Mills, as the iron lines in the region around minimum deviation λ 4,500 are few in number and faint; and as no air lines appear in the titanium spectrum, no self induction is required in the circuit. The wave lengths of the titanium lines are obtained from Rowland's Solar Spectrum tables.

The settings for the slit in the star focus and for the camera focus are functions of the temperature and are taken from tables prepared from actual tests of these variables under varying temperature conditions. In this connection I learned that the camera focus was determined by definition tests, which according to my experience are not sensitive enough to determine the focus accurately within two or three tenths of a millimetre. This may be due to some aberrations in our camera lens preventing the sharpest definition at any point, but, so far as the resulting spectra are any evidence, there is little difference between it and the Lick. With the method described by me in our last report, a change in focus of less than a tenth of a millimetre is easily recognized, and, owing to the importance of correct camera focus to prevent systematic displacements of the lines and to the ease and quickness with which the focus is determined, the method I have used appears preferable.

Campbell's method of determining the collimator focus and the star focus are very ingenious and reliable and exhibit, especially the latter, the care and attention to details that help to account for the remarkable work he has done and is doing. The collimator focus is determined by pointing its objective to the sky and holding against its wide open slit a small photographic plate. Star trails at slightly different settings soon give a very accurate value of the true focus. In setting the slit in the focus of the objective and

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correcting lens, the first step is to determine the difference in focus between the spectral lines and the dust lines, as it is only in rare cases that no astigmatism is present in the prisms and the two foci coincide. The camera focus is then set so that the dust lines are in focus, the slit opened widely and a number of spectra made at different settings of the star focus. The narrowest spectrum will evidently be the one in focus. I am indebted to Dr. Campbell for pointing out the above precaution in regard to the focus for the dust lines, as the necessity for it had not occurred to me.

In the measurement of the spectra, I was informed that practically every known method of reduction had been tried, and they are at present using the method developed by R. H. Curtiss of comparing the positions of the star lines with the same lines in a standard solar spectrum of the same dispersion. This method of course is only applicable to stars of the second type, whose spectra are similar or nearly similar to that of the sun. With early type stars I presume some method of reduction to wave length is used, but astrophysicists will, I am sure, look forward with interest to a discussion of the relative values of the various methods, which it is to be hoped Dr. Campbell will find time to prepare.

The measuring microscopes used are by Toepfer & Sohn, and similar to the Ottawa microscope except that they have no second movement at right angles as is the case in our machine. Two settings are made on each star and comparison line in each position of the plate under the microscope, the average variation being two units in the third place. The spectrograms are all of good quality for measurement, although their spectra of the brighter stars are no better than ours they have a very decided advantage when it comes to the fainter stars. Owing to flexure and to temperature changes, the definition becomes diffuse and washed-out looking with our modified Universal spectroscope, while, owing to freedom from flexure and to good temperature control, the spectra of faint stars made at the Lick are of practically as good quality as those of the brighter stars.

The spectra of stars as faint as the sixth photographic magnitude can be obtained with the Mills spectrograph, although upwards of two hours exposure is required. For stars fainter than the sixth magnitude, a spectrograph of lower dispersion is required, and at Mt. Hamilton they use a single prism instrument giving only one-fifth the dispersion of the Mills. The outside limit with this small dispersion is the eighth magnitude, and even these are only obtained with a very long exposure. This instrument is built up from the telescope truss and collimator section of the old Mills instrument, using a light flint prism as dispersing medium. The camera is stiffened by a tubular brace from the collimator section to the camera, and Dr. Campbell acquainted me with a curious fact in connection with the material of this truss. The collimator section is of steel, the camera section of brass. A brace or truss of steel was found to give no displacement of the spectral lines for change of temperature, while a brass truss gave a marked displacement. It is evident that, if necessary, a composite brace could be constructed that would neutralize any temperature displacement. The single prism instrument is used chiefly in special investigations, such as those on variable stars, which in their minimum phase go beyond the limit of the Mills spectrograph.

The spectrographic equipment and methods proved most useful and interesting to me, and the ideas and wrinkles gained have proved of the greatest value in our work and, even if nothing more had been learned, would have been worth the trip.

Another interesting feature of the observatory is the Crossley reflector. This telescope has quite an interesting history. Donated by Mr. Crossley to the Lick observatory it was, after much time and trouble, put into effective condition by Prof. Jas. E. Keeler, the late director of the Lick observatory. He made a large number of beautiful photographs of nebulae, discovering many new ones. Indeed it has been stated that it was principally owing to the enormous amount of labour connected with this work that his death was hastened. There is no question that he obtained magnificent results with a very poorly mounted instrument. Since Dr. Perrine has

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had charge of the reflector an entirely new mounting has been made after a quite original plan, which I am informed works admirably. That the work being done with the reflector has not suffered is evidenced by the discovery by Dr. Perrine of the 6th and 7th satellites of Jupiter. The polar axis is of tubular form built up of boiler plate with cast ends, on which are the journals rotating in bearings on separate piers. The body of this tube is eccentric with respect to the journals and the declination axis passes through it perpendicularly. The telescope is mounted on the declination axis at the side of the tube nearest the centre, thus the eccentric position of the tube helps to balance the weight of the mirror and its tube. The driving is effected by means of two long sectors into which two worms, each driven by the clock, gear. The one of these sectors that is idle is driven backwards while the active one is driven forward. As soon as the latter gets to the end of its run, an automatic arrangement unclamps it, clamps the idle one and the telescope's motion is continued without change. Owing to the length of these sectors and to the care with which they were cut and hobbled in place, the telescope drives so accurately that practically no guiding is needed for 15-minute exposures, and only occasionally in longer. The guiding is done by a double slide carrier, shifting the light plate rather than the heavy telescope. In satellite work, the motion of the satellite relative to the stars is computed, and the guiding cross wires moved at short intervals so that the plate exactly follows the satellite although making trails of the stars.

I was much interested in the plates, shown to me by Dr. Perrine, in which the 6th and 7th satellites of Jupiter were first seen. Also in a large number of beautiful photographs of nebulae and clusters which are soon to be reproduced in a book. The predominant form of nebulae as shown by these plates is spiral thus indicating the necessity of a revision of the Laplacian hypothesis. The star plates obtained, which are $3\frac{1}{4}'' \times 4\frac{1}{4}''$ in size, are measured by either one of two machines, the Repsold or the Stackpole, neither of which are entirely satisfactory according to Dr. Perrine's experience. The Repsold machine has a silver scale as a standard, the settings being made by a double micrometer microscope on the star image and the microscope being then tilted to point on the scale, thus giving the x co-ordinate, the y being obtained on a second scale at right angles. The Stackpole machine has a simple microscope provided with cross wires and the plate is set so that the star is at their intersection. The scale readings are made by two auxiliary microscopes on glass scales, no micrometers. I had the privilege of being with Dr. Perrine, while he was making plates for the redetermination of the position of the new satellites of Jupiter, and the reflector now seems to work very smoothly and regularly, and everything is arranged more conveniently than when Keeler did his work.

As from everyone at the Lick with whom I came in contact, I received many kindnesses from Dr. Aitken who is responsible for the micrometer work with the 36-inch. This work consists chiefly of the measurements of position angle and distance of double stars, and also the survey of the whole available sky at Mt. Hamilton with the 36-inch for the discovery of double stars. In such a systematic and thorough way is this work carried on that when he completes it there will be practically no more doubles to discover. Already by this method about 1,500 new doubles have been discovered, and these doubles are all comparatively close, to the best of my recollection none are noted which are farther apart than 5 seconds. His method is both simple and thorough. All stars of magnitude 9.0 or greater in the B.D. are entered upon cross-section sheets in their correct relative positions, each sheet being about 40 minutes wide in declination and covering about 15 minutes of time in right ascension. Every star on this list is carefully examined by the large telescope and, if double, is measured and recorded. Each double, both new and old, is measured at three different times, recorded on the sheets and transferred to a table. A card catalogue of the most important binary stars is kept, and measurements are made on each of these as occasion arises and when a measurement is required to get a complete orbit. On these cards are entered all previous measures, and also the

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approximate dates when future measures are desirable. His system of keeping track of what has been done and what is required to be done in double star work is very complete and well arranged to avoid loss of time and unnecessary duplication of work. I was much interested in comparing the star images at Mt. Hamilton with those I had seen at Ottawa, and Dr. Aitken kindly allowed me every convenience for so doing. The night I saw through the 36-inch, the seeing on their scale (5 perfect) was only fair, a little less than three, but even then was much steadier than any I had observed at Ottawa. A double of 0.25" distance was well separated in the telescope, whose theoretical separating power according to the formula $d = \frac{1.22 \lambda}{r}$ is about 0.15". The appearance of the star image within and without focus, making allowance for the difference of seeing, was practically the same as that given by our 15-inch.

On the next night Dr. Aitken was observing with the 12-inch Clark telescope, seeing fairly good, between three and four. A double star 0.4" apart, just within the theoretical limit, was separated, but not easily. The star images were beautifully small and crisp and the diffraction rings very clearly defined. The appearance within and without focus was practically identical, and I can readily believe the director's statement that this is one of the finest if not the finest objective ever made. The seeing was immeasurably better than any I have ever experienced in Ottawa.

A six and one-half inch meridian circle by Repsold is completely fitted with all the necessary accessories. There are full sized collimating telescopes at the north and south and also a full sized collimating lens for the azimuth mark about 100 feet away. This is placed directly below the southerly collimator. The irregularities of the pivots are observed by means of mirrors on each end of the axis, viewed through a telescope to the east and west. Tucker, the astronomer in charge of the positional astronomy, was away on a vacation during my visit, and consequently, I did not get any information concerning this branch of the work.

Numerous other pieces of research which arise from time to time are carried on at the Lick observatory and there are other lines of work which also have some time devoted to them. Photometric work is one of these, the instrument used being the Harvard type of wedge photometer on the 12" equatorial, the measurements being used to obtain the light curves of variable stars. Maddrill who was doing the photometric work claims the results with this type of photometer are reliable to about the tenth of a magnitude. The measurement of the position of comets is also on the programme of the observatory and is generally undertaken by the junior members of the staff.

The esprit de corps of the whole staff is very good. Everyone I talked with was enthusiastic about his own work and the work of the observatory and proud to be a member of its staff. The quantity of work turned out is enormous and its quality is universally recognized. The Lick observatory has of course a very great advantage over every other observatory in the character of the climate and its eminent suitability for astronomical observations. Not only are the number of good nights much greater than anywhere else, but the quality of the seeing is also superior to that elsewhere.

During the summer months the observers can be almost absolutely certain of having every night fine, and every one with experience knows what an advantage it is in many lines of work, to be able to obtain an observation whenever necessary. I spent four days most pleasantly and profitably at the Lick and could with much advantage have considerably extended my visit, but as my time was limited, it had to be made short. I left the observatory with regret, but with very pleasant memories of the kindness shown to me.

The Solar Observatory of the Carnegie Institution.

Pasadena was reached on Sunday, September 2, and an attempt was made to find Professor Hale, but without success. I found that he was in Santa Barbara for a few

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days, and I looked up Professor Ritchey, the Superintendent of instrument construction, who invited me to come around to the offices on the next day, Labour day. Although the shop was to be closed, the unloading of some of the heavy parts of the 60-inch reflector required his presence. On the following morning Professor Ritchey showed me the different points of interest in the instrument and optical shops of the observatory which are models for convenience and accuracy of working. The instrument shops are fitted with the most modern machine tools, lathes, milling machines, planer, shaper, grinder, drills, &c., and several men are continually employed. The principal line of work carried on here is the construction of the new instruments required at the observatory and laboratory, and, as so much of the work is new, requiring special apparatus, the facilities of the shop are taxed to the utmost. Besides much preliminary experimental work, the very complete and beautiful spectrohelio-graph in regular use at the observatory has been made. The globe measuring machine or helio-micrometer has been developed and finally completed. The solar and laboratory grating spectrographs with all their accessory appliances have been constructed here, to say nothing about the many smaller pieces of experimental apparatus continually required in any line of original physical or astronomical research. The work upon which most of the men were engaged at the time of my visit consisted of the driving clock, slow motions, and small accessories of the mounting of the 60-inch reflector which is now under construction. The mirror is being figured by Professor Ritchey, and the heavy parts are being made by the Union Iron Works of San Francisco.

The optical shop is a very interesting feature of the equipment as Professor Ritchey has introduced some novel methods in the grinding and figuring of mirrors, and the machine used in the finishing of the 60-inch mirror is arranged to give a number of different motions to the grinding or polishing tool and to the mirror. The figuring of the 60-inch mirror is being effected by a quarter-sector tool and Ritchey claims that he can do everything with it that can be done with the full sized tool with the advantages of less weight and greater ease of handling. Another noticeable point is the scrupulous care taken in cleanliness. Varnished walls, double windows, cloth packed double doors and close grained hard cement floors are some of the precautions employed. Whether such extreme care is necessary is questionable as the Brashear Company make surfaces of the most beautiful polish, and with never a scratch, without taking apparently a tithe of the care that is taken in Pasadena. The 60-inch mirror was being tested by zones at the centre of curvature, but they are at present figuring a 36-inch flat to test at the principal focus. I have no doubt that when finished, the mirror will have a figure as perfect as it can be made and that some beautiful results in nebular photography will be obtained by it, especially in the transparent and rarefied atmosphere of Mt. Wilson.

I examined with much interest the drawings of the mounting for this mirror whose design, due to Professor Ritchey, is ingenious and admirable. Although I am no judge of the quantity of material required, the polar axis and principal moving parts seemed to me unnecessarily heavy, but the error, if error it be, is certainly on the safe side. The mirror with its accompanying skeleton tube is to be carried in a fork on the north end of the polar axis, which is about 18 inches in diameter at the upper end, having a central hole some 9 inches in diameter, along which the beam of light is to be transmitted when the instrument is used in a Cassegrainian form. The weight of the mirror, tube, fork, polar axis, driving gears, worm and all moving parts is to be counterbalanced by a cylindrical float 10 feet in diameter, between the fork and the outer bearing, which will revolve in a semicircular trough filled with mercury. The upward thrust on this float is nearly equal to the weight of the moving parts, and there consequently should be little friction in the bearings. The driving worm wheel is 10 feet in diameter, it is to be cut in place on the axis and finished perfectly smooth by grinding with a worm. It is hoped that owing to its large

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diameter and to the care with which it is to be finished the driving will be uniform and regular.

An erecting shop has been built in the yard beside the main shop, in which the reflector is to be entirely set up and placed in thorough running order before being taken up on the mountain. A travelling electric crane is installed to handle the heavy parts, two or three of which, such as the fork and polar axis, weigh in the neighbourhood of ten tons each. In order to get such massive pieces to the summit of the mountain, a trail is being built up the mountain at great expense, and actual transportation is to be accomplished by a special automobile, each wheel driven by a separate electric motor.

Many interesting details of shop methods were kindly explained to me by the foreman, Mr. Jacomini, particularly those relating to the cutting and grinding of worms and worm wheels, the method of making and grinding curved slits, and the cutting of bevel gears. Many useful shop wrinkles were obtained, such as the use of a small motor for a portable emery grinder, and of a spring tool for cutting threads.

I examined with much interest the measuring and computing division of the work of the observatory which is now under the charge of Professor W. S. Adams, and is all done at the Pasadena office. There are a number of measuring microscopes for spectra by Gaertner and a plate measuring machine of the same maker, but the most interesting machine in the division is the globe measuring machine, or helio-micro-meter as Professor Hale calls it, for determining heliographic positions of spots, faculae, flocculi, &c. The original form of the instrument consisted simply of a ruled globe divided by meridians and parallels into sections of a degree square, on which the image of the solar photograph was projected by a lens of the same focal length as the one with which the negative was taken. In such a case, placing the globe with its equator, poles, and prime meridian corresponding to the position of the equator, poles and prime meridian of the sun when the negative was made, the position of any point on the negative can be at once read directly on the globe, saving considerable measurement and computation.

In the improved form the plate and globe are placed side by side and illuminated strongly by electric light. At the distance of 60 feet, the focal length of the concave mirror of the Snow telescope, two concave mirrors reflect images of these into two telescopes, side by side, directly above the globe and plate, and these images are or can be superposed on one another by totally reflecting prisms into a single eyepiece. The globe itself has no graduations, but has an equator and principal meridian ruled on it, while the position of the globe with respect to two axes at right angles to one another is determined by graduated circles which can be read by telescopes near the eyepiece above mentioned. A cross wire is set on the spot whose position is required, and the globe is then rotated by means of slow motion handles until the intersection of equator and meridian coincide with the cross wires and therefore with the spot. The reading of one circle gives the heliographic latitude and of the other the heliographic longitude of the spot, without any computation whatever.

Prof. Hale returned from Santa Barbara on September 5th, and at first proposed to go up the mountain with me, but later found that he would be unable to leave Pasadena. He was very busy during the time of my visit with his investigation into the cause of the characteristic phenomena in sun spot spectra, which has since been published, and there was being installed for laboratory researches in connection with this investigation a transformer for an electric furnace with a capacity of 50,000 watts and also a transformer capable of delivering 5,000 watts at 1,000, 2,000, 4,000, 8,000, 16,000, 32,000, or 64,000 volts for powerful spark work. Professor Gale, who looks after the physical side of this and similar investigations, was, however, going up to the observatory, and I had the pleasure of his company and of Mr. Rainer of the National Physical Laboratory of England, who was visiting Pasadena at this time.

The only present means of reaching the summit is by horse or mule back and, after taking the street cars to the foot of the trail, one has to ride on an animal the

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remaining 9 miles of winding mountain trail through the frequently precipitous canyons to the summit. The trip, which required about 4 hours to make, was quite a pleasant one and the summit was reached about six o'clock. We at once went to what is called the Monastery, the bachelor quarters of the observers, and were introduced to the staff at that time working upon the mountain. I will refer to each more particularly when I come to speak of the work done on the mountain. The Monastery consists of a series of bed rooms with a common living room or library and dining room. The library has the principal astronomical and scientific periodicals on file together with a fair collection of the most useful books, which are, I believe mostly from Professor Hale's private library. The buildings are distributed over a considerable area on the mountain top, the Snow telescope house being about three-eighths of a mile from the Monastery with the laboratory and other temporary buildings between. The view from every part of the peak is magnificent, mountains after mountains to the limit of vision on the one side, and on the other the cities of the plain, Los Angeles and Pasadena, with the Pacific beyond.

The number of observers on the mountain at the time of my visit was seven. Of these Mr. Ellerman, who was the senior officer, is astronomer in charge of the solar work on the top of the mountain, Professor Gale has charge of the conjoined physical research with Mr. Olmstead as his assistant, Mr. Palmer is assistant to Mr. Ellerman, looking after the bolographic work with the Snow telescope. In addition to these regular officers of the observatory, Mr. Abbot of the Smithsonian Astrophysical Observatory and his assistant Mr. Ingersoll were working during the season on Mt. Wilson in determinations of the solar constant, while Professor Nichols of Columbia, another guest for the summer was doing some work on the absorption of gaseous vapor.

I was most interested in the work of Mr. Ellerman, as it was much on the same line as we propose to carry on ourselves, but I will speak first of the work being done by other members of the staff and by the guests.

Professor Gale and Mr. Olmstead were at work in the laboratory obtaining spectra of the elements most affected in sunspots. Such spectra were made with a high and low temperature arc, or in the arc and the flame of the arc, to compare the difference between the lines in the two cases and to further compare the lines so affected with those affected in sun spots. They use, for all this work, the Littrow type (with combined collimator and camera objective) of grating spectroscope, which is found to be both convenient and accurate. A number of spectra can be made on one plate, which is placed directly below the slit, as it is movable vertically by rack and pinion.

Mr. Palmer is using a bolometer to determine the energy curve at different points of the spectrum over a diameter of the sun. The bolometer is arranged to give a continuous record of the energy as the sun's image drifts across the slit. The image is in this case formed by the Snow telescope and, as steadiness of the solar image is not necessary, the work is usually done in the middle of the day when the definition is too poor for spectroscopic or spectroheliographic observations.

Mr. Abbot with Mr. Ingersoll as assistant has quite an elaborate apparatus, installed in a temporary building on the summit, for the determination of the solar constant and of variations in it. A continuous bolographic record of the solar spectrum from the ultra violet to the intra-red, inclusive, is obtained and the energy curve thus derived is integrated and calibrated by various methods so as to give very accurate values of the solar constant. Mr. Abbot believes the values he obtains are correct to one per cent and he has hence been able to detect a variation in the value of the constant between 2.0 and 2.3 calories. He has not as yet been able to connect this variation into any periodic relation with other phenomena such as spots, prominences, or other disturbances. The temperature changes on the earth's surface that are of a general as distinguished from a purely local character, follow markedly this

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variation in the solar constant, and Mr. Abbot states that one should be able to predict, considerably in advance, the probable temperature.

The work with the Snow telescope, which had been done conjointly by Messrs. Ellerman and Adams, the former taking the photoheliographic and spectroheliographic work and the latter the spectroscopic study of sun spots and of the solar rotation, was of great interest to me and I took every opportunity during my stay of being with Mr. Ellerman while he was making the daily photograph. Unfortunately, owing to Mr. Adams absence, there was no work done on spot spectra, nor any plates made for the measurement of the solar rotation. Even if he had been present, however, there were no sunspots of sufficient size to make successful spot spectra.

The solar definition on Mt. Wilson is at its best between one and two hours after sunrise and is also frequently good about an hour before sunset, but in the middle of the day, in general, somewhat unsteady. The form of coelostat house used, consisting of walls of canvas louvres painted white with inner movable canvas walls, seems to answer the purpose admirably as the temperature keeps quite cool inside the house and the free circulation of air prevents any stratification in the path of the beam. Thus any disturbance must be due principally to the air between coelostat and sun. This disturbance again is minimized by having the coelostat mirror at as high an elevation above the heated surface of the ground as practicable, and by having all the soil near the pier covered as far as possible with shrubs and trees to diminish radiation.

During the hours previously mentioned, the solar definition is very good indeed, much superior to any I have seen in Ottawa, so good indeed that the principal difficulty is not in the boiling or unsteadiness of the image, but is caused by change in figure of the mirrors due to the heating, by the sun's rays, of the silver surface. This change of figure is quite regular and appears principally on the coelostat mirror which becomes convex instead of plane and consequently lengthens the focus appreciably. The amount of increase depends upon the length of time the mirror has been exposed to the sun and also upon the freshness of the silver coating, but there seems to be a maximum increase of about 6 inches in 60 feet or nearly one per cent. An exposure of two or three minutes to the sun lengthens the focus by about an inch and, as, at the times of best definition, the angle of incidence on the coelostat mirror is large, the astigmatism thereby produced will affect the definition, the amount of course depending on the change in focus and the angle of incidence. Professor Hale has shown that the convexity is due to an actual bending of the mirror and has suggested as a remedy making the mirror nearly as thick as its diameter. The effect in practice is kept at a minimum by shading the mirror by a canvas screen, except during the times when a plate is being exposed or the image is being focussed, and also by blowing air on the surface by electric fans. The silvering of the mirrors is also frequently renewed, as a fresh coating absorbs much less heat than one which has become oxidized or tarnished. The change of focus during an exposure on the spectroheliograph, which may last for two or three minutes or in some cases with iron and hydrogen lines considerably longer, is compensated for as much as possible by setting the slit midway between the focus at the beginning and the estimated position of focus at the end.

The process of making a set of photographs is as follows. The coelostat house proper is rolled back on its wheels, its roof moving over the roof of the telescope house, until the coelostat mirror is fully uncovered to the sun. The coelostat and secondary mirrors are uncovered and placed in such position as to send a full beam to the concave mirror situated 100 feet south, which is then uncovered and the image focussed on the photoheliograph shutter, the concave mirror being moved backwards and forwards on a track by an assistant. The coelostat mirror is shaded and a plate placed behind the shutter, which consists of a narrow adjustable slit in a thin piece of board. The mirror is uncovered and, after the focus has been again examined, the

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concave is moved the correct distance to bring the focal plane on the sensitive surface and is diaphragmed to about 3 inches. The release of the shutter allows a spring to draw the narrow slit rapidly across the plate. Process plates are used on account of their greater contact.

The whole photoheliograph is moved out of the way and the sun's image is refocussed on the slit of the spectroheliograph. This instrument is of a very ingenious and yet simple design and works to perfection. In essence, it consists of a slit spectrograph of two prisms, the deviation being made 180° by means of an adjustable mirror. In place of the observing telescope is a second slit in the focal plane of the camera lens, and by moving the mirror any line of the spectrum may be thrown on this slit. The sun's image is focussed on the first slit and a plate placed almost in contact with the second slit. The plate and the solar image are stationary while the spectroscope is moved smoothly and uniformly past them. Thus, as any width of any line may be transmitted through the second slit, the solar image is reproduced on the plate in the light of the particular element calcium, hydrogen, or iron, which is set on the second slit. The greater part of the weight of the moving parts is counterbalanced by mercury floatation, while the balance of the thrust is taken by steel balls running in grooves. A screw motion driven by an electric motor moves the carriage containing slits, prisms, mirror and lenses smoothly and uniformly at any desired rate across the sun's image. For the calcium line about $1\frac{1}{2}$ minutes exposure is required, for iron $\lambda 4045$ and hydrogen H_δ about 3 times as long.

A plate is taken for orientation by making successive exposures on the same plate, the solar image being allowed to drift 30 to 40 seconds between exposures. The second slit is then set on the centre of the H line, this being effected by using the electric arc which gives the H and K lines very strongly. Two plates are made of this region, one of iron $\lambda 4045$ and one of H_δ , $\lambda 4102$. This series of exposures is repeated on every morning and evening that the conditions permit of satisfactory results.

The spectrograph for making spot spectra, and for making adjacent spectra of opposite limbs of the sun for determination of the solar rotation, is of the Littrow form, one lens of 18 feet focal length acting as both collimator and camera, the plane grating of 4 inches aperture being so inclined as to diffract the light back upon its own path to the camera, which is placed 6 inches above the slit. The instrument is attached to the ceiling above the spectroheliograph and the sun's image can be thrown on the slit by simply tilting the concave. The spot spectrum is made through a diaphragm in front of the slit and the spectrum of the photosphere through a second diaphragm, which places a strip of solar spectrum on each side of the spot spectrum. Considerable linear dispersion is available with the focal length of 18 feet, but Professor Hale thinks it advisable to increase the focal length to 25 or 30 feet in order to get full photographic resolution.

The same spectrograph is used for obtaining the velocity of the sun's limbs to determine the rotation period. The opposite extremes of a solar diameter are reflected to adjacent positions on the slit by a pair of reflecting prisms at each side, and the double shift of each line from the advancing and retreating edge is measured. The limbs at the sun's equator are brought into juxtaposition on the slit by rotating this reflecting arrangement only, so that the edge of the limb is generally not tangent to, but inclined across the slit. A method by which not only the reflecting arrangement but the whole spectrograph could be rotated would be preferable, and also a means of obtaining accurately opposite limbs at the extremities of any desired parallel of latitude. As Mr. Adams, who had done most of the work with this instrument, was away, I was unable to obtain any information as to his success and the probable accuracy of the determination.

Mr. Ellerman very kindly showed me a large number of plates of spot spectra, and of those made with the spectroheliograph and photoheliograph, all of which were very interesting and instructive. The plates of spot spectra were very good, widened and weakened lines being well shown. A comparison of spectroheliograph plates taken

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at the same time in hydrogen and calcium light, and also in those taken at intervals of a few hours, proved very interesting. The comparison is very easily made in the stereo-comparator, which depends upon the principles of binocular vision for exhibiting slight and faint differences between two plates. On two plates of the calcium flocculi on the sun's surface, taken about ten hours apart, the globular surface of the sun and the different elevations of the calcium clouds were well shown, while, in a single eyepiece, arranged so that the two images could be seen in as rapid succession as desired, was substituted for the binocular arrangement in the comparator the forms of the flocculi taken at the same time in calcium, iron, and hydrogen could be readily compared.

After the Snow telescope house, the most interesting place on the mountain is the laboratory, which is a substantial cement building well equipped for spectrographic research. The spectroscope or spectrograph used is another grating instrument of the Littrow form, and is placed along an inner wall pointing towards the centre of an annular pier about 10 feet in diameter. Around this annulus are placed different means for producing emission spectra such as the induction coil with variable capacity and self induction, an accessory to the coil being an air pump for exhausting tubes; an ordinary arc lamp for obtaining metallic spectra; a synchronous arc for obtaining the alternating arc at any desired phase; Crew's rotating arc and a chamber for obtaining the arc under high pressure. Each one of these sources is directed towards a mirror at the centre of the annulus, which reflects the light to the spectroscope slit, while sunlight may be obtained from a heliostat outside. The usefulness of such an arrangement for spectroscopic investigation is self-evident as one can, in a moment, turn from spark to enhanced spark, to ordinary, synchronous, or rotating arc, to arc under pressure, or to sunlight without having to, as under ordinary conditions, erect each one of these in the optical axis of the spectroscope.

In the spectroscopic laboratory are also very complete dark and enlarging rooms for all branches of photographic work, and two or three small rooms in one of which is the stereo-comparator, and in another a machine for measuring spectra, for, although the measuring and computing is to be done in Pasadena, there is always demand in experimental work for preliminary or tentative measurements of plates.

The arrangement at Mt. Wilson as regards living is very simple, and, although it is objectionable as separating the observers from their families for the greater part of the time, has advantages on the score of cheapness. The colony there differs from that at Mt. Hamilton, which is essentially complete in itself, having houses, a school and forming a community with the director as chief, in being entirely of a bachelor character, the families of the observers living at Pasadena. This seems in many respects unsatisfactory, as the trip up and down the mountain is quite an undertaking, occupying about five hours in the ascent and three to four hours in the descent, but it is probably the best plan that can be evolved at present.

I can not close this description of the Solar observatory without expressing my gratitude to Professor Hale and every member of his staff whom I met, for their uniform kindness and willingness to assist me in every particular possible during my visit. It is not necessary to speak of the enthusiasm with which the work is carried on, nor of the esteem and respect each member entertains for the director, who, with his highly specialized qualifications for the work in hand, is at the same time most widely read and broad in his interests. He impressed me not only as one of the most able men I have ever met in his specialty, but he is also charming for his geniality and kindness, while his method for the management of the great work he has undertaken can not be excelled. The few days spent at Pasadena and on the summit were not only very pleasant ones, but the insight obtained into the methods of carrying on the work will be of the greatest use to us when we commence our own work in solar research.

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Lowell Observatory.

Flagstaff, which is a place of some 2,000 inhabitants on the elevated plateau of Arizona, was reached on Sunday, September 9th, about 3 p.m. The observatory is situated on a hill half a mile to the west of the town, about 300 feet high, the altitude of the observatory being about 7,200 feet. I walked up to it and had a short talk with Mr. Slipher, who has charge of the spectrographic work, and arranged to spend the following day and night at the observatory. He was very kind in showing me the spectra he has obtained, in explaining to me his method of working and the class of work he is at present engaged in. The quality of his star spectra does not differ essentially from those obtained with our Brashear spectroscope, his short exposure ones being slightly less sharp and his long exposure ones sharper than ours. In the Lowell spectrograph the flexure is probably much less and the temperature control closer than in our instrument, and this will explain the better quality of spectrum obtained of the fainter stars.

He is considering the advisability of obtaining a new single prism instrument for stars of the first type with broad lines. High dispersion is worse than useless for such stars, as it makes the already broad lines so wide and diffuse and so weakens the contrast as to render them hardly recognizable, let alone measurable. For this purpose he wishes to obtain a perfectly homogeneous prism with as slight absorption as possible, especially in the violet, in order to obtain the largest possible number of the hydrogen series. He showed me many peculiar spectra of the first type in some of which even the H lines were barely recognizable. Some of these showed traces of a second spectrum, and one or two were very complex and peculiar. He seems to be getting excellent results, but like every one else engaged in radial velocity work is far behind in his measurement and reduction. He uses the method of reduction employed by Frost and Adams, which is practically the same as that employed here until very recently, while his measurements are made on a Gaertner microscope essentially the same as that employed at the Yerkes except that it has a position circle on the microscope head to determine the inclination of the lines. This position circle is used for determining the rotation period of the planets where, if the slit is set parallel to the equator, the approach of one limb and the recession of the other will cause the lines to be inclined. From a measurement of this inclination the velocity and consequently the rotation period can be obtained.

Much work on the spectra of the planets has been done following out the well known purpose of the observatory and some beautiful planetary spectra have been secured. I was much interested in a magnificent spectrum of Saturn and his rings, in which the different angular velocity of the inner and outer parts of the ring first demonstrated by Keeler at Allegheny was very clearly shown.

Mr. Lampland, who has made such success in the photography of the planets especially of Mars, was also very kind in showing me some fine examples of his work. Any one with experience in that line knows that the photographing of such objects is a very difficult matter owing to the amount of magnification of the image in the prime focus necessary and to the corresponding magnification of the inevitable atmospheric tremor, and that it is only under exceptional conditions of steadiness that any success is possible. His success is possibly partly due to his perseverance and to exposing in rapid succession, thus taking advantage of the frequently very short intervals of really first class seeing.

On Monday night I had the privilege of being with Messrs. Slipher and Lampland at work. The air at Flagstaff owing to the dryness and high altitude is exceptionally transparent and the naked eye view of the heavens is very brilliant. The quality of the seeing, however, was not to my mind equal to that on Mt. Hamilton, and this I judge principally by the estimate formed by the observers themselves of the steadiness using the same scale, calling perfect seeing 5, seeing 3 at the Lick was much steadier than seeing 3 at the Lowell observatory. The same thing was true in regard to the blue image used in stellar spectroscopy. Such image seemed much larger and

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more unsteady at the Lowell than at the Lick observatory. This may be due in part at any rate to a difference in the performance of the correcting lenses. It was indeed the great difference in the character of this blue image as seen at the different observatories that led me to investigate the character of the image given by correcting lenses.

The Yerkes Observatory.

Leaving Flagstaff on Tuesday, September 11, via the Santa Fe system, Chicago should have been reached on Thursday, but owing to a derailment ahead of the train we were nearly twenty-four hours late and I was consequently a day later in reaching Williams Bay than I had anticipated. However, Professor Barnard, who had most kindly invited me to stay at his house during my visit, met me at the station and took me up the lake to his house which is very prettily situated close to the observatory and overlooking lake Geneva. I was very kindly received by Mrs. Barnard, and most hospitably entertained during my stay. After dinner Professor Barnard took me to the observatory and introduced me to the members of the staff, and I received here from all, the same kindly treatment as was accorded me wherever I visited.

The Yerkes observatory is prettily situated on lake Geneva, a well known Chicago summer resort, about 90 miles from the city. It is a handsome building of white terra cotta and forms a very effective picture. The grounds, however, are entirely uncared for, and the appearance would be much improved by suitable landscape gardening. The building is one storey, of considerable length, with the large 90-foot dome at one end and two of about 30 feet diameter each at the other. The offices, library, &c., are on both sides of the central corridor, connecting the large with the two smaller domes. In the basement below are instrument and optical shops, the spectroscopic laboratory, &c.

In the 90-foot dome is the 40-inch refractor with its various attachments and with every convenience for facilitating as far as possible the handling of this massive instrument. At the Lick observatory the floor and dome are moved by water power, but here they are actuated by electric motors, whose controllers are placed close to the south side of the pier. By these means the movements are made quite rapidly and easily and are under complete control. The telescope itself, though much heavier than the 36-inch, can be moved from the eye end by the hand, though not very easily, and can be clamped and moved in slow motion by hand wheels, as in smaller telescopes. However, electric motors in the clock room inside the pier allow the telescope to be turned rapidly in right ascension and declination. Four ropes running vertically at the south side of the column serve to control these motors and by pulling down on one or other, move the telescope quickly east or west around the polar axis and north or south around the declination axis. Thus the telescope can be quickly and easily moved to any desired position or reversed. Besides these motors, another automatically winds the clock when run down, and two small ones placed on the tube and declination axis respectively, which gear into the slow motion screws in declination and right ascension, give a very slow motion in either co-ordinate by simply turning a switch at the eye end.

The 40-inch, unlike the Lick, is used both day and night, in the day time for solar work, photoheliograph and spectroheliograph pictures, and at night for stellar observations. The programme is arranged to have the equatorial in use every fine night from dark to dawn. Both here and at the Lick observatory the work is so divided that, as a rule, one person does not use the instrument for a longer period than one-half the night, being relieved at midnight by a second man. This is an admirable plan, where it can be followed, as it allows the telescope to be worked to its full capacity without overtaxing the observer's strength.

The 40-inch is used on two nights a week for spectrographic (radial velocity) work, two nights for Burnham's double star work, one and one-half nights for general

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micrometric work by Barnard, and the other one and one-half for photometric work and for making photographs to be used in the determination of stellar parallax. The spectrograph used has been fully described by Frost in the *Astrophysical Journal*, and is especially distinguished for its very massive and rigid construction, and for the large size of its prisms. It is only with telescopes so large and heavy as the Yerkes that such a heavy instrument could be used and some of its parts are to my notion unnecessarily heavy. Part of the extra weight is necessary on account of the large prisms, which are of sufficient size to transmit a pencil of 51^{mm} aperture. As recounted by Prof. Frost in his description of the instrument, considerable difficulty was experienced in obtaining homogeneous prisms and even those now in use, although a great improvement on the first ones, are not entirely free from action on the light and the resulting definition is not as good with full aperture as when a diaphragm of half the aperture is inserted. Professor Frost is of the opinion that, if he were making a new spectrograph, he would use considerably smaller prisms to transmit a pencil of about 30^{mm} diameter. The advantage of the large prisms lies in the wider slit that may be used for the same purity of spectrum, an advantage that, in the case of telescopes of such great focal length as the Yerkes, should not lightly be foregone.

The spectrograph, as described, was only adapted to use three prisms, but has since that date been modified to permit the use of a single prism, and I believe most of the work now done is with this dispersion. Prof. Frost tells me that perceptible flexure is entirely absent in the Bruce instrument, and that I can readily believe owing to its massive construction which is in this respect an advantage. Also, owing to the great weight of the spectroheliograph the change from one instrument to another is accomplished with little change in the balancing, but when the change from either of these attachments to the micrometer or other visual appliance is required, or vice versa, some 500 pounds of counter weights have to be handled, making the operation somewhat laborious.

The spectrograph is enclosed within a double walled aluminum case and is heated by coils of German silver. The current is turned on and off these coils by hand as required. The case is not fitted with an automatic arrangement for controlling the temperature, as there are always two observers in the room when in use, and one can attend to the temperature while the other guides. But in my opinion a good automatic control is preferable, as likely to maintain the temperature much more constant, which is extremely important for accurate results. The image given by the 40-inch with correcting lens for photographic light does not seem nearly so good as is obtained at the Lick. There is much diffuse light around the central image and this is probably the reason why the exposure times required at Yerkes are about double those at Lick. The difficulty probably lies in the correcting lens as the object glass visually gives excellent definition.

Professor Frost has two or three different lines of work with the spectrograph under way. The chief one is the determination of the velocities of those Orion type stars within the limits of the instrument, which is about the fifth mag. with a dispersion of 3 prisms and about the seventh with a single prism. He is also engaged in determining the velocity curves and comparing them with the light curves of some Algol variables and finally has a list of Hydrogen or first type stars ready to be observed. But like every other worker in the line of sight, he has accumulated a large number of plates ahead of the measurement and reduction, and complains of the impossibility of getting assistance in this work. He is also endeavouring to obtain velocity curves for some of the spectroscopic binaries discovered at Yerkes.

During the period of my visit, he was engaged on the orbit of β Cephei the spectroscopic binary of a period of only about four hours, which has since been discussed by him in the *Astrophysical Journal*. He very kindly showed me many interesting spectrograms he had obtained and was most kind and helpful in many ways. One of the most pleasant parts of my trip was the privilege of talking over with him and

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others engaged in the same line of work as myself, the small details of the work and in comparing experiences, and many helpful ideas were obtained in this way.

I spent one evening with him in the dome while he was making spectrograms, and saw the ease with which by the slow motion motors, the large telescope is guided. The motion with these motors is so gradual and uniform that no vibration is set up and the guiding can be very accurately done. I found it difficult, owing to the amount of diffuse light around the image to guide, but have no doubt a little experience would soon render it easier. The observer and the engineer in charge of the 40-inch, take turns at the guiding, while the other attends to the rising floor and dome motors and looks after the temperature of the outer case.

Professor Burnham, unfortunately, I did not meet, as he only comes to the observatory on Wednesdays and Thursdays, and uses the telescope on these two nights in micrometer measure of double stars in preparation for his great general catalogue of double stars, soon to be issued by the Carnegie Institution of Washington. Professor Frost told me that he was a wonderful observer, making as many as 60 measures in a night.

Professor Barnard uses the 40-inch on one and a half nights per week in micrometric measurements of the positions of the satellites of Jupiter, Saturn, Uranus and Neptune, in a triangulation of the principal star clusters and in several smaller miscellaneous pieces of work. I was with him a couple of nights while he was observing with the 40-inch, and I could not help but admire the systematic way in which he set about his work, the quickness with which the measures were made and recorded, and the way in which the instrument was handled to obtain the maximum amount of work. He took the greatest pleasure in showing me objects which would exhibit the great power of the object glass, but owing to the invariably poor seeing we were unable to get a fair test of its capabilities.

Mr. Parkhurst, during the time of my visit, was measuring the brightness of the satellites of Saturn, on the 40-inch, with one of the Harvard type of comparison wedge photometers, in which an artificial star is brought to equal intensity with the real star by means of a photographic wedge. The 40-inch is used one night a week for photometric work by Mr. Parkhurst, who also uses the 24-inch reflector and the 12-inch refractor for the same purpose.

Mr. Jordan, for half a night per week, makes photographs of a narrow zone, near the equator on plates 8 x 10 in size, guiding by means of a double slide plate carrier and a guiding eyepiece at the edge of the field. These plates which are exposed for about an hour each to get stars of the 12th or 13th mag. are sent to Kapteyn, who is having them measured in accordance with a regular scheme for the determination of stellar parallax.

Mr. Fox, who had just returned from Potsdam, after a year spent with Hartmann, has charge of the solar work at the Yerkes, and makes a daily photo and spectroheliogram of the sun's surface. He is at present measuring the spectroheliograph plates in an attempt to determine the period of the solar rotation. The position of the spots and flocculi are determined by the projection of the negative on a globe ruled in degrees. The negative is first reduced to about 2 inches in diameter, and this is strongly illuminated by an arc lamp and condensing lens and its image is projected on the globe by an objective of 12-inch diameter, and the same focus as the 40-inch objective. In this way the photograph of the sun is projected upon the globe in the same way as if it were the actual image, and its pole can be placed at the corresponding position of the sun's pole and the position of spots or flocculi estimated to tenths of degrees. It is, however, questionable whether as much accuracy can be obtained by this method as by measuring in polar coordinates and reducing, but it is certain that the former takes only a small fraction of the time.

The spectroheliograph works on a similar principle to that at Mount Wilson, but, owing to the fact that it is attached to a moving telescope, it cannot be used in exactly the same way, by moving the instrument as a whole across the fixed solar image and

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in front of a fixed photographic plate. The spectroheliograph, which is a heavy piece of apparatus, weighing some 700 pounds, is attached rigidly to the telescope tube, and after the instrument is adjusted, the telescope clock carrying tube and spectroheliograph at the solar rate, the sun's image is made to drift across the slit, which is set east and west, by the slow motion electric motor in declination. A small shaft coming down the tube communicates a similar motion to the sliding plate holder and plate, which is moved at the same rate across the second slit as the sun moves across the first slit. The instrument, though from its nature not so simple and efficient as the Mt. Wilson spectroheliograph, gives excellent results. The photoheliograph is simply a plate holder, having in front a sliding shutter containing a narrow slit placed so that the plate is in the prime focus of the 40-inch. Process plates are used, and the objective is diaphragmed to about $2\frac{1}{2}$ inches aperture, the exposure being given by a slit about $\frac{1}{20}$ -inch wide, moved across the plate by two strong springs. Here also very good definition is secured, though the colour curve of the 40-inch objective at the part of the spectrum for which process plates are sensitive is very steep, and the difference in focus for the extreme limits to which the plate is sensitive must be in the neighbourhood of 3 inches. However, stopping down the objective must so diminish the angle of the cones of light that the resulting aberration must be within the limits of visibility so far as photographic resolution is concerned; indeed a simple calculation will show that the lateral aberration for a longitudinal aberration of $1\frac{1}{2}$ inches would be about $\frac{1}{200}$ -inch, and this is not of much account in ordinary solar definition. As, however, only the extreme limits to which the plate is sensitive, $\lambda 4600$ to $\lambda 4000$, would have that amount of aberration, and as the sensitiveness of the plate at these limits is much less than at the centre, the resulting aberration would probably not exceed one-half of that stated above, and would certainly not be in evidence in solar definition.

Considerable time is required to change from spectrograph or micrometer to spectroheliograph, and the whole operation of making the change and exposing the plates occupies about an hour. I was with Mr. Fox while this operation was being carried through, and was much interested in watching the various steps of the process. The changing from one attachment to another is much facilitated by the special carriages holding each instrument which can be wheeled up to the telescope, placed in a certain hour angle and declination with the rising floor in its highest position, and readily attached to the tube. Every device possible for minimizing the labour of changing and adjusting has been adopted, rendering the process quite safe and easy. After the change has been made and the tube rebalanced, the declination slow motion motor is attached to the plate holder carriage by a shaft running down the tube, which is easily connected to another shaft on the spectroheliograph, which by suitable gearing drives the plate at the correct speed.

Mr. Fox kindly showed me many spectroheliograph plates, some of which were not excelled by any on Mt. Wilson, but taken as a whole the latter are probably superior. Some very interesting examples of calcium flocculi were seen, and the method of obtaining the solar rotation period from some of the more persistent of these flocculi, which maintain their form and position for a longer time than sun spots, was explained. Mr. Fox also gave me an account of some of his work with Dr. Hartmann, of whom he speaks in the highest terms, and I was particularly interested when he spoke of Hartmann's new spectrocomparator, and the new type of spectrograph camera lens.

The two-foot reflector with which Ritchey made his beautiful nebula pictures is in the east dome and is at present being used chiefly for photographic photometry by Parkhurst, the determination of stellar light intensity by the diameter of the photographed image. Parkhurst is obtaining excellent results by this method, but it seems to be applicable chiefly to reflector images. He is at present working on a new method of photometry by the measurement of the density of extra-focal discs. In this method a photographic doublet objective is used and the plate moved a few millimetres out of

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focus. The result is that the star light is spread out into a disc and the density of this disc will depend on its diameter, the length of exposure, and the brightness of the star. Here, however, there may be difficulties as regards the behaviour of the photographic plate to light of different intensity. The relation between the light action and the resulting capacity is not a simple one, and what is worse for this case, is not even a constant one, so that before satisfactory results can be obtained considerable photographic investigation must be carefully carried out. Mr. Parkhurst has a Hartmann microphotometer for measuring the opacities of these discs, but owing to faulty centering of the Luminer Brodhum cube there is overlapping of the images and consequent loss of accuracy in its use.

I had a long talk with Prof. Barnard over the stellar photographic lens question, and I was also much interested in seeing some of his magnificent results with the Bruce telescope, obtained at Mt. Wilson last year. As pictures of the Milky Way, they are unexcelled and he is now hoping to get satisfactory heliogravures made for reproduction. In the Bruce telescope he has found a very satisfactory instrument, and one very convenient in use. It consists of three photographic cameras of 10-inch, 6-inch and 2½-inch aperture, respectively, attached to a 5-inch visual guiding telescope and equatorially mounted on a bent column to allow of passing the meridian at high altitudes without reversing. The field given by his objectives is not as large as that given by our Brashear 8-inch, but the definition at the centre is perhaps a little better. I had taken with me three negatives by our 8-inch lens of an hour's exposure each, with foci differing by ½ mm, to see whether Barnard was still of the opinion he had previously expressed, viz.: that I had been using the lens with the plate too far within the focus. However, when he saw the negatives, he admitted that the focus was correct, but that the large star images at the centre were due to a penumbra around the image that only showed with fairly bright stars, with faint stars it was not visible, and with those very bright it had become fully exposed, and thus much increased the diameter of the image. Such a penumbra is not visible in the images given by his objectives and the extra wide field in ours is probably obtained at the expense of residual chromatic aberration. In talking with Dr. Brashear and Mr. McDowell on this question, they stated they were certain it was not spherical aberration and they could only explain it as due to chromatic effect. I left the plates with them to show to Hastings to try and determine the cause.

Prof. Barnard thinks that this penumbra should be got rid of, even at the expense of diminishing the field, but I can not say that I entirely agree with him. It seems to me that it depends upon the purpose for which the lens is to be used. For pictorial work and for star positions the wide field seems to be an advantage, though the penumbra from the spreading of the light will cause the loss of the fainter stars. Hence if the purpose is to get a limited field of the faintest possible objects, then this resultant aberration should be cured, but if it can only be cured with sacrifice of field, it seems to be preferable for most purposes to let it remain as it is, especially as the seeing here is rarely transparent or steady enough to attempt very faint objects.

The Bruce telescope is mounted in a small building in front of the observatory, and quite close to Professor Barnard's house, and every fine night that this indefatigable worker is not using the 40-inch, he is at work with the Bruce, photographing parts of the sky where he has not as yet obtained photographs which he deems wholly satisfactory. Another branch of photographic activity he follows energetically whenever opportunity offers, is the photographing of comets, and he has obtained some beautiful negatives of the more conspicuous of the recent comets. He suggested this as a useful application of our photographic doublet, and advised it being mounted separately with a guiding telescope in a similar manner to the Bruce.

I also had a talk with Mr. Wallace, the photographer at Yerkes, who is carrying on two or three lines of photographic investigation. The most important of these is the relation of the temperature during exposure on the sensitiveness of the photographic film. He was led to this investigation by the experience of Professor Frost

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in spectrographic work, who found that the exposure time in zero weather was much less, nearly 50 per cent, than in summer, although possibly part of this may be due to the greater transparency of the air in winter. Wallace claims to have definitely proved that plates are much more sensitive to light action at a low than at a high temperature. This seems in direct contradiction to the general law of increase of chemical action with increase of temperature, which of course holds when development takes place. So far as Wallace's experiments have gone, however, they seem to substantiate his theory, and he was only waiting for winter weather to finally complete his tests.

He is much interested in the tests of colour sensitiveness of photographic plates, and is an ardent advocate for this purpose of the transmission grating, as opposed to the prism, on account of the normal spectrum given by the former. Indeed he is so strong a believer in the advantage to be gained in accuracy and the standardization of results by the use of the former that he is making, and presenting to everyone likely to use such an article, replicas of one of Rowland's plane gratings, which give very fine spectra.

He has devised a very simple spectrograph for testing plates in which this grating is used and in which at any time a plate may be tested without any setting up or adjusting of apparatus. His laboratory is well supplied with sensitometers, photometers and other appliances for investigations in photography, and such work is a very useful adjunct of a modern observatory, in which a large part of the work depends upon the application of photography.

The workshop and optical shops in the basement of the observatory are well equipped for instrument work, but are evidently not so much used since the departure of Professor Ritchey for Pasadena. The spectroscopic laboratory, also in the basement, is well suited for general spectroscopic investigations, although naturally not so well arranged and equipped as the one on Mt. Wilson, which is a further development by Prof. Hale, and a model of convenience.

The spirit of the staff of the Yerkes observatory, like that of Mt. Hamilton and Mt. Wilson, is one of enthusiastic devotion to their chosen profession. Every member works enthusiastically along his own line of research and there seems to be a unity of interest among all, which is a tribute to the kindness and tact of the director. The three large observatories I visited, the Lick, Solar and Yerkes, seem alike fortunate in their directors, as well as in the personnel of their staffs, and their success and the amount and quality of work done is in a large measure due to the harmony and good feeling which exist in all three.

My thanks are due to every member of the staff at Yerkes for their efforts to make my visit a pleasant and profitable one, but especially to the director and to Professor Barnard. To Professor Frost for the assistance and advice in spectrographic matters and to the insight into their methods of work he so willingly afforded me, and to Professor and Mrs. Barnard for their hospitality and efforts to make my visit a pleasant one.

Allegheny Observatory.

Leaving Yerkes on Wednesday morning, September 19, the afternoon was spent in Chicago, and I left in the evening for Pittsburg, reaching there the next morning. I immediately went to the Brashear instrument works and renewed my acquaintance with Dr. Brashear with the greatest pleasure. The morning was profitably spent in discussing various optical problems of interest with him and Mr. McDowell, especially those relating to the field of the photographic doublet they made for us and the optical parts of our new spectrograph. I saw the prisms and the new single material camera lens and pointed out to Mr. McDowell the necessity for enlarging the rear element to prevent cutting off part of the pencil, which contention has since been justified. I spent considerable time during my stay in Allegheny in their shop, as it was considerably more interesting than the observatory, which has as yet hardly made

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more than a start at astronomical work. They were not at the time of my visit engaged in any large optical work, but were expecting orders for several large objectives and mirrors. Dr. Brashear, whom I found the same whole-hearted, kindly gentleman and his good wife, did their utmost to make my visit to Allegheny a pleasant one. Dr. Brashear took me up to the observatory and introduced me to the director, Dr. Schlesinger and to Dr. Curtiss, and with them we inspected the various interesting features of the observatory. Since my previous visit to Allegheny, the Keeler Memorial telescope has been completed and mounted, and spectrographic work is now being carried on by Dr. Curtiss. The whole telescope, mirror and mounting, was made by the Brashear Company, the design of the mounting being due to Wadsworth, and is, so far as I can learn, a very satisfactory instrument. It is arranged in the cassegrainian form for spectrographic work, the spectrograph being placed below the mirror. The spectrograph is a single prism instrument of Curtiss' design, constructed by Brashear, the form being somewhat similar to the remounted Mills, except that it is of a triangular instead of oblong form, with angles of approximately 120° , 30° and 30° , the prism being near the obtuse angle, and the two adjacent sides of the box which, in this case, is of brass, instead of steel, form the collimator and camera tubes respectively. The instrument is attached to the reflector similarly to the new Mills at two points of support, one near the centre of gravity of this box consisting of a pivoted axis passing through the box, its ends being moveable in guides for placing the slit in the star focus, the other consisting of a cylindrical bearing concentric with and forming the slit end of the collimator tube moving in a collar attached to the same truss which carries the guides above mentioned. Thus the spectrograph proper is self-contained and held without any constraint by its supports. This form of instrument should be the least liable to flexure difficulties, but how it behaves in that respect I do not know. It had only been in operation a short time, and owing to their having no measuring microscope, no plates had been measured, so no idea of its effectiveness and accuracy could be obtained.

The form of mounting there adopted, which admits of changing the position of the slit by the movement of the whole spectrograph rather than of the collimator tube only, is necessary for reflectors owing to the rapidly changing positions of the star focus with change of temperature of the mirrors, this frequently requiring the focus to be changed during an exposure. If the collimator tube only were moved there would be great danger of some systematic displacement of the lines, but this cannot occur in the movement of the instrument as a whole.

The method of attaching the spectrograph to the reflector quite close to the mirror is convenient, on account of the slight change of position necessary for the observer in guiding. It makes no difference, of course, in the flexure, which depends only on the angular motion and not on the motion of translation of the spectrograph. The spectrograph, when in use, has a close-fitting outside wooden case, lined with felt, and the spectrograph itself is covered with felt on the outside to smooth down irregular temperature changes. German silver wire is coiled around on the felt on the inside of the case, and the current is turned on and off these coils by an electric contact thermometer in conjunction with a relay. Dr. Curtiss informed me that he preferred to have the heating coils uniformly distributed over the case rather than trust to convection currents to distribute the heat from coils in one part of the case, and in this contention our own experience with the heating of the new spectrograph bears him out.

The dome is arranged to be turned by electric motor, and this much diminishes the labour of the observer, as the hand motion does not work very easily. Besides this reflector, only the 13-inch refractor belonging to the original Allegheny observatory is mounted. While I was there the steel work of the dome for the 30-inch refractor was being erected. Dr. Brashear, to whom the new Allegheny observatory owes its existence, as it is by his efforts and influence that the money was obtained to build and equip it, had just secured sufficient subscriptions to complete the dome and

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equatorial mounting for the 30-inch refractor, and had hoped to have it in operation in 1906. When, however, they started to polish and figure the crown and flint discs, which had been ground some time previously, it was found that the glass was not sufficiently homogeneous to make a perfect objective, and they were obliged to reject them after putting \$5,000 worth of work on them. Besides this severe loss to the Brashear Company, it necessitated delay until other discs could be obtained, as frequently a long time is required to obtain perfect discs of so large a diameter as 30 inches.

Wadsworth's plans for the new Allegheny observatory were very ambitious, and lay chiefly along the line of solar research, which he claimed, and rightly, to be the work most suited to the smoky atmosphere of Allegheny county. It is also, I believe, the purpose of Dr. Schlesinger to undertake solar investigations as soon as the necessary equipment is obtained. In the meantime radial velocity work with the Keeler Memorial and the one prism spectrograph is being carried on as continuously as the weather will permit.

In Drs. Schlesinger and Curtiss, the observatory possesses two able men, and should maintain the high reputation already attained under Langley and Keeler. The kindness extended me by these two gentlemen made my visit to the observatory a very pleasant one, and the opportunity of discussing with Dr. Curtiss the spectrographic problems we had previously corresponded about, proved very helpful. I cannot, however, close this short description of my visit to Allegheny without expressing my appreciation of the kindness of Dr. Brashear and his family. For encouragement and helpfulness in the commencement of my astronomical work, I, like many other astronomers, already owed him more than it is possible to repay, but he always seems to be glad to add to such a debt, and my present visit was no exception to the rule.

Washington.

In Washington, which I reached on Sunday afternoon, September 23, I had three places to visit, the Bureau of Standards, the Naval observatory and the instrumental branch of the Coast and Geodetic Survey, and as my time was getting short, my visit to each was necessarily hurried.

The Bureau of Standards I found a most interesting place, and well worthy of a much longer visit than I could afford. As it is a comparatively new institution, I found it equipped with the very latest apparatus for measurements of the highest precision, installed in buildings especially adapted for their particular purposes.

The director of the bureau, Dr. S. W. Stratton, has succeeded in obtaining a most capable staff of physicists to carry on the work of the institution, and it promises to play a most important part in the scientific and technical activities of the United States. Its main purpose is, I presume, in accordance with its name, to furnish standard measurements of all physical quantities whenever required, either for scientific or commercial purposes, but with that, if we are to judge by the bulletin issued, is combined much original investigation of the highest rank.

I found on reaching the Bureau on Monday morning that the director, to whom I had a letter of introduction from Dr. Brashear, was not in the building, and I was conducted to Dr. Rosa, chief of the electrical branch, who, after showing and explaining the very complete electrical equipment for the measurement of resistance, capacity, self-induction, current, potential, &c., introduced me to Mr. Fischer, who has charge of the linear measurements and the standards of length.

This branch of the work, in which I was particularly interested from its application in the scales and screws of astronomical measuring engines, was thoroughly shown to me by Mr. Fischer who took great pains in making me acquainted with everything of interest. The various comparators employed for standardizing scales with the auxiliary apparatus for ensuring the highest accuracy of measurement were first shown, and these were followed by the unlocking of the vault containing the

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primary standards of length of the United States. Mr. Fischer's description and short history of each of these standards was very interesting, and one could easily spend a day at this branch of the Bureau's work alone. The underground passage in which steel tapes are standardized also possesses much of interest on account of its application to our surveying work.

The engineering department and the machine shop, which is completely equipped with separately motor-driven machine tools and where a number of machinists are continually employed, was next visited. Another interesting branch of the work was the photometric laboratory, where all the most recent apparatus for the measurement of light intensities is installed, and where much valuable research on photometric problems has been undertaken.

The spectroscopic laboratories under Drs. Nutting and Coblentz were also of much interest to me. Dr. Nutting, who has done considerable work on line structure, kindly showed me the complex nature of some of the mercury lines by an echelon spectroscope. A useful wrinkle obtained here, was the employment of a special transformer, using ordinary alternating current for spark work, rather than an induction coil. Dr. Coblentz also showed me the spectroscope and appliances used in his work on infra-red emission and absorption spectra.

There were many other interesting departments in the bureau which I had not time to see, but which would be well worth an extended visit.

The Naval Observatory.

The Naval observatory is situated in the same section of the city as the Bureau of Standards, and is only a short walk from the latter. The group of buildings comprising the observatory is situated in extensive and well kept grounds, and the white buildings with the green surroundings make a pleasing picture.

The acting superintendent introduced me to Professor Skinner, who has charge of the work done with the equatorial telescopes, and he showed me around the buildings, introducing me to the various officers. The most interesting parts to me after the equatorials, were the time system and the meridian circle work. The time system is not so complete as ours, and although they possess a Riefler sidereal clock, the means for the maintenance of constant temperature are primitive compared with the complete system installed by Mr. Stewart. However, they propose to move the standard clocks into a separate building, where presumably, a better system of temperature control will be installed. They have many ingenious arrangements for the comparison of clocks and chronometers, which is in the Naval observatory, a very important part of the work. All the chronometers of the navy are rated here, and there are always a large number of chronometers under regulation.

The meridian circle work is an important part of the observational work, and a great deal of useful work in that line has been done at Washington. The meridian circles are placed in separate buildings of galvanized iron with azimuth marks about 100 yards distant. A travelling-wire micrometer for the Warner and Swasey meridian circle, also made by Warner and Swasey, had just been received, but had not yet been attached to the instrument. It looked a very workmanlike micrometer, and had for the contact wheel, instead of the usual hard rubber, one of glass with inserted platinum strips for the contacts.

The twenty-six inch equatorial is placed in a separate building, which, besides the dome, contains two or three offices for the observers. The object-glass by Clark, was originally provided with a mounting by the same firm, but this has since been changed to one of Warner and Swasey's, which, like all their mountings, is very convenient and efficient. The equatorial room has a rising floor moved by hydraulic pressure, and this is a necessary and most useful adjunct to a refractor of over 18 or 20 inches aperture. The work done with this telescope is, so far as I know, wholly micrometric and consists chiefly in measurements of the positions of satellites of

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the planets, of the minor planets and of comets. The smaller equatorial in the main building is used in the same line of work.

I had an opportunity of observing on Monday night, the character of the image given by the 26-inch and of the quality of the seeing at Washington. At every observatory I visited I had examined the appearance of the image within and without focus as well as at the focus. Theory calls for the appearance of the central disc and rings to be nearly the same outside as inside the focus. This is not the case with the Ottawa instrument, where the appearances within and without focus are somewhat different. However, an examination of every telescope I saw, the 36-inch Lick, the 24-inch Lowell, the 40-inch Yerkes, the 26-inch Naval and the 18-inch Flower, gave appearances almost identical or at any rate quite similar to that seen at Ottawa. A single exception was the 12-inch Clark objective at Mt. Hamilton, whose figure, according to Professor Campbell, is almost absolutely perfect, and which gave almost identical appearances within and without focus. Whether the zonal differences of focus, which presumably are the cause of this dissimilar appearance of the extra focal images, are sufficiently great to affect the quality of the image at the focus (so far as regards visibility and separating power) is questionable, for each one of the objectives mentioned above is considered to be of the very first quality, and they all have separating power up to their theoretical limit. Every one of these except the 18-inch Flower and the 15-inch Ottawa, which are by Brashear, are of Clark's figuring. The Brashear objectives, however, are by no means inferior, as Professor Eric Doolittle has been able to resolve double stars even under the poor atmospheric conditions of Philadelphia, of a separation considerably within the theoretical limit for an objective of 18 inches aperture. This shows that the figure must be so nearly perfect that the light is concentrated within the central disc, and into the central part of this disc in a manner equal to that called for by theory, and this would not be the case if the outstanding zonal aberrations were of any appreciable magnitude.

In this connection it may be of interest to recount an experience of Dr. Brashear in this line, which he told me along with a wealth of other experiences and incidents in connection with optical work. This was in connection with an objective of moderate aperture, I think about 6-inch, which he made for a certain purpose, and which required it to be of as perfect a figure as possible. The objective was completed and tested by Dr. Brashear and Mr. McDowell, and sent to the purchaser. The sending of course meant, as every one who has had an experience with Dr. Brashear's methods knows, that the objective was as good as could be made. However, a notification was shortly received that the objective would not answer, that the appearance of the extra focal images was not the same, and that the figure was therefore not perfect. To satisfy both themselves and the purchaser, they therefore figured a second objective of the same size as the first, so that this condition of similar extra focal images was satisfied, and sent this one to the purchaser, asking that the two be carefully compared and the best one kept. Shortly afterwards the one giving similar extra focal images was returned.

The old Clark mounting of the 26-inch, which had lain around the observatory for many years, has recently been utilized by Professor Peters of the staff, for the mounting of one of Brashear's photographic doublets for stellar photography. The method of driving of the old Clark has been ingeniously modified by using an electric motor for furnishing the driving power, the governor being used to control the action of this motor. Although it makes a curious looking mounting, I am told that it works very efficiently, and answers admirably for the purpose for which it is used, photographing the minor planets.

Coast and Geodetic Survey.

I had not much time to spare, but looked up Mr. Fischer, head of the instrument division of the survey, whom I had previously met in the inspection of boundary monuments. He very kindly showed me around the building, introducing me to the

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heads of the various branches, and I had a very pleasant time there. Although the instrument shop is by no means modern, and the machine tools look considerably out of date, they are able to turn out work of the highest class and in considerable quantity. The principal instrument they were then engaged on, was a machine for computing and predicting the tides. They are at present using a small one of a limited number of elements, the purpose of the machine, being of course, the synthesis of the components of the various causes that go to form the tide of any place. The residuals between the computed and actual values though not large, will probably be considerably diminished by taking account of further influences, and the machine is being made to use up to about three times as many elements as the old. Both the principle and the mechanism are rather complex and the workmanship required is of the very highest order, so that the machine will probably require some time to complete.

Mr. Fischer also showed me their instrument room, in which, however, at that time very few instruments were in stock, nearly all being in use in the field. Another interesting room was that containing the circular dividing engine, which is used for graduating the circles of their surveying instruments. Although not a new machine, it gives accurate graduations, and the details of its working were of much interest. I also obtained from Mr. Fischer many interesting items of instrument construction, such as the methods of working and use of invar, aluminum bronze and other special structural materials.

The Flower Observatory.

Philadelphia was reached on Tuesday evening about six o'clock, and after registering at the hotel, an attempt was made to locate the observatory. It was a curious indication of the relative importance of astronomy in the average business man, when I found it impossible, either at the hotel or anywhere in the neighbourhood, to find the location of the Flower observatory, one person indeed directed me to a horticultural pavilion in one of the parks in which was a tower for overlooking the city. The difficulty may be partly accounted for by the observatory being in one of the suburbs. I was finally driven to calling up, by telephone, a leading educational man, I forget his name, and obtaining the desired information from him.

I reached the observatory about nine o'clock and, thanks partly to the kindness of Dr. Brashear, who had written of my coming, I was most kindly received by the family of Professor Chas. L. Doolittle, the director, who was himself observing with his reflex zenith telescope. Dr. Eric Doolittle, who came in shortly after I arrived, showed me the 18-inch equatorial of the Flower observatory, which he uses in measurement of the position angle and distance of double stars. He has already published two volumes of his measures, which are recognized as of high accuracy. He speaks in the highest terms of the quality of the 18-inch Brashear objective, and of the convenience of the Warner & Swasey mounting.

On the next day the director showed me the whole observatory, which, besides the equatorial and reflex zenith telescope, is fitted up for the use of the students of astronomy in the University of Pennsylvania, this observatory being the headquarters of the astronomical department. The work of the director in latitude determinations is well known, and his explanation of the special instrument he uses was of much interest. I spent the balance of the day in his library in the discussion of various astronomical matters of common interest, including the requirements of astronomical libraries.

In this connection, I may mention that at all the observatories I visited, I paid special attention to the libraries and obtained a good idea from them and from the various librarians of the character of the astronomical publications of which we are most in need, and these needed books and publications we are now obtaining as rapidly as possible. Undoubtedly the most complete astronomical library is at the Naval

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observatory, but the Lick has a good working library, and is considerably ahead of the Solar and Yerkes in that respect. It has, of course, been much longer established and, moreover, the funds at the Yerkes available for additions to the library are extremely limited.

I spent the short time I remained in Philadelphia very pleasantly at the Flower observatory, and I retain very pleasant memories of the kindly nature of my reception and entertainment.

Harvard College Observatory.

I reached Boston early on Thursday morning, and at once went to the Harvard observatory, where I was kindly received by Director Pickering, and shown the nature and examples of the work, especially along the photographic line, done there. As at the Lick and Yerkes, photographic reproductions (generally in the form of transparencies which are frequently enlarged) of the most characteristic negatives obtained in their work, are collected in a large room at the observatory. There are some very fine examples of almost every kind of astronomical photography in this collection, which is a most interesting and instructive one, and I spent considerable time in study and comparison of these examples. The director also showed me their store-room, containing thousands of star negatives, a complete history of the sky for several years back. It is not once only, but several times, that the usefulness of this collection has been proved as corroborative evidence of later discoveries and this usefulness will by no means diminish as time goes on. He then introduced me to Messrs. Wendell, Gerrish and King, who each, during my stay, were most kind in showing me everything of interest in their particular lines.

Besides the continuous photography of the sky which is being carried on by several equatorially mounted cameras on every clear night, the most important work carried on at Harvard is stellar photometry, and the quantity of this work turned out is wonderful. The director and Professors Bailey and Wendell are those chiefly engaged in it, and many different types of photometers have been and are employed in this work.

The director uses a comparison wedge photometer, the original form of those sent out to different observatories including the Lick and Yerkes and still makes many observations. The photometer is used in connection with an equatorially mounted mirror, which reflects the starlight in a horizontal direction to the photometer eyepiece placed at a convenient height for the observer seated in an ordinary chair. An assistant finds and sets on the stars which according to a well arranged programme are close together in the sky, and records the settings of the wedge while the observer has only to make the settings. In this way measurements are made very quickly and a great deal of work can be done in a night. Professor Pickering told me that he had completed his millionth setting some time back, which is quite a record. The quickness and accuracy with which the settings were made would have been more surprising if one had not reflected that upwards of a million previous observations had given the observer considerable practice and experience.

Professor Wendell, who took great pains in showing me his method of observation does his work with the equatorially mounted refractor, of 15 inches aperture, which is mounted in the main observatory building. Instead of, as the director, comparing the star with a variable artificial star he compares two stars close together in the sky by means of a polarizing photometer. In one form, which is used for stars very close together, a double image prism forms two images of each star, and the comparison is made by the equalizing of the intensity by the rotation of a Nicol prism. The other form of photometer follows exactly the same principle, except that stars at a much greater distance may be compared. A pair of prisms, whose distance may be varied at will, are inserted up the tube of the refractor, and these bring the two images close together in the eyepiece, where they are analyzed as before.

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Professor Wendell claims by this method to work to hundredths of a magnitude and its applicability to periodic variables is evident. The variable may always be compared with the same standard stars near it, and results strictly comparable with one another obtained. During the night I visited the observatory, he was measuring the intensity of some planetary satellites. He showed me the light curves of a number of variable stars, the intensities being determined by the method above described.

A third method of determining the magnitudes of stars is a photographic method in which the intensity of the star images on negatives are compared. By this method a great many variable stars in different star clusters have been discovered, and it promises to give very fruitful results. Not the diameters of the images are measured as in the method described by Parkhurst,* but their intensities are compared.

Mr. Willard P. Gerrish, to whom many of the ingenious and original mechanical devices employed on the telescopes are due, showed and explained to me the method of mounting the Common 60-inch reflector. The weight of the moving parts is counter-balanced by the upward thrust of water in which the large hollow polar axis turns. The image is sent by auxiliary mirrors upward along the prolongation of the axis into the eyepiece in the observing room. All movements of the telescope are by electric motors, and the measures of these motions, both in right ascension and declination, are communicated by endless tapes into the observing room where the graduations, which are placed on the tapes themselves, are read through windows in a board side by side, beside which are the switches for the setting motors. The arrangement is most ingenious and convenient, and will permit the required observations to be made with the maximum comfort of the observer, which is essential for the best results.

Mr. Gerrish also explained to me the contrivances by which the equatorial movements to which the various cameras are attached, are driven in synchronism with the sidereal clock. By altering the rate of the clock slightly and changing the adjustment of the axis, the following may be made to correct for refraction over a sufficient range to prevent any drift of the images on the plate. Many other interesting features of the observatory especially relating to the photographic side of the work were shown to me, but my whole visit was too short to obtain the greatest benefit from the many interesting features.

Conclusion.

Perhaps the most pleasant part of my visit to these observatories was the opportunity afforded of making the acquaintance of many of the most noted astronomers of the country. I can only say that I found every one willing in every way to give me help, advice, and the benefit of his experience. It would be, I am sure, impossible to meet in any profession a more companionable or genial class of men, and I carry with me the recollection of many pleasant hours of congenial intercourse. The actual practical benefits from such a trip can hardly be estimated, as one obtains therefrom confidence in himself and his work, knowledge of how best to attack the problems in hand, and the benefit of the experience of others in the smaller details of the work which are never published and which can only be obtained by actual contact and converse with the workers.

THE NEW SPECTROGRAPH.

As outlined in last year's report, the Brashear Universal spectroscope was modified to obtain satisfactory velocity determinations, until a modern spectrograph designed solely for that work could be constructed. My experience with the Brashear instrument had been of service in pointing out what features were desirable and

* Astrophysical Journal, Vol. XXIII, page 79.

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what undesirable in a spectroscope for purely radial velocity work, and I set myself the task of designing an instrument which could be used with both high and low dispersions and which, with the limitations of size and weight imposed by the size of the telescope, would be as efficient and accurate as possible.

In this task I did not scruple to avail myself of the most suitable parts of the designs of other spectrographs, and I am indebted especially to the various Brashear spectrographs, and to the Bruce spectrograph of the Yerkes observatory for many of the details of the instrument. Nevertheless, the groundwork of the design and many of the details are new and were developed from a consideration of the requirements, and from a knowledge, founded on experience, of the essential features in the design of a spectrograph for radial velocity work.

The requirements for this instrument may be briefly summarized as follows:—

1. The entire weight with temperature case and all attachments must not much exceed one hundred pounds, which limits the spectrograph proper to about fifty pounds.

2. To prevent as far as possible differential temperature effects, the spectrograph should be constructed of one material.

3. Facility of construction and smaller differential changes of focus with temperature, make brass and bronze preferable for this purpose.

4. The design of the instrument to be such that no direct bending stress shall be applied to any part, but all stresses due to its weight and attachments to a moving telescope, shall act in the direction of extending or compressing members of the framework.

5. The spectrograph to be so devised that it may be used with equal facility with one or three prisms and with linear dispersions of from 60 to 10 tenth-metres per millimetre.

6. The importance of constant temperature in spectrographic work requires an automatic thermostat arrangement for maintaining the temperature constant within 0.1° C.

Many other smaller matters looking towards the convenience and accuracy of its working might be mentioned, but such details will appear in the description.

The Optical Parts.

The angular aperture of the telescope, 1 to 15, determines that of the collimator whose length therefore depends on the aperture of its objective and of the prisms. In choosing the dimensions of the latter, I was guided by the experience of others. Frost found difficulty in obtaining homogeneous prisms with an effective aperture of 51^{mm} . The Mills spectrograph has prisms of 38^{mm} , while the Potsdam, Bonn, Pulkowa and Lowell have prisms of about 30^{mm} aperture. The use of large prisms permits a greater slit width for equal purity of spectrum, and, as my work with the correcting lens has shown, this is a decided advantage as regards exposure time. Considering this advantage on the one hand, and the greater absorption and greater weight of prisms and mechanical parts for large prisms, as well as the possibility of nonhomogeneity of the material on the other hand, the size decided upon was 35^{mm} , which fixed the length of collimator at 525^{mm} .

The glass chosen for prism material was the dense flint (O – 102) of the Jena Glass Works, which has been used on all recent spectrographs. It has high dispersion with remarkable transparency, and is probably the best prism material at present available. The dimensions of the prisms necessary to transmit the full pencil depend upon the angle of incidence, and this will in turn depend upon the deviation of the central ray and the index of refraction. The wave length of the central ray, that at minimum deviation, was chosen at $\lambda 4415$ as making the best compromise between the shortness of exposure required at $\lambda 4500$ with the better quality of the spectrum

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for measurement around H_γ . For mechanical reasons the total deviation of the three prisms was taken as 180° , and this required a prism angle of $63^\circ 45'$. For convenience of reference, the formulae required for obtaining the most suitable dimensions of the optical parts will be collected together here.

Let a = angle of prism.

δ = deviation.

a = aperture of collimator.

l = length of side of prism.

t_1 = thickness of refracting edge.

t_2 = thickness of base of prisms at limiting positions of the pencil.

i = angle of incidence.

r = angle of refraction.

μ = index of refraction.

f = focal length of camera.

θ and s = angle and linear distance between any two dispersed rays.

Then at minimum deviation

$$i = \frac{a + \delta}{2} \quad r = \frac{a}{2} \quad (1)$$

$$\sin \frac{a + \delta}{2} = \mu \sin \frac{a}{2} \quad (2)$$

$$l = a \sec \frac{a + \delta}{2} \quad (3)$$

$$t_2 - t_1 = 2 a \sin \frac{a}{2} \sec i \quad (4)$$

$$\frac{d\theta}{d\lambda} = \frac{d\theta}{d\mu} \frac{d\mu}{d\lambda} \quad (5)$$

$$\frac{d\theta}{d\mu} = 2 \sin \frac{a}{2} \sec i = \frac{2 \sin \frac{a}{2}}{\sqrt{1 - \mu^2 \sin^2 \frac{a}{2}}} = \frac{2}{\mu \tan i} \quad (6)$$

Using Hartmann's simple interpolation formula for the prismatic spectrum

$$\lambda = \lambda_0 + \frac{c}{\mu - \mu_0} \text{ where} \quad (7)$$

c , λ_0 and μ_0 are constants we get

$$\frac{d\mu}{d\lambda} = - \frac{c}{(\lambda - \lambda_0)^2} \quad (8)$$

The resolving power R and purity P , are obtained from the following relations:

$$R = (t_2 - t_1) \frac{d\mu}{d\lambda} = a \frac{d\theta}{d\lambda} \quad (9)$$

$$\frac{ds}{d\lambda} = f \frac{d\theta}{d\lambda} = f \frac{R}{a} = \frac{R}{\beta} \text{ where } \beta = \frac{f}{a} \quad (10)$$

$$P = \frac{\lambda}{d\psi + \lambda} R \quad (11)$$

where d = slit width and ψ = angular aperture of collimator.

These are all the formulae required for determining the optical constants of a prism spectrograph.

The constants of the particular melting of O-102, from which the prisms were figured, as furnished by the makers, are as follows:—

Wave Length.	Index of Refraction.
·00006563	1·6413
·00005893	1·6467
·00004862	1·6603

From these values applied in the interpolation formula above given, we obtain the three constants.

$\lambda_o = \cdot00002190.$
 $\mu_o = 1\cdot61146.$
 $c = 6\cdot115595.$

and from these constants the following indices of refraction and the $\frac{d\mu}{d\lambda}$ for each wave length were at once obtained by substitution in the above formulae (7) and (8).

Wave Length.	Index of Refraction.	$\frac{d\mu}{d\lambda}$
4862	1·6603	1829
4650	1·6667	2343
4415	1·6701	2636
4342	1·6721	2822
4102	1·6796	3490
4000	1·6833	3983

We have now sufficient data to calculate the required angles of the prisms, the length of their sides, and the resolving power. From formula (2) we obtain $\alpha = 63^\circ 44\cdot5'$ for a deviation for $\lambda 4415$ of 60° . The prisms, however, were made of an angle of approximately $63^\circ 50'$, and this gives a deviation of $60^\circ 10\cdot6'$, therefore, from (4) $t_2 - t_1 = 2 \times 3\cdot5 \sec 62^\circ 3' \sin 31^\circ 55'$.

$= 7\cdot89$ cms. for one prism.
 $= 23\cdot66$ cms. for three prisms.

The length of a side of the prism from (3)

$l = 3\cdot5 \sec 62^\circ 0\cdot3'.$
 $= 7\cdot46$ cms.

Owing to the dispersion and consequent spreading of the pencil, the three prisms were made with sides 75, 80, and 85^{mm} long, respectively, and of 40^{mm} high. While the separate prism for the single prism attachment was made of the same dimensions as the first one above.

In order to obtain any desired linear dispersion within the range of the instrument, three camera objectives of 525, 375, 250^{mm} focus, each of 45^{mm} aperture were ordered from the Brashear Company, but of these only the long focus one has yet been supplied. The problem of obtaining a satisfactory camera lens, one that will stand the critical test for definition and flatness of field required in spectrographic work, is a difficult one and involves greater difficulty for short focus, large angular aperture lenses than for long focus. But I will speak more particularly concerning camera lenses when I come to describe the tests of the instrument. Here it may be of interest to give the resolving power, the purity of the resulting spectrum, and the angular dispersion together with the linear dispersion for each camera when the three prisms are used.

These values were computed by formulae (9), (10) and (11) above.

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THREE PRISMS.

Wave length.	Resolving power.	PURITY OF SPECTRUM FOR SLIT WIDTHS IN MILLIMETRES.				LINEAR DISPERSION MILLIMETRES PER TENTH-METRE FOR CAMERA FOCUS.			
		·025	·0375	·05	·075	525	375	250	$\frac{d\theta}{d\lambda}$
4862	43260	9770	7130	5534	3843	15·4	21·6	32·3	25·5''
4570	55420	11885	8533	6656	4623	12·2	17·1	25·6	32·7''
4415	62350	13058	9358	7292	5059	10·7	15·0	22·5	36·7''
4340	66750	13790	9874	7690	5456	9·9	13·9	20·8	39·3''
4102	82550	16304	11634	9046	6259	8·1	11·3	17·0	48·6''
4000	94210	18234	12994	10093	6979	7·0	9·8	14·7	55·5''

As each of these values for a single prism is exactly one-third of the above, it is evident that at $H\gamma$ we may obtain values of the linear dispersion from about 10 to 60 tenth-metres per mm. It is also evident, taking for granted the relation between slit width and exposure time obtained in the investigation on the 'star image' given below, that there is need for an investigation into the most efficient form of spectrograph—one with low dispersion and long camera, or one with high dispersion and short camera. The same purity of spectrum may be obtained in the latter case with a slit upwards of three times as wide, and hence, even allowing a very large margin for the greater absorption and reflection, the exposure time should be much less. It is proposed, as soon as the two shorter focus cameras are obtained, to make a thorough investigation of the above problem with reference both to early and solar type spectra.

The Frame of the Spectrograph.

Considerable thought was bestowed on the design of the frame of the instrument as, owing to the limitation in weight, it was necessary to choose as self-contained, compact and rigid a form as possible. To my mind, the most important point in the design was to adhere as closely as possible to the simple direct truss form, so that all stresses induced by change of position of the telescope act along the members of the frame. Thus any flexure will only arise by the actual extension or compression of the parts and not by any lateral bending. The design of the Brashear instruments was used as a guide with the difference that, in the triangular, tubular, tripod form of frame, the three tubes were brought much closer together, entering at the prism box into a well braced solid casting and, there almost touching the collimator tube. This is shown in the photographs, figs. 1, 2 and 3, in which the same letters are used throughout to designate different parts. They are indeed so close together that for all practical purposes they may be considered as meeting in a point, and no bending of the casting can alter the position of the line of collimation. This casting A, is continued to act as the base of the prism casting, and the support of the objective end of the camera in the three prism form, and its outer end, which is generally unsupported in other spectrographs, is rendered very rigid in every direction by the diagonal truss B B, which begins at opposite ends of a diameter of the upper ring casting II, continues past the end of the casting A, to which it is rigidly fastened by screws, and again is united almost at a point by a rigid oblong casting C'. When the spectrograph is used with a single prism the camera passes close to the end of

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this truss to which it is firmly united with a rigid tie D. The convergence of these two tubes prevents any lateral motion, and evidently motion in the other direction can only take place by actual extension or compression of the tubes. This diagonal double tubular truss is an essential feature of the design, and serves two purposes—first to stiffen and render practically invariable in position the outer end of prism box and inner end of camera when used with three prisms, and second, to act as a tie to the outer end of the camera when used with a single prism, preventing motion of the camera both tangentially and laterally.

For the same reason that at one end of the tripod shaped truss the tubes are brought as close together as possible, at the other end they are separated as far as possible. The spectrograph truss is made comparatively short in order to be compact and self-contained, and is attached to the solid casting at the eye end of the equatorial by what may be called the telescope truss. This is, as shown, composed of two ring-shaped castings, the lower one E, to which the spectrograph proper is attached, the upper one F, fastened to the telescope, united by three tubes which in the normal position of the spectroscope form prolongations of the three tubes of its frame. The upper ring which is of the full outside diameter of the end casting of the telescope, is attached to the latter by three swivel bolts at the outer edge of the ring and radial with the three tubes. The lower ring has a depression turned in it into which a corresponding elevation on the spectrograph ring fits, thus admitting of turning in any desired position angle. Three clamps G G, also radial with the tubes, admit of rigidly fastening in any desired position. Thus, as stated above, when in normal position, the three tubes are practically continuous from the end of the collimator to the end of the telescope tube, forming an exceedingly rigid truss, while at the same time the spectrograph can be freely rotated and is entirely self contained and compact.

As previously stated, in order to prevent differential expansion with change of temperature, the spectrograph should be made of one material, and brass was chosen for the purpose, both on account of greater facility of construction, and to avoid, as far as possible, change of collimator and camera settings with change of temperature, it being a well known fact that the expansion of a brass tube more nearly compensates the change in focus of the usual type of lenses than iron. To obtain as much stiffness as possible, hard drawn tubing was used for the truss and the castings were of aluminium bronze, which is much stiffer than ordinary bronze or gun metal. The patterns were deeply ribbed, giving the maximum of strength with the minimum of weight.

In the collimator section of the spectrograph, the three inclined tubes are $1\frac{1}{4}$ inches in diameter, of heavy gauge and are very carefully fitted into their bearings in the ring H and prism base casting A. These of course, form with the diagonal tubes B B, of 1 inch diameter the essential part of the truss, but considerable stiffness is added by the heavy central tube 2 inches in diameter, in which the collimator tube moves, and by the tube I, $1\frac{3}{8}$ inches diameter, which carries the comparison apparatus, slit diaphragm, and guiding telescope. After this section had been put together, it was placed between the lathe centres, the axis being the line of collimation, and the upper face of the ring and the lower face of the prism base casting were turned off perpendicular to this axis and perfectly true, ensuring perfect collimation in any position angle, and giving at the lower end a true surface to work from. The telescope truss was similarly constructed and trued, the tubes, however, in this case being of steel, $1\frac{1}{4}$ inches in diameter, as, since this is independent of the spectrograph, no temperature effect need be feared.

Prism Casting and Cells.

As the photographs, figs. 3 and 4 show, both the prism castings and the cells are made very substantial in order to prevent any relative motion of the prisms with

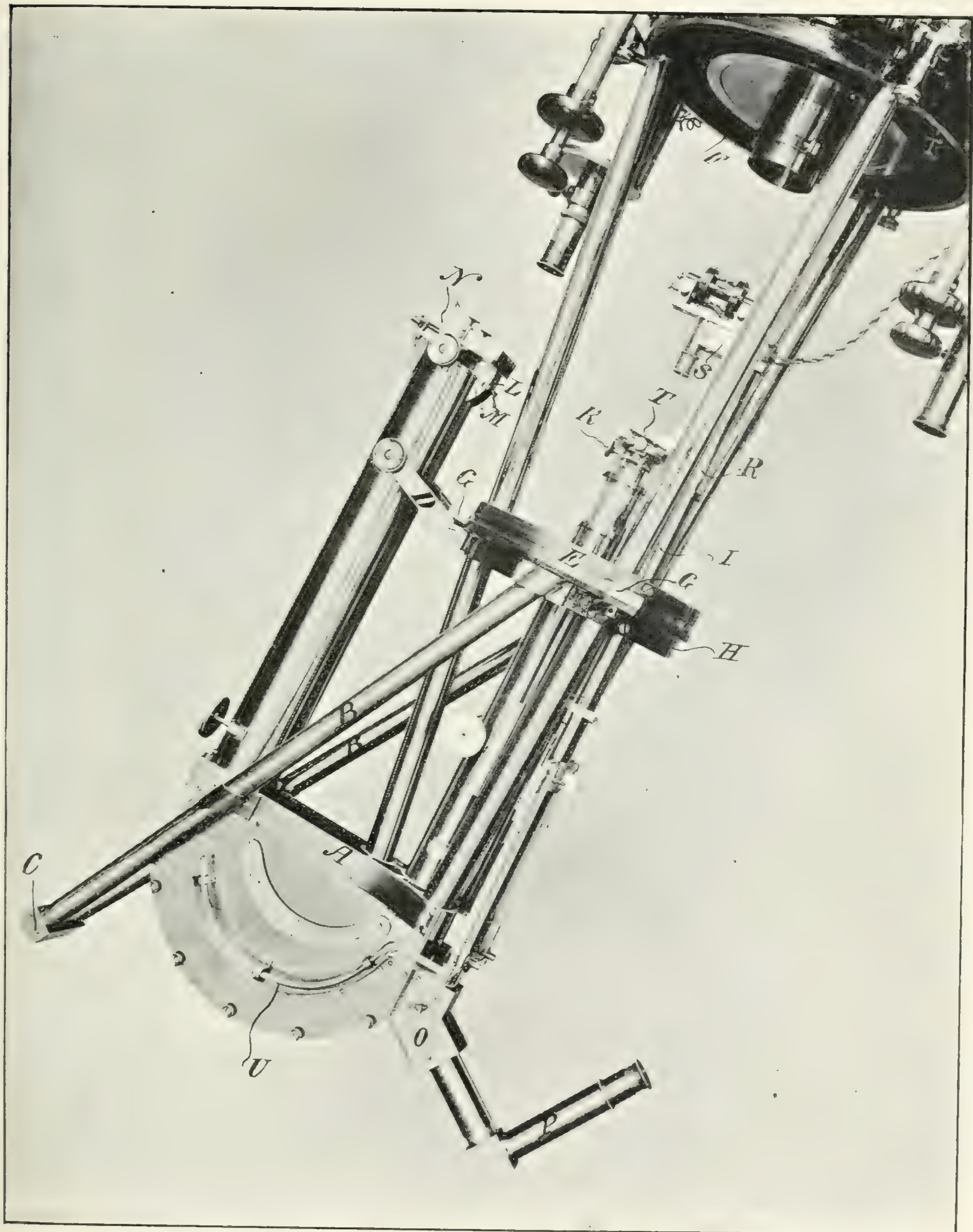


FIG. 1. Spectrograph—Three prism arrangement.

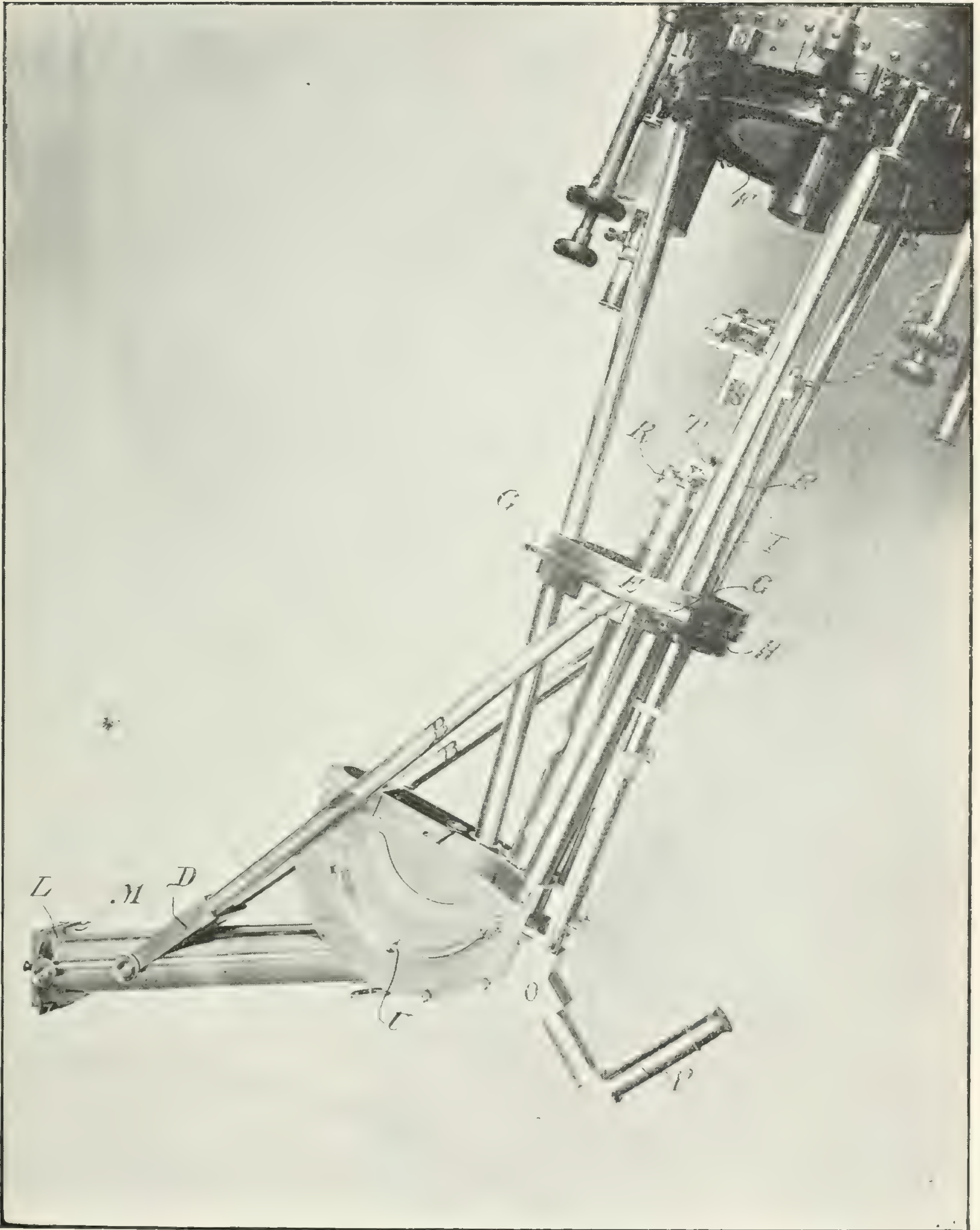


FIG. 2.—Spectrograph—One prism arrangement.

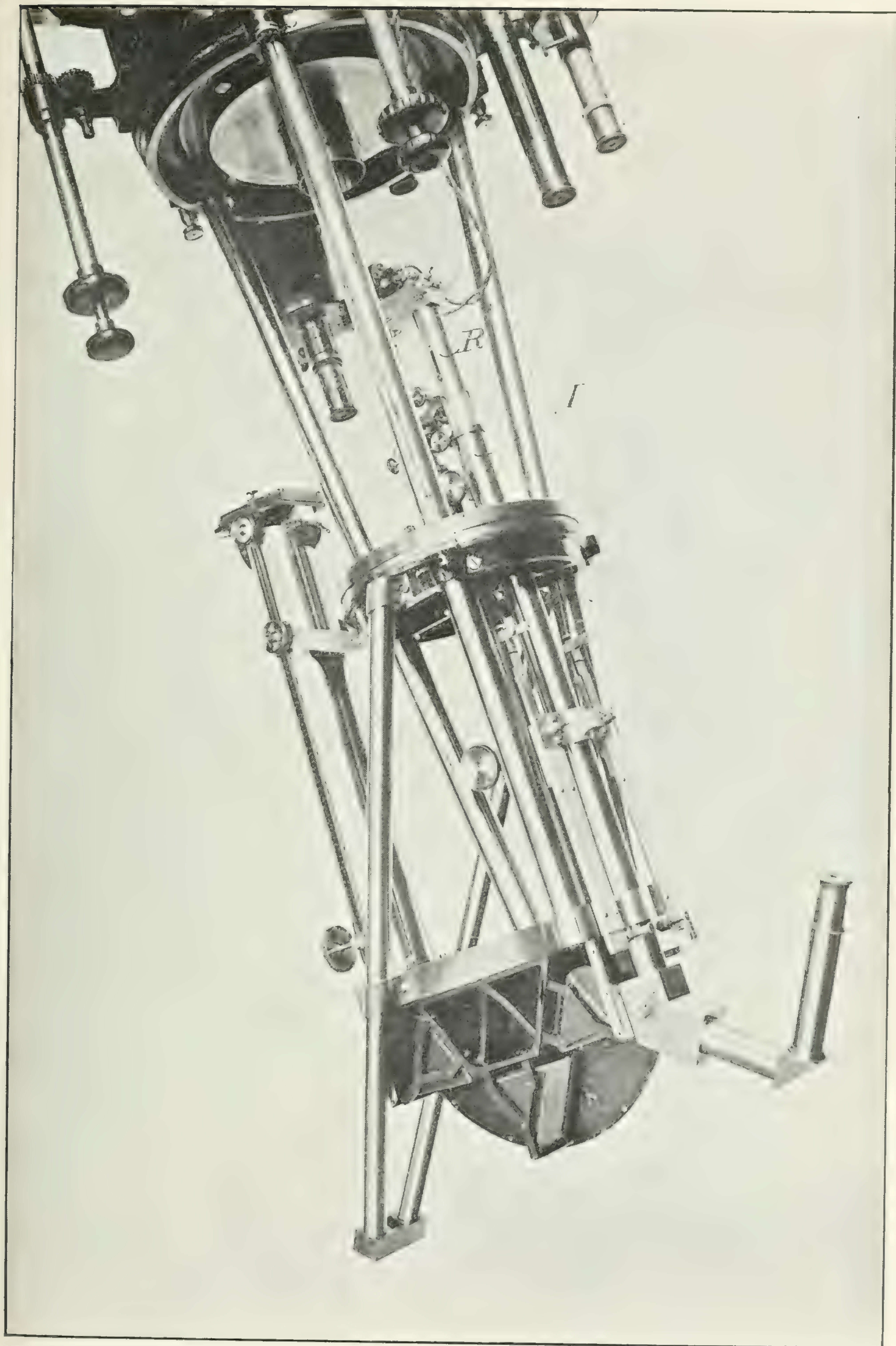


FIG. 3. —Spectrograph with side of prism box removed.

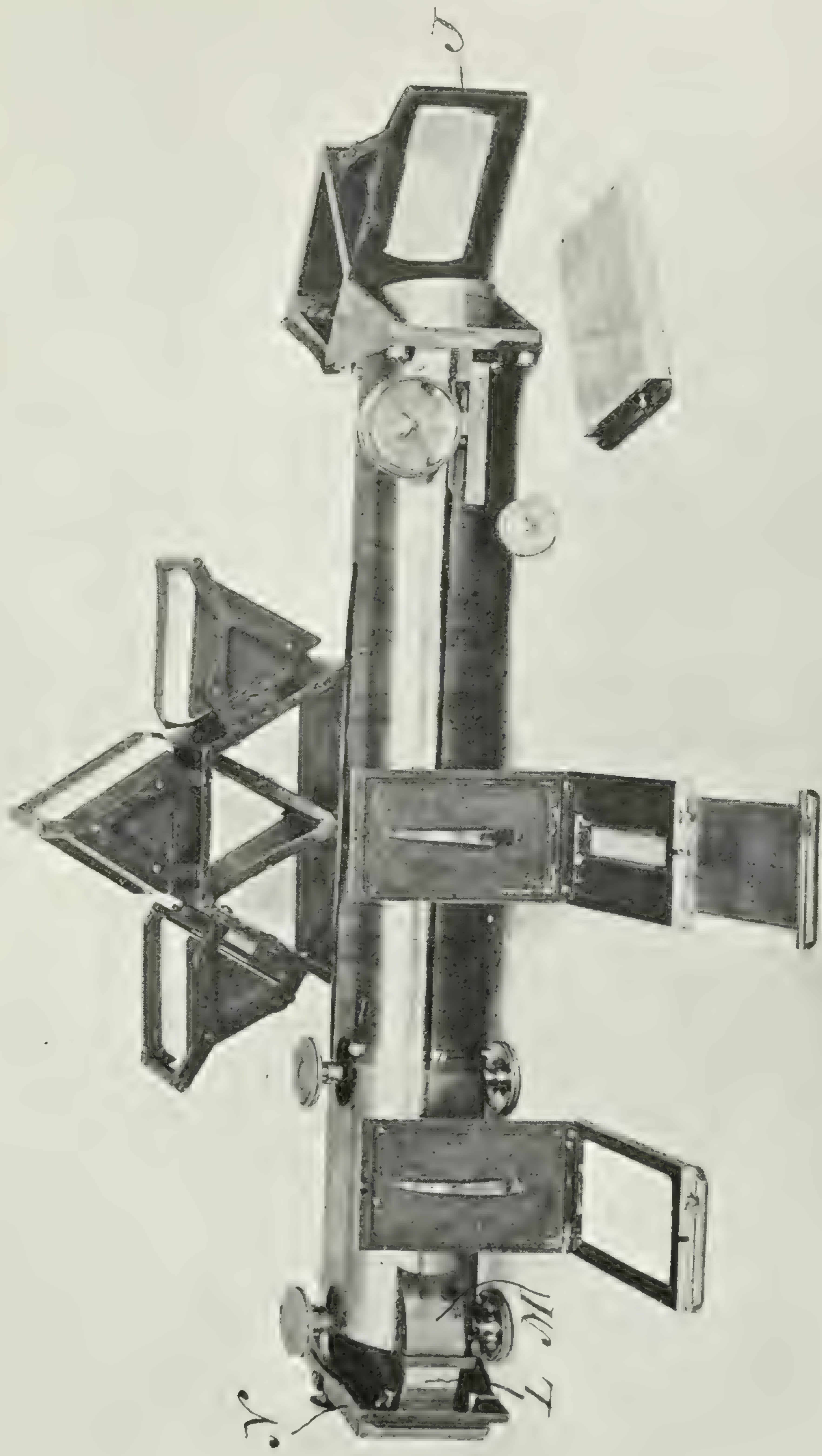


FIG. 4.—Camera, prisms and plate holders.

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respect to the rest of the instrument. The braced form of the prism castings for both single and three prisms, together with the amount of metal present, not only effectually prevents any flexure, but also adds considerably to the rigidity of the base casting to which they are firmly screwed. The prism cells are made in one piece, well ribbed for stiffness, and the outer edges, near the refracting edges of the prisms, are connected by a carefully fitted rod J of brass. The prisms are fastened in the cells by gentle pressure produced by three screws pressing on a plate on top of the prism, a piece of blotting paper being placed between glass and metal on both sides. To ensure the maintenance of the prisms in the correct position, narrow brass strips are screwed to the base of the cell abutting against the prism, and preventing it from shifting. Thus only enough pressure need be exerted by the screws to prevent looseness, and no effect on the definition need be feared.

The Collimator.

The collimator objective, made by Brashear, is of Hasting's triple 'Isokumatic' construction, of 35^{mm} aperture and 525^{mm} focus, and is mounted in a tube of 1½ inches diameter, about 19 inches long, which moves by rack and pinion, the position being read on a scale, over a range of about 80^{mm} in the central 2-inch tube. This movement is to allow adjustment for any change in the star focus due to temperature changes, or changes in the correcting lens. The collimator tube, when adjusted, is firmly clamped within the central tube at both ends to avoid any chance of displacement. It is bushed at the upper end to 1¼ inches internal diameter to receive the slit tube, and this bush can also be firmly clamped on the slit tube, when the adjustment of the collimator focus is completed. The slit, which was very satisfactorily made for us by Brashear, has inclined, 3.5°, reflecting, speculum metal slit jaws. In accordance with my specifications, the edges of these jaws were brought to a sharp edge instead of being left, as has frequently occurred, about half a millimetre thick. I feared in such case a possible displacement of the spectral lines, owing to uncertainty in position of the effective slit aperture, and corresponding uncertainty of the camera focus. The slit tube is graduated in millimetres and is provided with a tangent screw K, for adjustment in position angle, to enable the lines to be made exactly perpendicular to the length of the spectrum, which allows greater convenience and accuracy in measurement. The interior of the collimator tube is thoroughly diaphragmed to prevent reflections.

The Camera.

Up to the date of writing, only the long focus 525^{mm} camera objective has been completed and consequently only the one camera has been made. As the objective is not achromatic, but composed of two separated single elements of the same glass, light crown, the plate has to be tilted, 5.5° for the three prisms, 16.5° for the single prism, and the camera requires a somewhat different construction from that usually followed, where the tilt is only two or three degrees. This, as is clearly shown by the photographs, consists of one cylinder L, to which the camera back is attached, capable of rotation in a second M, which is attached to the tube, through an angle of some 30°. The inner cylinder is graduated and can be firmly clamped in any desired position by clamp screws on the axis of rotation, as well as on the sides of the cylinder. Besides this motion of rotation, the camera back N, into which the plate holder slides, can move transversely some 15^{mm} on ways, and can be rigidly clamped in any desired position. This motion has been found very convenient, as it allows any number of spectra up to 10 or 12, to be made side by side on the same plate, for comparison in focussing, or for other purposes. The plate holders slide in ways, and are securely fastened down to the same focal plane every time by a pair of clamp screws so that no chance for displacement occurs.

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The camera tube, 3 inches in diameter, is made in two pieces, the plate holder end, about 6 inches long, attaching to the objective end, about 15 inches long, and firmly secured with six steel screws well shown in fig. 4. The purpose of this is to enable the camera end, which is troublesome to make, to be used with objectives of different focal length, each fitted into a tube of the required length, some 6 inches shorter than the focus. A pair of large clamp screws both at top and bottom of the camera section, serve to firmly clamp the tie braces D D to the camera. With the long focus camera, the lower screws are used, but with the shorter focus, the upper ones will be required. The objective end of the camera tube is attached to a flanged casting, which is fastened in turn to the prism base casting when three prisms are used, or to the prism casting when a single prism is used, by four screws, and can consequently be quickly and easily attached and detached. This flanged casting is bored out to receive the 2½-inch tube about 17^{cm} long, containing the two crown lenses, one at each end. This objective tube is focussed by a rack and pinion, the settings being read on a scale with vernier to tenth millimetres, while a clamp screw allows it to be rigidly fastened at any desired setting.

The Plate Holders.

The design of the plate holders was changed from the usual type, in which the plates are supported at the ends only, as tests had shown that successive plates did not occupy the same position with regard to the camera back. Experience with the Brashear spectroscope had shown the necessity of accurate camera focus, for a position of the plate only 0.1^{mm} from the focal plane would, under the presumption that the distance between the centres of intensity of star and comparison light on the collimator and camera objectives is only 5^{mm}, cause a displacement of the spectral lines equivalent to a velocity of 1.8^{km} per second. Such a displacement of the starlight can under the present condition of the correcting lens easily occur, as it is impossible to obtain uniform illumination of the camera objectives, and an almost unnoticeably non-central position of the star image on the slit causes a displacement of the centre of intensity to a greater extent than 5^{mm}.

Even if this were not the case, the importance of accurate focus from the standpoint of definition would be a sufficient incentive to any improvement in the methods of obtaining and maintaining it. With the original design of plate holder, any differences in curvature in the successive plates used would change the position of the sensitive surface and, even if one plate were in accurate focus, the next might be as much as 0.1^{mm} or even more in front of or behind such position. Hence the new plate holders were designed so that the plates are supported as close as possible to the measurable portion of the spectrum. This is effected, as shown in the photograph, fig. 4, by opening the plate holder in the middle similarly to the English book form, placing the plate face downward, resting on a raised portion at the edge of the opening—13^{mm} wide and 76^{mm} long—in the front half of the holder, which is closed by the usual slide. On closing down the back of the holder, a spring presses the plate firmly on this projection. The raised portion consists of two strips at each side of the opening each about 50^{mm} long, and no curvature in the glass can in this case cause any appreciable deviation of the sensitive surface, as the spectrum is nowhere more than 6^{mm} from the support. Great care was taken that, in each of the four holders, the distance of the strips from the front surface, which, is clamped against the camera back, is exactly the same, and this was ensured by taking a light cut over the surface of the strips on each holder after they were finished, the setting of the milling machine remaining unchanged throughout. The size of plate used is 2" x 3½" and the opening in the front of the holder and in the camera back allows a spectrum three inches long and any width up to half an inch to be photographed.

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The Guiding Arrangement.

Guiding in the old spectrograph was effected by the starlight transmitted through the slit and reflected from the first surface of the first prism into the guiding telescope. The image could be maintained central, only by keeping it at maximum intensity, and this is not easy, except with stars of about the 3rd magnitude. With brighter stars the image appears too large, and with fainter it is difficult to see. Hence, in the new spectrograph, it was decided to use the light reflected from the slit jaws for guiding, and I am indebted to Prof. Frost's spectrograph for the idea of combining the two methods and using either at will. I did not, however, after obtaining the opinion of other spectroscopists, consider it necessary to use symmetrically inclined slit jaws, for, though unexceptionable in theory, they offer difficulty in practice and introduce complication in uniting the separate images in the guiding telescope. When it is considered, that even in an ordinary inclined slit, the edges of the two jaws, when closed and when open to any width, are always in the same plane perpendicular to the line of collimation, and that the parts of the jaws away from the edges have no action on the light transmitted through the jaws, it is not likely that any error can be introduced by using the simpler form. In order to be able to guide by transmitted light, the guiding telescope must be in line with the light reflected from the front prism surface, and must hence make an angle twice the angle of incidence, or $124^{\circ} 06'$, with the optical axis of the collimator. In order to get the reflected light from the inclined jaws in the same direction, a system of right angled prisms and mirrors must be used. The foundation of the guiding system is a brass tube T, $1\frac{3}{8}$ " diameter, parallel to the collimator tube, with a distance of $2\frac{3}{8}$ " between centres, and of approximately the same length. At the lower end, this tube carries a box O into which, at the proper angle, the bent guiding telescope P is attached, and which contains a mirror moveable on a pivot at one end into the proper position to reflect light coming down the tube, and which can be turned out of the way of light coming from the prism surface when desired. Into the upper end of this tube slides a second tube R, moved by rack and pinion, and firmly fastened in any desired position by a clamp screw. This carries at its upper end an oblong box S, about 6 inches above the slit, projecting over it, but not far enough to intercept any of the starlight. Within this box at the outer end is a right angled prism, which receives the starlight reflected from the inclined jaws and sends it horizontally to a second prism placed over the centre of the tube, which, in turn, reflects it downward through the tube to an achromatic objective of 1 inch aperture and 10 inches focus. This lens is placed at its focal distance along the optical path from the slit, and sends a parallel pencil down the tube to the mirror, and thence into the guiding telescope. By simply turning the mirror, guiding can be done by the light transmitted through the slit opening or by the part of the light reflected from the polished surfaces of the jaws. It may be stated that guiding is almost entirely performed by the second method, although the first is useful for determining the zero slit opening and for examination for dust, &c. The bent guiding telescope can be rotated to any angle, and thus allows a comfortable position for guiding in any position of the telescope. There is very little loss of light in the optical parts, and one sees the whole slit and diaphragm mechanism as well as if observed direct.

The Comparison Apparatus.

The principal requirements for a serviceable comparison arrangement are convenience in use and permanence of adjustment. Owing to the change in star focus with change of temperature and to the corresponding change in slit position, the upper part of the guiding attachment, the comparison apparatus, and the diaphragms in front of the slit for limiting the star and spark light must be moveable vertically, and this is effected very conveniently in the present instance by attaching them to the moveable tube R, in the upper part of the guiding tube I. A stop on the lower part of

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the diaphragm is brought almost into contact with the slit head and all three attachments are then in the correct position.

The arrangement of the comparison apparatus is well shown in the different figures and, by swinging between centres in the end of the box containing the guiding prisms, can be turned down always into the same position ready for use, or up out of the way of the starlight, being held there by a spring catch. There are four sets of spark terminals mounted in a drum-shaped arrangement, Fe, Fe-V, Ti, and Cr, and any one of these is brought into position and adjustment by rotating the drum, the position being determined and the contact made at the same time by clamping a pair of screws. A small condensing lens, mounted in a tube below the terminals and in the axis of collimation, serves to form an image of the spark on the slit, a uniform illumination of the collimator lens being further ensured by a piece of ground glass in the upper end of the tube about a centimetre below the spark gap.

In the old spectrograph, the iron spark was used for a comparison spectrum, and, as is well known, gives, when no self induction is included in the circuit, many air lines and considerable continuous spectrum as well as the purely metallic spectrum. The continuous spectrum considerably diminishes the sharpness and contrast of the lines, and settings cannot be so accurately made. When the new spectrograph was brought into use, a plate condenser containing 36 plates about 10 inches by 12 inches in size, arranged so that either 12, 24, or 36 plates could be used as desired, was constructed and placed in parallel across the spark gap to intensify the spark, replacing the Leyden jars which were continually breaking down. A coil of self induction, consisting of 100 turns in three layers of heavy rubber insulated No. 12 wire, wound on a hollow cylinder, into which iron can be placed if required, was also constructed and placed in series with the spark gap. The air lines and continuous spectrum, even with iron which causes more trouble than titanium or iron-vanadium, were then entirely eliminated. A test of the most suitable spectrum led to the choice of iron-vanadium as the lines, although not so plentiful between $\lambda 4600$ and H_{β} , are much more suitable between $\lambda 4400$ and the extreme violet than in the titanium spectrum. In our single prism work, lines are needed all the way from H_{β} to K, or even lower.

The Slit Diaphragms.

From experience with the old spectrograph the necessity of absolute independence of slit head and diaphragm arrangement was impressed upon me, and in consequence the attachment T, containing the diaphragms, was fastened to the sliding tube R, above mentioned, and does not touch the slit head or collimator tube at any point. Moreover, as star and comparison spectra are always made of practically the same width, the trouble entailed by an adjustable diaphragm, which is always getting out of adjustment and in which the edges of the tongues are not at right angles to the slit and are thereby objectionable, was obviated by using a fixed diaphragm with an opening on one side 0.25mm wide for the star spectrum, and two openings 1.0mm wide separated by 0.35mm for the comparison spectrum on the other side. This slides, in ways parallel to the slit, into a small carriage which is moved transversely to the slit between stops by a pair of knurled wheels whose rotation through about 40° brings in one direction the star window, and in the second direction the comparison windows directly over and within a half millimetre of the slit. These separate windows are adjustable laterally so that the star spectrum can be made exactly central with regard to the comparison spectrum, and this adjustment when once effected is permanent. If different widths of spectra are required, or different widths of windows for the same width of spectrum, as will be the case when camera objectives of different focal length are employed, separate slides, which are easily constructed, are made for each. In this way the diaphragm can be changed in two or three seconds and always be in correct adjustment and position. All that is necessary in changing from star to comparison spectrum is to turn one of the knurled wheels above mentioned as far as it will go (about 40°) and push down the comparison arrangement, the whole

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not occupying more than a couple of seconds, while a reversal of this process changes back from comparison to star spectrum. During the whole exposure of star and comparison neither slit nor collimator tube is touched, nor can any pressure be exerted on them by the above changes, and furthermore, the star spectrum is always centrally situated between two equally exposed symmetrical comparison spectra, thus increasing the accuracy and convenience of measurement.

Method of Focussing.

The method of focussing the camera was fully described in the last report, and need not be referred to here, except to state that it is determined by observing the relative displacement of the lines of adjacent spectra, one made through the half of the camera lens and prisms near the refracting edge, and the other through the half near the base. The slit diaphragm used for this purpose has slits of such width that three spectra each, 1^{mm} wide are made side by side and in contact, the centre one through the refracting-edge-half and the two outside ones through the base-half of the prisms. Thus the displacement or non-displacement when in focus of the lines, can be at once accurately determined by mere inspection. A small opening is made in one side of the prism box, directly underneath the collimator, into which a half circle diaphragm slips and which can be turned to occult either half of the collimator lens. Thus by inserting this diaphragm and the proper slit diaphragm, and making two or three exposures on the same plate, the camera being moved transversely in the ways, as previously described, the focus can be accurately determined to 0.05^{mm} in less than 5 minutes. This is done on practically every evening the instrument is used and ensures, with the new form of plate holder, almost absolute accuracy of focus, at least within considerably less than 0.1^{mm}.

Automatic Temperature Control.

In accurate spectrographic work, the importance of constant temperature of the prisms and of the metal parts of the frame work can not be overestimated, as poor regulation is very likely to introduce systematic displacements of the spectral lines, which in extreme cases may amount to several kilometres per second. If, for instance, just before the comparison spectrum is exposed, the heating current be turned on for two or three minutes, so that the temperature within the case rises one or two degrees, this being quite possible with hand regulation, the expansion of the metal parts may displace the position of the comparison lines, and, if the heat is then turned off, without similarly affecting the position of the star lines. Some experiments made with the single prism attachment of the new spectrograph where, owing to the extended nature of the frame the effect would naturally be large, showed a displacement of the lines in adjacent spectra,—one made when the temperature had been stationary for some time, and the other when the heat had been on for five minutes with a rise of temperature of 1.5° C. in the outer case,—of 0.005^{mm} equivalent with the low dispersion employed to about 10^{kms} per second. Moreover, some of the spectra obtained with the Brashear spectroscope show much broadened comparison lines, more so than can be accounted for by flexure, probably due to poor temperature regulation. Some of the measures also show variations from the expected result of greater amount than can be normally accounted for.

Considerable thought was therefore expended on the question of temperature control, and the method used by Hartmann for the Potsdam spectrograph with modifications in the arrangement of the heating coils, was finally adopted as giving the simplest and most practicable solution of the problem. In this method the temperature is automatically controlled by a pair of electric contact thermometers disposed with their long curved bulbs, one on each side of the prism box. One of these with its accompanying guard to prevent accidents is shown at U. The capillaries, of about half a millimetre bore, rise one on each side of the guiding tube to which the scales and

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supports are firmly attached. The capillaries are open at the end to admit the platinum wires whose positions are adjusted to any desired scale reading by rack and pinion. A platinum wire sealed into the lower end of each capillary forms the second contact and each of these are connected in series with a 300 ohm relay and a couple of dry cells, a spark coil of about 1,500 ohms connected across the gap preventing any oxidation at the mercury surface due to excessive sparking. The scale readings are large, 1° C, occupying a space of about 2.5^{mm} and thus a difference of temperature around the bulbs, which, owing to their large surface quickly respond, of considerably less than 0.1° C causes contact to be made or broken and the relays to act, breaking or making the heating coil circuits.

The outside temperature case, seen in fig. 5, which is constructed of sheet aluminum lined throughout and all the joints broken with felt to prevent rapid conduction and loss of heat, completely encloses the whole spectrograph and is firmly attached by screws to the three clamps by which the spectrograph is fastened to the telescope truss. Thus the spectrograph can be rotated in position angle and attached to or detached from the telescope without removing the case. The removal of a few screws, however, allows the case to be detached from the spectrograph. Windows to read the prism box thermometer and the electric contact thermometers with doors to admit to the focussing screws, &c., are provided in the sides of the case wherever necessary. It is also arranged with a removeable extension for use with the single prism.

The heating coils are composed of No. 28 German silver wire wound on thin wooden frames and, to prevent accidental contacts and short circuiting, the wire is single silk covered. At first the coils were limited to a single large coil on each side of the case opposite the prism box, as in Hartmann's arrangement, but it was found that, as the temperature outside fell, the temperature in the prism box thermometer dropped about 0.1° C. per hour, even though the heating arrangement appeared to be working perfectly, and although resistance was cut out of the circuit to meet the increased demand. It appeared that this fall in temperature could only be due to the fact that the heating coils were directly opposite and close to the thermometer bulbs and that, although the temperature remained constant at the bulbs and around them, in the other parts of the case it diminished, causing by conduction, &c., the drop observed on the thermometer with its bulb inside the prism box. The application of the guards of bright sheet metal between coils and bulbs advocated by Hartmann, did not do much to remove the difficulty, and moreover, the sensitiveness of the control was much diminished, the time of response of the thermometers being lengthened from about half a minute with moderate heating current, to two or three minutes, thus introducing, to my mind, greater danger of systematic error than the gradual drop before referred to. Additional coils were then inserted to practically cover the two sides of the case and this seemed to remove the greater part of the trouble, for, although there is a slight fall in the prism box thermometer when the room temperature diminishes, if the heating current is turned on when the room is at maximum temperature, this soon ceases and the temperature then remains constant. Although the heating coils are disposed as symmetrically and uniformly as possible, there is no doubt that stratification and non-uniform temperature occur in different parts of the case, and, when the telescope is changed from one star to another, this may give rise to some changes in temperature in different parts of the case. This change of position of the spectrograph is avoided as much as possible by rotation of 180° in position angle whenever the telescope is moved from one side of the pier to the other, the spectrograph being always used with the camera above the collimator. But the only certain remedy for stratification and local inequalities of temperature is some means of stirring the air inside the case. This will entail some difficulty in arranging, owing to the limited space available and to the necessity for keeping the weight at a minimum, but it is hoped before long to instal some such device, the most promising appearing

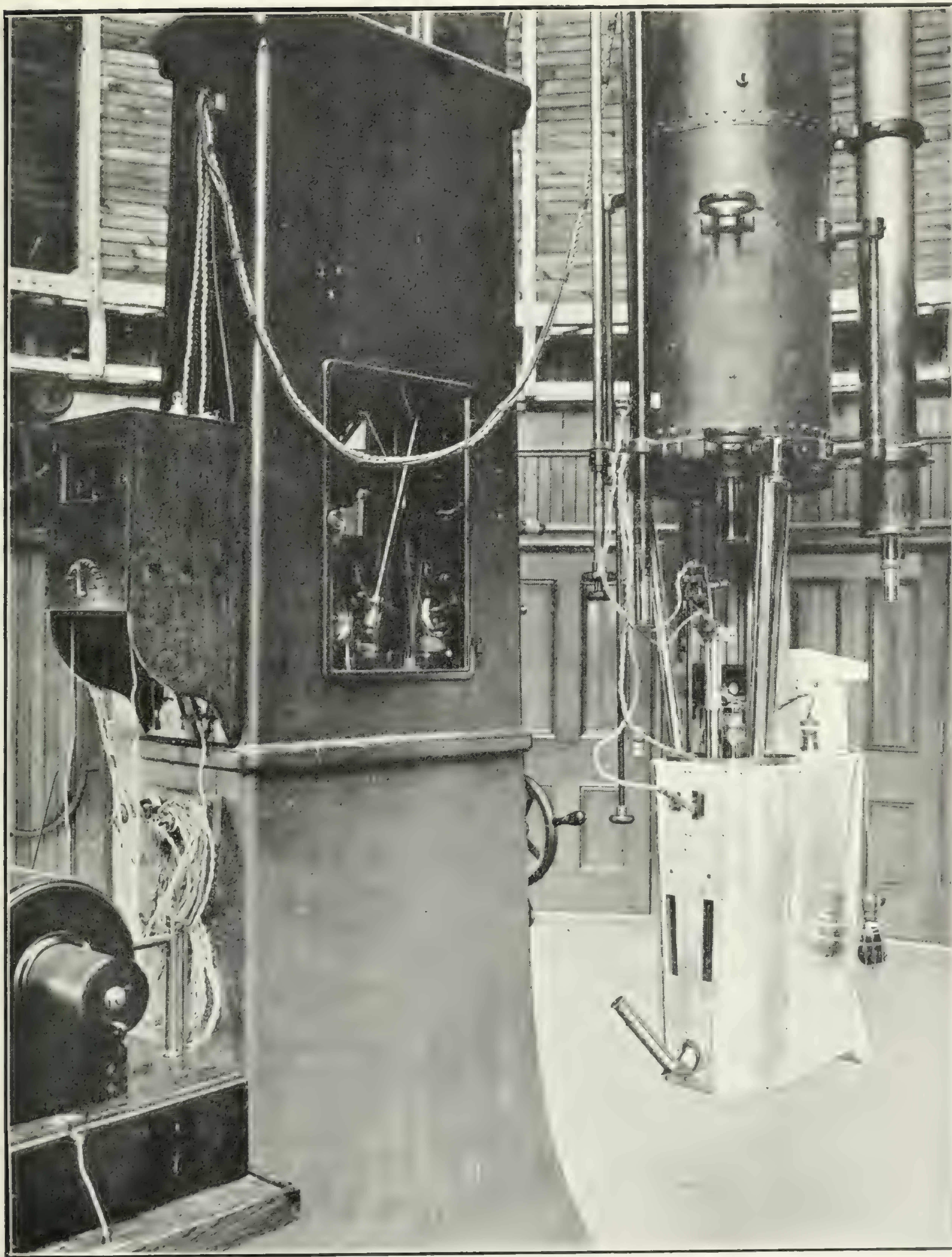


FIG. 5. —Spectrograph ready for work.



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to be to place the heating coils in an external case and by means of a fan to force the air through the spectrograph case over these coils, the current in which is turned on and off automatically.

The arrangement of the heating circuits is very convenient and all connections can be made and the heat turned on in a few seconds. A neat box seen in fig. 5, containing the two relays, connected with the two thermometers, each of which controls the coils on its own side of the spectrograph only, two dry cells for actuating the relays, and a variable resistance for altering the amount of heating current is placed on the south side of the telescope column about six feet from the floor, entirely out of the way of all moving parts. The eight wires, two to each thermometer and two to each side of the heating case, lead from binding posts, properly connected to the relay, battery and resistance terminals inside the box, up the south end of the column and then loop across to the inner side of the tube below the declination axis and down to the eye end. Here they are connected to plugs in three hard rubber blocks which, when shoved into corresponding jacks in the top of the spectrograph and the two halves of the case, complete all the necessary connections, while the relay and heating currents are turned on by a pair of knife switches below the relay box. The wires from the terminals of the induction coil follow the same course up the column and down the tube and there are hence no wires whatever running across the floor to be tripped over or short circuited. The loop in the wires, which are here bound together by tape, from the top of the column to the tube is just sufficiently long to allow free movement of the telescope into every position and can never get in the way or become entangled in any of the moving parts. If it should occur that the spectrograph can not be attached to the telescope when the temperature control is required, a second set of eight wires, leading from the same terminals on the top of the box, end in a similar set of plugs and contact can be completed with the spectrograph in any part of the room in exactly the same way as before. When these wires are not in use they are coiled up out of the way. Two further conveniences are the ten step adjustable rheostat on the relay box allowing the heating current to be varied to suit the difference between internal and external temperatures, and the miniature electric lamps on the top of the relay box in series, one with each of the heating circuits, and showing, by its glowing, when the heat is applied.

It is the custom here, on the nights when the spectrograph is to be used, to place the control in action about 4 p.m., when the temperature in the dome is about its maximum and, if the thermometers are set at this temperature, one may be reasonably certain that the prisms will be in a steady state when observing is commenced, and moreover the initial fall in temperature above mentioned, will have occurred and a constant temperature will be maintained for the balance of the night. Indeed the whole temperature regulation works so well that it requires no attention whatever, while any defect or interruption of its action would be indicated in any case by the cessation of the intermittent lighting or extinguishing of the small lamps, which occurs every few seconds. Owing to these very short intervals and to the small range of response less than 0.1° , the temperature within any particular part of the case must remain very nearly constant. Moreover, the tubular portions of the truss, including the camera, are covered by a layer of felt to completely smooth out any remaining irregularities and ensure no differential expansions during the period of exposure.

General Mechanical Construction.

Before describing the adjustment and tests of the optical parts of the spectrograph, a few words may be appropriately said in regard to its construction. With the exception of the slit head, for which the reflecting jaws could not be made here, the whole instrument was constructed by the mechanician of the observatory, Mr. Alex. Mackey, and I cannot speak too highly of the quality of the workmanship. It is constructed throughout in the best manner, and reflects the greatest credit on his skill.

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Considering the amount of work on such an instrument, it was finished in a remarkably short time, and we may consider ourselves fortunate in having so able a mechanician. Our thanks are due to the Brashear Company for the high quality of the optical parts, and for their endeavours to supply us with a wide field camera objective. They have been successful with an objective for use with the single prism which gives the whole visible spectrum in good focus, but have not so far succeeded in making an equally good one for use with three prisms. They hope, however, to solve the problem and to construct an objective that will meet all requirements as to field. If such can be obtained, it will much increase the amount of observational material without increasing the time of exposure as, with a camera lens giving 8° of field instead of 2.5° the usual limit, three times the length of spectrum is measurable and this is a decided advantage in the case of stars with few lines.

Adjustment of Prisms.

Although presumably the bases of the prisms are perpendicular to their refracting edges, I did not know that any special care had been taken to make them so and felt that it would be safer to adjust them so that all their surfaces would be perpendicular to the plane passing through the optical axis of collimator and camera. As the base of the casting A had been turned perpendicular to the plane in question, and, as the bases of the prism castings were milled exactly perpendicular to their sides, the procedure adopted was to lay the castings on their sides on a thoroughly levelled surface plate after the prism cells had been screwed in place, and made as true as possible, and then to place each prism in position on its cell and observe the reflected image through the telescope of a transit, placed in the same horizontal plane. If the reflection of the object glass of the telescope in the polished surface of the prism appeared central with respect to the cross wires, it was presumed that the surface was in adjustment, but if not the cell was shifted on the casting, the bearing parts being filed or scraped where necessary, in order to bring both refracting surfaces truly vertical. After this had been done for all four prisms, the fixity of position was ensured by pinning the cells to the base castings so that in case of removal, they would always go back to the same position.

Focus of Collimator.

The focus of the collimator was determined both by Schuster's method and by Hartmann's extra-focal method. Schuster's method, which is well known and was described in last year's report, gave a value of scale setting of 10.8 as the focal position of the slit, with a probable error of between 0.1 and 0.2. The collimator was taken out of the spectrograph and a small photographic plate was held securely with its sensitive surface against the widely opened slit, a piece of tissue paper being interposed to prevent scratching. The collimator was held in the brackets of the small finder on the equatorial and a diaphragm with a couple of small holes, near opposite ends of a diameter, was placed over its objective. Several extra-focal exposures were made on a star and the measures of the resulting negatives gave a focal setting of 10.6. This is in very close agreement with the value by Schuster's method, as, owing to the thickness of the tissue paper and the wide separation of the slit jaws, the plate would probably be at least 0.1^{mm} from the plane of the edges and the extra tenth millimetre would readily be accounted for by the lower temperature under which the Hartmann method was used. The collimator setting was therefore fixed at 10.8. After the prisms had been adjusted for minimum deviation at $\lambda 4415$, which was done in the usual way by an observing telescope, the slit was so adjusted in position angle by the tangent screw K that the lines were exactly perpendicular to the length of the spectrum. This adjustment was obtained by trial photographs, and when finally correct the slit was firmly clamped in place. It may be mentioned here that the deviation of $\lambda 4415$ was found to be for the three prisms about 180.5° instead of 180° ,

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but the difficulty was overcome by slightly inclining the camera towards the collimator so as to bring $\lambda 4415$ central. This is due to the prisms being ordered in round numbers with a refracting angle of $63^\circ 50'$ each, instead of the slightly smaller computed value and possibly to some of them being of a greater angle even than that.

Tests of Camera Objectives.

As previously mentioned, three objectives each of 45^{mm} aperture and of 525, 375 and 250^{mm} focus respectively, were ordered from the Brashear Company, with the proviso that the extent of flat field, within 0.1^{mm} , should be as great as possible, about 8° if obtainable. The field previously obtained from the best triplet objectives does not exceed about $2^\circ 30'$. A more recent objective made by Zeiss for Hartmann according to the latter's plan, of the same material as the prisms, the spectrum being obtained in focus by inclining the plate, gives a flat field of 14° . This is considerably greater than needed, as with three prisms about 8° is all that can be obtained without losing so much light at the edges of the field, due to vignetting of the beam, as to be quite useless for star spectrograms. One of these objectives was later ordered from the Zeiss Company, and has recently been received and the result of its test is given below. I therefore suggested to Dr. Brashear that they try to make us some objectives after this plan, and he transmitted the problem to Hastings. The latter did not believe there was anything of value in the idea of using the prism material for the objective, but preferred the plan of using crown glass of the lowest dispersion, separating the two elements and obtaining flatness by introducing oblique astigmatism, which will evidently not affect the sharpness of the spectral lines.

An objective made after this plan was received from the Brashear Company, and as soon as the prisms and collimator were mounted, was tested for field, both with the prism train and with a single prism.

The form of the field was obtained by a modification of Hartmann's method of testing objectives* which is also practically the same method as used here in obtaining accurately the camera focus. Hartmann's method uses small apertures and extra focal measurements and gives better results when used at some distance from the focal plane, while in the method used here the two images of spectral lines are obtained side by side, one through the front and one through the rear half of the camera objective, by means of a suitable diaphragm and the best measurements are obtained when close to the focus as then the lines are well defined. The displacements, though small, correspond to a distance of only two or three-tenths of a millimetre from the focal plane, and consequently accidental errors of measurement of the displacement only introduce small errors, in general considerably less than 0.1^{mm} , in the position of the focus. A great advantage of this latter method is that the form of the field can be determined quite accurately without any measurement or computation whatever, by simple inspection of the displacements with a hand magnifier. As is well known, either of these methods will only give accurate results when the optical system of the spectrograph is free from aberration, and results so obtained should be checked by other methods. This was done in these tests by making a series of spectra at foci differing from one another by 0.1^{mm} , side by side on the same plate, through the full aperture of the system, and judging the position of best focus by comparison of the definition. As the two methods, so far as could be judged by the definition test, gave identical position of the focal plane, it is evident that the results obtained may be used with confidence.

The diaphragm used is inserted just below the collimator lens and is in the form of a semi-circle, which, revolving around its diameter allows, in one position, a semi-circular pencil to pass through the refracting edge, and in the other through the base of the prism and through corresponding portions of the camera objective. When two spectra are made side by side, one through each position, the displacements of the two

* Ztscht. für Instrumentenkunde, Jan. 1904.

parts of any line gives by its magnitude and sign the distance and position of the focal point for that line, and hence the focus for any part of the spectrum can be obtained and the form of the field determined.

Four objectives were tested.

1. The Hastings-Brashear, single material 525^{mm} focus;
2. The same with enlarged rear element;
3. The Hartmann-Zeiss Chromat of 525^{mm} focus;
4. The Ross Homocentric 10 inches focus.

In each of these cases double test spectra as above described for positions within one millimetre on each side of the focus were made, the displacements were measured and the focal positions computed by similar triangles according to Hartmann's method. These positions were plotted on cross section paper and a continuous curve drawn through them gives the form of field. In fig. 6 are shown,

- (A) Curve for No. 1 with single prism from D to $\lambda 3800$
- (B) Curve for No. 1 with single prism from $\lambda 5000$ to $\lambda 3800$.
- (C) Curve for No. 1 with three prisms from $\lambda 4862$ to $\lambda 4102$.
- (D) Curve for No. 3 with normal separation of the elements.
- (E) Same with elements in contact.
- (F) Same with 1.5^{mm} increased separation of elements.
- (G) Same with 2.25^{mm} increased separation of elements.

The Hastings-Brashear objective is composed of two positive elements of light crown glass, separated from one another a distance of about one-third the focal length. As there is no correction for colour, the plates have to be inclined towards the violet about 5.4° when used with three prisms and about 16.4° with single prism. This objective, as curves (A) and (B) show, gives almost ideal results with a single prism, the field being almost absolutely flat over the whole range of spectrum obtainable, and would probably extend considerably farther on each end if necessary. There is a portion about $\lambda 4700$ where the focus is about 0.2^{mm} shorter than the rest, this being probably due to differences in the ratios of dispersion of the prism and objective material, but, as this comes in a position where there are practically no available star lines and as the deviation is very small, it is unimportant. The angular field between D and $\lambda 3800$ is about 6° , but there is no doubt that the flat field extends considerably farther on each side. However, for stellar spectroscopy, owing to the steepness of the colour curve of the objective and correcting lens to the red of H_β , and to the violet of H_ϵ , to the diminished sensitiveness of the plates in these regions, and to the increased absorption of the glass and of the terrestrial and stellar atmosphere beyond H_ϵ , the usable portion of the star spectrum is limited to the portion between H_β and K and no field beyond that is required. Hence this objective has been adopted for work with the single prism and gives admirable results. It has a further advantage that owing probably to the combination of inclined field, increased dispersion, with increase of temperature, and to the use of a brass camera tube, the focus is practically invariable under all conditions of temperature so far observed.

When, however, the same objective is used with three prisms the field is by no means so satisfactory. As shown in curve (C) it is flat in no place but forms a continuous curve not very different from a circular arc. By compromising somewhat, the field is usable between about $\lambda 4325$ and $\lambda 4550$ an angle of slightly over 2° . This is of about the same order as that given by the ordinary cemented triplet lenses, but is not as good as I should like to obtain for use in spectra with few lines, or even in second type spectra when the measurement is made by means of Hartmann's Spectro-Comparator. Moreover, when this objective is used with three prisms, part of the pencil, near the margins of the field, is intercepted by the cell of the rear element owing to the considerable separation of the elements and to the displacement in position of the pencil in passing through the prisms. This diaphragming begins at about 1.5° from the axis and increases until at the edge of the field, about 4° from the axis,

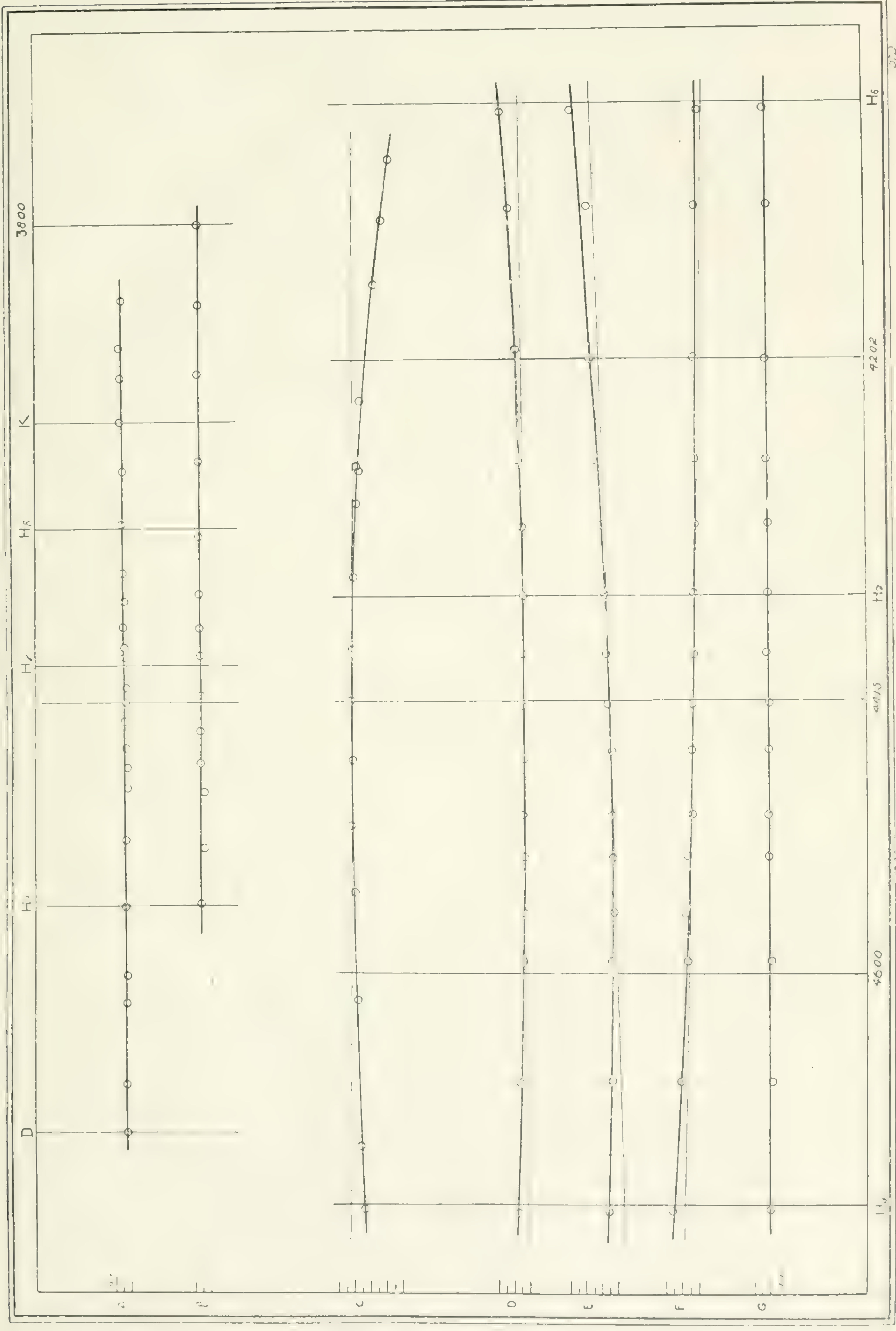


FIG. 6. Tests of Camera Objectives.

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about half the light is cut off. When used with one prism, however, the slight displacement of the pencil and the smaller angular deviation of the usable rays allows it to pass through uninterrupted.

Mr. McDowell very kindly made and figured for me a new rear element with an increased aperture of 15^{mm} which allows the full pencil, even at the margin of the field, to be transmitted. This, which is listed as No. 2, was also tried with three prisms, but, although it gave a more intense spectrum at the edges, the form of field was almost identical, as was to be expected, with the original lens. The plates obtained were not measured or computed, but inspection showed that there was no material difference. The field of the lens with separations of the elements of from 2^{cm} less to 6^{cm} greater than the normal 17^{cm} was also tested, but no improvement was noticed, indeed so far as could be judged by inspection there was very little change either way. Evidently, therefore, this form of objective can not be made to give satisfactory results when used with the dispersion of three prisms, although when used with a single prism the field is all that could be desired.

The favourable report of the performance of the objective made by Zeiss, according to Hartmann's ideas, of the same material as the prisms and described by the latter* led me to have one ordered for our use of 525^{mm} focus and 45^{mm} aperture. This objective has recently reached here and been carefully tested. It gives with three prisms, by slightly changing the normal separation between the elements, a field, fig. 6, *G*, which is practically perfect over the 8° required between H_3 and H_5 . With the normal separation the field, fig. 6 *D*, is slightly convex towards the lens, but by putting in a separating ring 4.5^{mm} wide instead of 2.25, the field becomes flat. The slight original convexity is probably due to slight differences in the average values of the constants of the prism material used in the computation, and the actual values of the melting from which the prisms were made. Indeed the indices of the prism material were some .002 less than the tabular values. The plate required inclining about 15.3° towards the violet, nearly as much as in the crown lens with single prism. Evidently the inclination of the plate, if the Zeiss lens were used with a single prism, would be about 45°. The lens has not yet been tried with a single prism as the camera does not permit so great an inclination. Besides, it is likely that the field with a single prism would be decidedly convex, and moreover, the field of the crown lens has a moderate inclination and can hardly be improved upon for single prism work.

The problem of flat field camera objectives, so far as regards those of moderately long focus, may then be regarded as satisfactorily solved, but for the shorter focus lenses required 375^{mm} and 250^{mm} focus, with angular apertures of $f/8.3$ and $f/5.6$ respectively, the same can not yet be said. Such objectives will be principally needed for three prism work, and whether the Hartmann-Zeiss Chromat can be adapted for such large angular apertures is a question I cannot yet answer. If it can, it seems to offer the most hopeful solution of the problem, giving a good field with moderate absorption and only four reflecting surfaces. Failing this, the only hope seems to lie in some of the modern commercial photographic objectives modified to satisfy the requirements in spectrographic work, which are considerably different from those in ordinary photography. In order to obtain a large angular field of moderately good definition, the spherical aberration in most of these lenses is only partially removed and this residual aberration, though unimportant in ordinary photographic work, can not be tolerated in the case of spectral lines which require the sharpest possible definition. Three such lenses in our possession, the Cooke Series III. $f/6.5$, the Goerz $f/7.7$ and the Zeiss Satz-Anastigmat $f/6.3$ were tried with the spectrograph, but the definition, owing to the before-mentioned aberration, was not sufficiently good for spectrographic work. The Ross Homocentric lens is, however, advertised as being free from such aberration, and through the kindness of W. J. Topley & Co., a Homocentric of 10 inches focus $f/5.6$ was loaned me for testing. Although no actual tests

* Zeitschrift für Instrumentenkunde, Sep. 1904.

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of the aberration of any of the lenses were made, still the definition tests furnished fair evidence, and the Ross lens gave excellent definition over the 8° of field required. Although no actual measurements of the form of field were made, it evidently is convex towards the lens and of a somewhat similar form and order to the Hastings-Brashear single material objective. Increase and diminution of the distance between the two elements did not apparently improve matters and the lens as it stands only gives from 2° to 3° of usable field. The Ross Company have, however, undertaken the making of an objective, in which, by neglecting the field beyond the 8° or 10° required, they hope to meet my requirements. It is to be hoped that, with Brashear, Zeiss and Ross working at the problem of obtaining a short focus spectrograph camera objective, something of value may result.

The New Spectrograph in Practice.

Since the spectrograph with temperature case was completed about 300 star spectrograms have been obtained, mostly with the single prism and the single material camera, giving a linear dispersion of about 30 tenth-metres to the millimetre at H_γ . Early type binary stars have been the principal ones observed for which this adaptation of the instrument is very suitable for, although the dispersion is only about three-fifths the old spectrograph, the accuracy of velocity determinations is, on account of the greater number of lines measurable, probably in many cases considerably greater while the exposure time required is only little more than half that previously necessary. In stars in which the hydrogen lines are alone visible only H_γ could be measured in the old spectrograph, while in the new H_β , H_γ , H_δ and if the exposure is sufficient H_ϵ are all measurable, and give generally accordant readings. In cases where the lines are very diffuse, especially if asymmetric, the different intensities of the spectrum at the different lines will evidently be liable to cause discordances. Even in these cases the mean value would be more trustworthy than the value obtained from a single line. The details of some measurements are given below and will give some idea of the confidence that may be placed in the results obtained.

When we come to the spectra of solar type stars in which the number of lines can be equally great in the two cases, the higher dispersion instrument gives, both on account of the less kilometer value corresponding to a given linear value on the spectrum and also on account of the greater purity of spectrum allowing more accurate identifications and wave lengths, much more accurate values. Indeed the probable error of the determination from a single line is only about half as great with the old instrument as with the single prism attachment of the new.

The above discussion refers only to the accidental errors of setting, &c., involved in any measurement of spectra, but takes no account of any systematic displacement which equally affects all the lines. Such systematic error may be due to several causes, the principal of which are three:—

1. Non-uniform illumination of the collimator and camera lenses by either star or spark light or both.
2. Differential displacement of the star lines with respect to the comparison lines due to changes of temperature, this displacement being caused by the change of deviation and dispersion of the prism, or by expansion or contraction of the metal frame work, or by a combination of both.
3. Displacement of the lines caused by flexure of the instrument due to its attachment to a moving telescope.

Symmetry of Pencil.

The distribution of the light in the pencil of rays coming from the slit may be far from uniform in the case of both star and spark light. The latter has usually been the principal one safeguarded, but my experience with the correcting lens has shown that an equal or greater asymmetry is likely to occur with the star light and this, when the

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image is not free from aberration, is more difficult to guard against. The illumination with the spark light can be readily made and maintained uniform, but if the star image has spherical aberration, which is probably generally present when visual objectives with auxiliary correctors are employed, or if the slit is not in the focal plane of the condensing system then symmetrical illumination of the collimator only occurs when the star image is exactly central on the slit. The fact that the slit jaws are seen by visual light, while the image itself is of blue light in either method of guiding, renders it difficult to get and keep the image exactly central, and consequently, it is probable that the illumination pattern on the collimator objective is rarely symmetrically disposed with respect to a diameter parallel to the slit. Owing, however, to the variations in the seeing and guiding, it is possible that the mean distribution over a long exposure may be sensibly uniform and any displacement avoided, although the definition will be poorer than would be the case under uniform illumination. The possibility still remains that the star light may, during the greater part of the exposure, have its centre of intensity to one side of the centre of the collimator lens, and in this case a systematic displacement of all the lines will occur *unless the camera is in exact focus*. Hence the remedy for this source of error lies in the first place in exact camera focus, in the second in obtaining a star image free from aberration and having it exactly focussed on the slit, and in the third in guiding as accurately as possible. As will have been learned from the preceding description of the spectrograph the first precaution has been most carefully followed. The second defect has already received here much attention,* and good hopes are held forth that the new correcting lens resulting from the investigation of the image given by the old one will give an image reasonably free from aberration. The third precaution is being carefully attended to here, both by the careful design and adjustment of the guiding mechanism and by the care used in following.

The uniformity of the spark pencil is ensured by the careful adjustment of spark gap and large aperture condensing lens in the axis of collimation and further by the insertion of a suitable diffusing screen.

Temperature Effects.

The previous description of the method of automatic temperature control and its efficient working should serve to remove any fear of trouble from this source, but it is proposed to install some means of air stirring in the outside case, and moreover, to try steel tubes for the diagonal brace in hopes of diminishing the displacement caused by any sudden rise in temperature whose amount was given above. The error caused by any possible displacement of this nature is guarded against by dividing the time of exposing the star spectrum into a number of intervals and distributing the comparison exposure equally among these intervals. Thus instead of at beginning and end only, the comparison spectrum is exposed at least four times.

Flexure.

Flexure was one of the difficulties which, in the design of the instrument, special care was taken to overcome as far as possible. Owing to the fact that no material is perfectly rigid there must always be more or less flexure and the only thing that can be done is, with the material at disposal, to render it a minimum. That this was successfully accomplished will be evident from the results of the tests given below. I have no means of comparison with the flexure of other spectrographs except the Bonn, the only one for which the flexure has been published.

The flexure was tested by making two comparison spectra side by side, each in different positions of the telescope, and measuring the shift of the lines.

* Appendix A "The Star Image in Spectrographic Work."

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With the telescope at hour angle 0 hrs. the displacement for a movement from declination— 20° through 130° to 20° below the pole was:—

For three-prism attachment $.0025^{\text{mm}}$, equivalent to 1.8^{km} .

For one-prism attachment $.035^{\text{mm}}$, equivalent to 70^{km} .

With the telescope at declination 0° , and for a movement from 0 hrs. to 4 hrs. in right ascension:—

Flexure with three prisms $.0018^{\text{mm}}$ equivalent to 1.3^{km} .

Flexure with one prism $.007^{\text{mm}}$ equivalent to 13^{km} .

For a movement from 0 hrs. to 2 hrs. in right ascension:—

Flexure with three prisms immeasurable.

Flexure with one prism immeasurable.

The flexure with three prisms even in the test of maximum flexure, swinging in the meridian from the position of camera over collimator to camera under collimator, causes only a very small displacement of the lines, but in the single prism attachment, owing to the extended nature of frame, the linear displacement is some 13 times as great while, owing to smaller dispersion, the kilometer value is forty times as great. A calculation of the displacement of the lines due to the actual extension and compression of the members of the truss, using the best tabular values of the constants obtainable, amounted to nearly as much as the observed value, showing that the design is probably as good as can be obtained. The case of maximum flexure is one that can never occur in practice, where we have to deal with movements of telescope and spectrograph on the polar axis only, and generally not exceeding two hours in duration. For such case, as the tests above show, no measurable flexure occurred even with single prism form when near the meridian, and any systematic displacement from this cause need not be feared. The division of the comparison exposure into a number of equally divided intervals over the star exposure will still further reduce the liability to error even if flexure were present.

In the only published flexure tests I have been able to discover, of the Bonn spectrograph which is of the same form as the Potsdam and Pulkowa instruments, the maximum flexure of this three prism spectrograph is equivalent to 70^{kms} per second and the flexure even for actual exposure conditions of one hour's duration is never less than about 7^{kms} , while no measurable flexure occurs in one hour exposure with either three or single prism Ottawa spectrograph.

The only safe test for absence of systematic error lies in the careful measurement of a large number of plates from one star known not to be a binary, but such test has not yet been made. A few plates of the standard velocity stars have been made, two or three measured, and, so far as this slight evidence can go, no trace of systematic error has been found.

There now follows the measurement of four star spectra made with the new spectrograph which gives an indication of the character of the results obtainable.

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α PERSEI.
THREE PRISM SPECTROGRAPH.

Mean of Micrometer Settings.	Measured Wave Length.	Normal W. L.	Displacement.	Velocity.	Mean of Micrometer Settings.	Measured Wave Length.	Normal W. L.	Displacement.	Velocity.
77 6832	4589 660	126	466	- 30 42	3086	4400 260	601	341	23 24
4391	4587 955	381	426	27 82	1748	4399 523	935	412	28 09
76 8133	4583 668	018	350	22 96	46 8821	4397 909	272	363	24 75
3078	4580 040	407	367	24 08	3246	4394 848	286	438	31 26
75 7327	4576 052	512	460	30 16	44 1940	4383 267	720	453	30 99
1096	4571 743	156	413	27 05	42 7832	4375 700	107	407	28 38
73 9121	4563 535	939	404	26 54	38 3535	4352 475	908	423	29 16
1567	4558 395	827	432	30 57	1666	4351 512	930	418	28 81
71 8120	4549 333	766	433	28 56	36 7100	4344 060	451	391	26 98
69 4682	4533 800	139	339	22 41	35 9521	4340 212	634	422	29 09
68 6341	4528 334	798	454	30 07	4484	4337 767	153	386	26 67
67 7258	4522 426	855	419	27 78	33 0144	4325 506	939	433	30 00
3456	4519 967	397	430	28 55	32 8591	4324 737	152	415	28 76
66 5988	4515 162	508	346	22 97	0162	4320 583	992	409	28 38
65 4857	4507 047	455	408	27 13	30 8379	4314 813	178	365	26 44
64 3891	4500 103	448	345	22 99	6568	4313 930	321	391	27 18
63 3002	4494 270	664	394	26 28	3858	4312 663	051	388	26 97
62 8115	4491 229	621	401	26 76	29 7029	4309 302	652	350	24 43
61 1626	4480 009	438	429	28 71	3578	4307 634	891	357	24 84
60 3209	4475 844	214	370	24 77	27 7308	4299 832	211	379	26 42
58 7496	4466 300	711	411	27 60	26 6206	4294 558	936	378	26 38
57 5198	4458 905	304	399	26 82	4758	4293 873	273	400	27 92
56 0778	4450 330	719	380	25 61	23 9838	4282 200	565	365	25 84
54 9307	4443 577	976	399	26 92	22 3278	4274 571	911	340	23 85
50 2344	4416 590	985	395	26 82	17 7917	4254 117	505	388	27 36
48 0797	4404 527	927	400	27 24	16 8377	4249 911	287	376	26 53
47 4854	4401 235	581	346	23 56	13 5713	4235 691	112	421	- 29 79

Mean. - 27 03
V_a + 25 10
V_d 10
Curvature - 28
Radial velocity. - 2 1

Θ AQUILAE.

ONE PRISM SPECTROGRAPH.

Line.	Wt.	Mean of Settings.	Corrected Setting.	Normal Setting.	Displt. in Revs.	Velocity.
λ 4864	2	73 0432				
H β λ 4861	1 $\frac{1}{2}$	72 9345	9002	8648	0354	+ 51 36
λ 4851	2	4820				
λ 4494	2	54 7632				
4481	1 $\frac{1}{2}$	0495	0265	9698	0567	- 64 81
λ 4466	2	53 1355				
λ 4341	2	45 2935				
H γ λ 4340	2	3185	2987	2387	0600	+ 62 64
H δ λ 4102	1 $\frac{1}{2}$	27 5148	4855	4219	0636	+ 55 20
λ 4099	2	2760				
K λ 3933	1 $\frac{1}{2}$	11 9599	9419	8514	0905	+ 68 15
λ 3930	2	11 5250				

Weighted mean. + 60 57
V_a - 28 10
V_d - 04
Curvature. - 28
Radial velocity. + 32 1

o ANDROMEDAE

ONE PRISM SPECTROGRAPH.

Line.	Wt.	Mean of Settings.	Corrected Setting.	Normal Setting.	Displt. in Revns.	Velocity.
λ4864	1½	72·9847
H _β λ4861	3	·8276	·8000	·8187	·0187	- 27·17
λ4851	1½	·4292
H _γ λ4341	1½	45·2720
λ4340	2	·2022	·2138	·2489	·0351	- 36·75
H _δ λ4102	1½	27·4185	·4604	·4965	·0361	- 31·48
λ4099	2	·2800
H _ε λ3969	2	15·4476
λ3970	½	·4760	·5576	·6035	·0459	- 35·71

Weighted mean - 32·29

V_a + 20·55

V_d + ·15

Curvature - ·28

Radial velocity.... - 12·9

α BOÖTIS.

ONE PRISM SPECTROGRAPH.

1907. May 24.

G. M. T. 15^h 22^m

Observed by J. S. PLASKETT.

Measured by J. S. PLASKETT.

Wt.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Wt.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.
1½	59·0211	4571·895	·865	·758	+ ·107	+ 7·01	2	43·7916	4318·962	·967	·817	·150	+ 10·41
2½	57·8276	4549·896	·848	·766	·082	+ 5·40	2	40·7733	4275·147	207	·922	·285	+ 19·98
1½	57·5687	4545·179	·135	·845	+ ·290	+ 19·14	1½	39·3174	4254·658	·720	·505	·215	+ 15·16
1	57·0595	4535·955	·923	·964	- ·041	- 2·71	2	37·9807	4236·203	·251	·141	·110	+ 7·79
1	56·8119	4531·496	·468	·355	+ ·113	+ 7·48	1½	37·5352	4230·126	·166	·845	·321	+ 22·76
1	56·5462	4523·156	·148	·985	+ ·163	+ 7·49	2	35·8247	4207·130	·137	·028	·109	+ 7·76
1	56·2021	4520·588	·583	·363	+ ·225	+ 14·92	2	35·4619	4202·320	·320	·161	·159	+ 11·34
2	52·8314	4462·100	·240	·967	+ ·273	+ 18·35	2	34·6641	4191·824	·827	·654	·227	+ 16·23
1	50·9497	4430·720	·836	·678	+ ·158	+ 10·70	2	33·9069	4181·964	·974	·947	·027	+ 1·22
2	50·7644	4427·678	·778	·420	+ ·358	+ 24·24	2	31·5872	4152·360	·424	·223	·201	+ 14·58
2	50·0134	4415·432	·500	·354	+ ·146	+ 9·91	2	31·2119	4147·654	·726	·587	·139	+ 10·06
2	49·3724	4405·080	·114	·951	+ ·163	+ 11·08	2	30·1597	4134·582	·686	·676	·010	+ ·72
1	48·7820	4395·642	·642	·426	+ ·216	+ 14·73	1½	29·6231	4127·983	·003	·029	- ·026	- 1·89
1½	47·8569	4381·008	·996	·961	+ ·035	+ 2·39	3	29·2754	4123·732	·866	·841	+ ·025	+ 1·82
2	47·0392	4368·235	·183	·841	·342	+ 23·48	2	29·1154	4121·782	·918	·639	+ ·279	+ 20·31
2	47·2546	4371·585	·537	·313	·194	+ 13·31	2	28·6926	4116·647	·793	·739	+ ·054	+ 3·94
1	47·1582	4370·080	·030	·867	·163	+ 11·18	2	26·6798	4092·580	·840	·626	+ ·214	+ 15·69
	45·2570	4340·912	·882	·634	·248	+ 17·14							

Weighted mean + 11·23

V_a - 15·95

V_d - ·02

Curvature - ·28

Radial velocity..... - 5 0

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MEASUREMENT AND REDUCTION OF SPECTROGRAMS.

The spectrograms taken with the Brashear spectroscope have all been measured and reduced in the way described in last year's report, by reducing all linear measures to wave lengths by the simple form of Hartmann's interpolation formula

$$\lambda = \lambda_0 + \frac{c}{s_0 - s} \text{ where}$$

s = linear value of line

λ = wave length

and c , λ_0 , s_0 are constants.

Owing to the known temporary use of this spectroscope, and to the fact of its being of an adjustable type, it was not thought worth while to develop a shorter method of reduction. But when the new spectrograph was completed and tested, and when it was found that the Brashear single material camera objective gave such excellent results in single prism work, a shorter method of reducing the measurements of the single prism plates was evolved. In the method previously used, every linear measurement had to be reduced to wave lengths by the above formula which involves the looking up of a logarithm and an anti-logarithm, a subtraction and two additions, all of seven figures, and besides this the constants of the formula, a matter of fifteen or twenty minutes work, have to be obtained. Hartmann, to whom we owe so many valuable methods and devices in spectrographic work, has, in A. N. No. 3703, described in full detail a method for avoiding most of this tedious and laborious work, which has been, in a somewhat simplified form, adapted for use here. In this method, instead of reducing each measure of each line to its wave length by the interpolation formula, tables are made in which the wave lengths of all star and comparison lines are reduced to their corresponding linear measures or micrometer readings by the same interpolation formula. The displacements in kilometres per revolution of the micrometer screw for every wave length are also computed and tabulated, and the differences between the tabulated micrometer reading for any star line and the actual measured reading, when reduced to the same zero, multiplied by this value gives at once the velocity in kilometres for that line. When a set of tables are once obtained in the manner described below, not only is the determination of the constants of the interpolation formula avoided, but all the laborious logarithmic computation is done away with, and the displacements are determined at once by simple subtraction after the measurement has been brought into coincidence with the standard by a graphical interpolation of exactly the same nature as required in the previous method.

When a spectrograph for radial velocity work has been brought into adjustment, such adjustment, so far as position of the prisms and focus of the collimator is concerned, remains permanent and the spectra produced, so long as the temperature is the same, are identical. If the temperature changes, the deviation and dispersion of the prism changes and there is also, in general, a slight change in the focus of the camera. In consequence the distance between any two lines in the spectrum is a function of the temperature only, increasing slightly with increase of temperature. Hence, in constructing the tables of micrometer revolutions of star and comparison wave-lengths above mentioned, we have to take into account this variation and construct tables for different temperatures. Owing to the smallness of the variation, it has been found sufficient to construct a table for every 10° C change of temperature and hence 6 sets of tables will be sufficient over the whole range actually occurring from -20° to $+30^\circ$ C.

It has been found in practice that, owing probably to accidental errors of setting on the comparison lines used as standards, the three constants of the interpolation formula obtained by choosing three lines with their known wave lengths and micrometer readings, substituting and solving in the formula, vary considerably in different spectra, even when made at the same temperature. It was necessary, therefore, in order to obtain a harmonious set of tables, varying continuously with the temperature,

to eliminate these accidental variations and the method pursued was one of averages combined with an assumed simple continuous change of the measurements and values of the constants with varying temperature. This assumption, though perhaps not strictly true, is convenient in use and, as it can not introduce any error in velocity determinations, has been used.

A number of spectra of the Fe-V spark, three or four at each temperature, were made at five different temperatures between 14.6° and 30° C. The greatest care was taken in making these spectra, not only in having the camera accurately focussed, but also in ensuring uniform temperature conditions in the prism, this being obtained by maintaining the whole instrument at constant temperature for several hours before exposure. About 20 good lines between λ 3930 and λ 4875 were measured on a selected plate at each temperature, and of these 20 lines, three were selected as standards for determining the constants of the interpolation formula. The choice was made, after trial of several sets, of the three in which the residuals between the computed and known values of the wave lengths of intermediate lines and of lines at the ends of the spectrum were a minimum. The three lines finally chosen as standards were λ 4594.216 V, λ 4395.382 V, and λ 4202.195 Fe. With these standards an attempt was made to reduce the above mentioned residuals by using the complete Hartmann interpolation

formula $s_0 - s = \frac{c}{(\lambda - \lambda_0)^\alpha}$ where α may be given any value, Hartmann has found in the Potsdam spectrograph that a value of 0.6 for α gives the lowest residuals. A trial was made here by Mr. N. B. McLean, who made all the computations required in this work, of three values of α, 0.5, 0.7, and 0.9, as well as unity, but the residuals were lower with the simpler form of α as unity than in the others.

In bringing together, in the table below, the micrometer readings of the three standard lines with the corresponding calculated constants for five temperatures, all the readings have been reduced to the same value for the standard λ 4395.382 of 48.7700. This brings the line at minimum deviation λ 4415 very near the reading 50 or the centre of the micrometer screw, which has an effective length of 5^{cm} and is of 0.5^{mm} pitch. Every star plate measured is so set on the stage as to bring λ 4395.382 as near as possible to reading 48.77, but any small deviations are of no moment, as their effect is removed in the curve drawing to be presently described.

TABLE OF CONSTANTS.

Temp. C.	4594.216	4395.382	4202.198	s ₁	log c	λ ₁
	s ₁	s ₂	s ₃			
14.6°	60.1754	48.7700	35.4949	184.8423	5.4709008	2222.025
17.4	60.1809	48.7700	35.4836	184.6549	5.4691204	2226.434
21.1	60.2038	48.7700	35.4469	184.4126	5.4668483	2235.390
25.4	60.2115	48.7700	35.4520	185.2279	5.4719729	2222.810
30.0	60.2098	48.7700	35.4366	184.3153	5.4659474	2238.319

As will be noticed in the table, there is a general progression of the values for the micrometer readings and constants with the temperature, but this is not uniform and, for one temperature, 25.4° C., the readings and constants are markedly variant. This is probably due to some slight difference in the inclination of the plate in this spectrum, and it was consequently omitted from the discussion. In order to obtain uniformly progressive values, the accidental discrepancies due to accidental errors of setting on the lines, deviation from the true camera focus and plate inclination, or inaccurate

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temperature determinations must be removed. If we form a table of the differences between the readings $s_1 - s_2$, $s_1 - s_3$, $s_2 - s_3$, and the ratio of the first two we should be

Temp. C.	$s_1 - s_2$	$s_2 - s_3$	$s_1 - s_3$	$\log \frac{s_1 - s_2}{s_1 - s_3}$
14.6°	11.4054	13.2751	24.6805	9.66476
17.4	11.4109	13.2864	24.6973	9.66467
21.1	11.4338	13.3231	24.7568	9.66450
25.4	11.4415	13.3180	24.7596	9.66474
30.0	11.4398	13.3334	24.7731	9.66444

able to form a progressive series. The ratios in the last column are nearly constant, but omitting the discrepant temperature 25.4° and allowing for accidental errors, a small regular decrease with increase of temperature is evident, which amounts to nearly 2 in the fifth place per degree. A comparison of the figures in the fourth column also indicates a change of about 0.007 revolution per degree. If we form an arbitrary series from the last two columns, using values averaged from them for the middle of the range together with the differences above quoted and computing $s_1 - s_2$ and $s_2 - s_3$ from them, we obtain the following values:—

Temp. C.	$s_1 - s_2$	$s_2 - s_3$	$s_1 - s_3$	$\log \frac{s_1 - s_2}{s_1 - s_3}$
0°	11.3630	13.2110	24.5740	9.66502
10°	11.3902	13.2538	24.6440	9.66482
20°	11.4173	13.2967	24.7140	9.66462
30°	11.4444	13.3396	24.7840	9.66442

Which combined with a reading of 48.77 for s_2 , give the following continuous values of s_1 s_2 s_3 from which the constants given for 10° 20° and 30° C were obtained.

Temp. C.	4594.216	4395.382	4202.198	s	$\log c$	L_0
	s_1	s_2	s_3			
0°	60.1331	48.7700	35.5591			
10	60.1602	48.7700	35.5162	184.8465	5.4715619	2218.782
20	60.1873	48.7700	35.4733	184.5845	5.4686837	2228.988
30	60.2144	48.7700	35.4304	184.3133	5.4657512	2239.261

With these constants, the micrometer readings corresponding to the normal wave lengths of the comparison and star lines used were computed and tabulated separately, small portions of these tables being given below. It is also necessary, as the velocities are now to be computed from micrometer instead of wave length displacements, to obtain the velocity value of one revolution of the micrometer screw in kilometres per

second for every star wave length used, and to tabulate these along with the wave lengths and micrometer readings of the star lines. In the formula,

$$s_o - s = \frac{c}{\lambda - \lambda_o}, \text{ by differentiation}$$
$$ds = \frac{c \, d\lambda}{(\lambda - \lambda_o)^2}, \text{ and by Doppler's principle}$$
$$v = 299860 \frac{d\lambda}{\lambda}, \text{ and substituting value of } d\lambda$$
$$v = \frac{299860}{\lambda} \frac{(\lambda - \lambda_o)^2}{c} \cdot ds$$

FE. V. COMPARISON LINES.

Wave Length.	Micrometer Readings.		
	10°	20°	30°
4875·674	73·3689	73·4161	73·4628
4871·453	73·1914	73·2386	73·2850
.....
4404·929	49·3612	49·3658	49·3676
4400·738	49·1040	49·1049	49·1058
4395·382	48·77	48·77	48·77
.....
3969·411	15·6591	15·5292	15·3986
3930·450	11·8081	11·6580	11·5072

STAR LINES.

Wave Length.	10°C		20°C		30°C	
	Micrometer Reading.	Velocities per Revn.	Micrometer Reading.	Velocities per Revn.	Micrometer Reading.	Velocities per Revn.
4861·527	72·7721	1454·4	72·8187	1452·8	72·8648	1451·2
4713·308	66·1128	1336·6	66·1505	1334·5	66·1879	1332·4
.....
4340·634	45·2589	1050·1	45·2481	1046·9	45·2387	1043·7
4325·939	44·2854	1039·1	44·2723	1035·9	44·2592	1032·7
4320·992	43·9547	1035·4	43·9405	1032·2	43·9264	1029·0
4318·817	43·8087	1033·8	43·7941	1030·6	43·7795	1027·4
.....
3964·875	15·2197	778·5	15·0874	774·5	14·9546	770·5
3934·825	12·2493	757·6	12·1015	753·6	11·9531	749·6

The process of reducing the measurement of a star spectrum now becomes considerably simpler. The measurement is performed as before, the plate being set in the first position, with the red end of the spectrum to the apparent right, so that when the V line 4395·382 is under the cross wire the screw reading is nearly 48·77. On the reversal of the plate this reading should become 51·23. After the mean of the two settings is taken, the table of comparison lines whose temperature argument is nearest to the temperature at which the spectrum was taken, or preferably the one in which the difference in the micrometer values for any two comparison lines agrees most closely with the measured value, is placed beside the mean readings and the differences between the two are plotted on cross section paper as ordinates, with micrometer readings as abscissae. Through the points thus obtained, a smooth curve is drawn which fills an exactly similar purpose to the curve used in the former method for obtaining the corrections to the star lines, and although open to the objection expressed by some observers of arbitrariness, nevertheless seems to me to be preferable

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to direct interpolation between adjacent comparison lines, as getting rid of some accidental errors. As it is impossible to know beforehand what effect any particular method of curve drawing will have on the resultant value, any fear of prepossession is removed. Corrections to the micrometer readings for the star lines are taken directly from this curve, and the corrected readings, which reduce the star lines to the same zero as the comparison lines, are then put down in a column next to the mean readings. The differences between these and the corresponding tabular values give the displacement due to motion in the line of sight which multiplied by the tabulated 'Velocities per Revolution' give the velocity at once. The work required is the same as in the previous method after the computed wave lengths have been obtained and all the laborious determination of constants and calculation of the wave lengths is avoided. Moreover, the liability to numerical mistakes is much lessened and these advantages are obtained without loss of accuracy in the resulting velocity. Any variation in the velocity obtained can only be due to differences in the drawing of the interpolation curve in the two cases and this can only be slight in any event.

Two or three measurements have been reduced by both methods, an example of which is given below, with resulting values within two-tenths of a kilometre of one another which has no significance in single prism work. Furthermore, as an evidence that the interpolation is linear and that the curve drawing is not likely to introduce error, the same measurement has been reduced by using tables for two different temperatures with exactly the same result.

The correction for curvature is applied by Hartmann to each star line of the spectrum and he has developed a system for all cases that may occur. In the case of the single prism plates the correction for curvature is so small, amounting to from $\cdot 00024$ to $\cdot 00032$ revolutions that the error introduced by using either 2 or 3 in the fourth place will amount to more than can occur by forming an average correction in velocity and applying it at the end of the reduction. The correction was obtained by measuring the abscissas and ordinates of the parabola formed by long spectrum lines and obtaining the constants of the equation. These equations which range from $x = \cdot 00096 y^2$ for $\lambda 4875$ to $\cdot 00127 y^2$ for $\lambda 3968$ where x is the correction and $2y$ the distance between the points of measurement on the comparison lines. This reduced to velocities gives for the usual distance between the tips in our measurement from $0\cdot 29$ to $0\cdot 26^{\text{kms}}$ per second, or in the mean $0\cdot 28$, and is of course, applied with the negative sign.

The various sources of errors in the measurement and reduction of spectrum plates were discussed in last year's report, and it may be said that further experience has not led to any modification of the statements therein. Any method based upon the use of an interpolation formula depends for its accuracy on a knowledge of the true wave lengths of the star lines, and such are not yet at hand. For stars of early type, it is probably the best means available for obtaining the velocity, but for solar stars in which, although the wave lengths of the single lines are known fairly accurately, the wave lengths of complex blends, such as inevitably occur in low dispersion spectra, are very uncertain and the error introduced correspondingly large. An evidence of this is given by the high residuals from the measurements of even such stars as Arcturus, where great care was taken in the choice of the least complex lines. It must be remembered, however, that the linear dispersion with a single prism is only one-third of the three prism instruments and is at $\text{H}\gamma$ about 30 tenth-metres to the millimetre, so that an accidental error of setting of $0\cdot 005^{\text{mm}}$ corresponds to over 10^{km} velocity. In consequence of these conditions, the production and measurement of solar type spectra with the single prism instrument is being held over as much as possible until the spectra-comparator is received. In this instrument trouble with blends and identifications is to a great extent avoided, as no accurate knowledge of wave lengths is required, and the displacement is obtained by direct comparison with some standard plate, whose velocity value is known.

The corrections for diurnal and annual motion are applied in the way described last year, and no further description is here required.

RADIAL VELOCITY.

Star η Boötis 812.
Date, June 10. 14^h 10^m
Hour angle, 30^m W.

Observer, J. S. P.
Measurer, W. E. H.
Computer, W. E. H.

Weight.	Micrometer Readings.		Means.	Corrected readings. Star lines.	Displacement in revolutions.	Velocity.
2	73·3385	6258	3399
2	72·8850	0812	8854
2	·7625	2008	7643	7928	·0207	+ 30·11
1½	57·7995	1625	8020	·8192	·0367	44·37
2	·7600	2035	7617
2	53·0638	9015	0646
1½	52·9870	9765	9887	·0120	·0379	43·35
2	·4055	5582	4071	·4314	·0358	40·68
2	·2082	7605	2073
2	49·3580	6003	3623	·3923	·0294	32·29
2	48·7390	2292	7384
2	·7635	1982	7661	·7971	·0244	26·62
2	45·9942	9685	9963	·0320	·0289	30·59
2	·2560	7080	2575
2	·2462	7095	2518	·2885	·0304	31·92
1½	44·2682	6958	2697	·3084	·0230	23·95
2	43·0715	8900	0742	·1162	·0361	37·04
3	42·0985	8695	0980
1	41·3328	6288	3355	·3818	·0391	39·37
2	40·5550	4030	5595	·6077	·0317	31·66
1½	39·7640	2005	7652	·8154	·0331	32·79
2	·7412	2280	7401
2	·0528	9075	0561	·1078	·0312	30·68
1½	37·8018	1632	8028	·8566	·0221	21·45
2	·3295	6320	3322	·3870	·0259	25·02
2	35·4572	5108	4567
1½	·2082	7565	2093	·2689	·0322	30·42
2	34·6832	2745	6878	·7486	·0287	26·98
2	31·9600	0008	9631	·0313	·0272	24·88
2	30·9460	0135	9498	·0213	·0250	+ 22·63
2	27·3125	6570	3112

$\odot = 438^{\circ} 27' \cdot 1$
 $\angle \odot \quad 33 \cdot 8$
 $\quad 439^{\circ} 00 \cdot 9$
 $\lambda = 198^{\circ} 01 \cdot 8$
 $\odot - \lambda = 240^{\circ} 59' \cdot 1$

$\log \sin (\odot - \lambda) \quad 9 \cdot 94175$
 $\log b \quad 1 \cdot 4193$
 $\quad 1 \cdot 36105$
 $b \sin (\odot - \lambda)$
 $\quad \quad \quad c \quad - 22 \cdot 96$
 $\quad \quad \quad \quad \quad + \cdot 44$

$V_s = + 31 \cdot 34$
 $V_n = - 22 \cdot 52$
 $V_d = - \cdot 04$
 $\text{Curv.} = - \cdot 28$

Radial vel. + 8·5

RADIAL VELOCITIES.

As mentioned in last year's report, the principal work undertaken with the spectrograph has been the determination of the radial velocities of stars and this work, except a few plates of the standard velocity stars given in last year's report, some plates of hydrogen stars and a few plates of Mira Ceti, a discussion of which is given below, has been confined to plates taken for the determination of the velocity curves and orbits of spectroscopic binaries. Of the 150 odd binaries so far discovered, only some 20 have had their orbits determined, and these have been chiefly of the solar type, which admit of accurate velocity determinations. About two-thirds of the binaries known are, however, stars of earlier type with few lines, often only the hydrogen series, which are in some cases diffuse. None of these stars admit of very accurate velocity determinations, and it is only in cases where the range of velocity is great and the corresponding percentage error small that accurate elements can be obtained. However, by increasing the number of observations and thereby obtaining

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mean values of the velocity for different phases in the orbit, fairly accurate values of the elements may be obtained in the stars with only a moderate range of velocity. It has been found necessary to carry out this procedure in the case of some of the binaries under observation, and so, although about six hundred spectra of 12 binary stars have been obtained, the necessity of obtaining in many stars a great number of plates and the impossibility of keeping the measuring and reducing up to date, have prevented more than one orbit being completed, although measurements have been made on several others.

Hence, as it seems useless to publish the measurements of those not finished, only those of α Draconis, which has been satisfactorily completed and of ι Orionis, of which preliminary elements have been obtained, will be here given, and the others which are under measurement, η Piscium, σ Andromedae, α Corona Borealis, η Bootis, ϵ Herculis, δ Aquilae, θ Aquilae, η Virginis and γ Geminorum, will be given as soon as they are completed.

 α Draconis.

α Draconis R. A. $14^h 1.7^m$ Decl. $+64^\circ 51'$, Magnitude Visual 3.6, Photographic 4.0 was the first spectroscopic binary star to be observed here, the first plate being made on July 2, 1906, and the last on July 5, 1907. With the old Brashear spectro-scope, 37 plates were made between July 2, 1906 and February 21, 1907, and with the new single prism spectrograph, 9 plates between May 22 and July 5, 1907. Of these plates, 45 have been measured, the other being rejected as unsuitable for measurement. All the measurement and reduction have been carefully and ably performed by Mr. W. E. Harper, assistant in spectrographic work. Some duplicate measures of plates giving large residuals from the velocity curve have been made by Mr. N. B. McLean and myself, but the velocity values except in the case of one plate have not been materially altered by the remeasurement.

The spectrum which is of Vogels Ia2 and Miss Maury's VIII *a* type possesses, in the measurable region with the Brashear instrument, generally only three lines Fe Ti 4549.642 Mg 4481.400 and H_γ 4340.634. In some of the spectra three or four other faint metallic lines are measurable. With the new single prism instrument in addition to the three above given H_β and H_δ and in strongly exposed plates H_ϵ can also be accurately measured. Of these lines the Mg line is the best defined, and it is usually given about twice the weight of the others. Although about twice as many lines are obtainable with the new spectrograph as with the old, the linear dispersion is only three-fifths as great and the probable errors in the two cases are likely not much different from one another. The best criterion as to the relative accuracy of a single determination of the velocity is obtained from the residuals between the velocities computed from the elements of the orbit and the observed velocities. These give a probable error of $\pm 3.4^{\text{km}}$ per second, while if three largely discrepant values are omitted it reduces to $\pm 2.9^{\text{km}}$.

Considering the character of the spectrum and the dispersion employed, this result may be considered satisfactory and the determination of the elements probably as close as can be obtained from the observations. However, if many more observations were available a correction to the elements might be obtained, although it seems probable from the agreement already obtained that this would be small.

The data as regards the conditions of temperature, slit width, focus, &c., are given in the Record of Observations, which is followed by the measures of the different plates. The results of these measurements are collected in the table below from which the velocity curve is obtained.

RECORD OF SPECTROGRAMS.

Star.	No. of Negative.	Plate.	Date.	Middle of Exposure. G. M. T.	Duration.	Hour Angle at end.	COMPARISON SPECTRUM.			TEMPERATURE.				Slit Width.	FOCAL POSITION.			Observer.	Remarks.	
							Beginning.	End.	Kind.	Room.		Prism Box.			Star Focus.	Collimator.	Camera.			Seeing.
										Begin- ning.	End.	Begin- ning.	End.							

α Draconis	321	Seed 27...	July 2	16 20	60	4 20 W.	s.	20	Fe. Spark.	68.0	66.8	23.9	23.9	.025	18.0	15.2	5.95	P	Fair..	Clouds.
"	328	"	" 4	17 45	60	6 00 W.	17	17	"	58.3	56.3	21.4	21.4	.025	18.2	15.2	5.95	P	"	
"	376	Seed R...	Aug. 15	14 10	50	5 00 W.	20	20	"	71.2	69.2	26.0	26.1	.020	18.6	15.2	5.95	H	Good.	
"	381	"	" 24	13 50	55	5 30 W.	15	15	"	66.2	65.5	23.2	23.1	.025	18.6	15.2	5.95	H	Fair..	
"	385	"	Sept. 5	15 30	60	7 55 W.	16	16	"	60.8	59.8	20.8	21.0	.025	12.4	15.2	5.68	H	Hazy.	
"	386	"	" 6	13 50	75	6 30 W.	16	16	"	71.0	68.0	24.4	24.4	.025	18.6	15.2	5.68	H	"	
"	389	"	" 10	14 15	60	7 00 W.	16	16	"	70.8	69.6	26.7	26.7	.025	15.0	15.2	5.75	H	Good.	
"	394	"	" 19	13 35	75	7 00 W.	16	16	"	76.3	70.0	25.5	25.6	.025	20.0	15.2	5.75	H	"	
"	398	"	" 27	13 10	60	7 00 W.	20	20	"	64.1	61.0	21.2	21.2	.025	15.0	15.2	5.70	H	"	
"	404	Seed 27...	Oct. 3	12 30	60	6 50 W.	20	20	"	64.0	62.0	21.1	21.1	.025	10.0	15.2	5.80	H	Hazy.	
"	412	"	" 18	12 30	70	9 00 W.	25	25	"	54.0	52.0	16.4	16.4	.025	18.5	15.2	5.80	H	"	
"	416	"	Nov. 1	12 58	60	9 05 W.	20	20	"	40.5	38.5	8.6	8.7	.025	18.5	15.2	5.79	H	Good.	
"	417	"	" 1	13 55	50	10 00 W.	25	25	"	38.5	37.5	8.7	8.6	.025	18.5	15.2	5.79	H	"	
"	418	"	" 1	15 50	60	11 55 W.	20	20	"	36.0	35.7	9.3	9.1	.025	18.5	15.2	5.79	H	Fair..	
"	422	"	" 6	12 45	90	9 28 W.	25	25	"	44.2	42.2	12.1	12.0	.025	18.5	15.2	5.70	H	"	
"	423	"	" 6	14 00	60	10 30 W.	25	25	"	42.2	41.2	12.0	12.1	.025	18.5	15.2	5.70	H	Good.	
"	424	"	" 6	15 00	60	11 30 W.	25	25	"	41.2	40.0	12.1	12.0	.025	18.5	15.2	5.70	H	"	
"	426	"	" 6	17 05	60	10 25 E.	25	25	"	39.0	38.4	11.1	11.1	.025	18.5	15.2	5.70	H	Fair..	
"	428	"	" 8	12 40	60	9 15 W.	25	25	"	41.7	39.7	9.6	9.7	.025	18.5	15.2	5.70	H	Good.	
"	429	"	" 8	13 40	60	10 15 W.	25	25	"	39.7	38.0	9.7	9.8	.025	18.5	15.2	5.70	H	"	
"	430	"	" 8	14 50	80	11 35 W.	25	25	"	38.0	36.5	9.8	9.7	.025	18.5	15.2	5.65	H	Fair..	
"	431	"	" 8	16 00	60	11 25 E.	20	20	"	36.5	35.5	9.7	9.6	.025	18.5	15.2	5.65	H	"	
"	435	"	" 16	14 10	50	11 10 W.	20	20	"	32.2	31.2	3.6	3.6	.037	18.5	15.2	5.72	P	Good.	
"	438	"	" 19	13 15	90	10 45 W.	20	20	"	40.5	39.7	9.9	10.0	.030	18.5	15.2	5.72	H	Cloudy	
"	447	"	Dec. 7	16 37	45	9 00 E.	20	20	"	2.0	3.6	19.1	19.1	.030	19.0	15.2	5.60	P	"	
"	457	"	" 11	18 40	50	6 40 E.	20	20	"	9.3	8.7	0.7	1.0	.033	19.0	15.2	5.68	H	Hazy.	
"	458	"	" 13	13 27	50	11 50 E.	20	20	"	24.0	21.5	0.5	0.5	.037	19.0	15.2	5.75	H	"	
"	462	"	" 17	17 40	50	7 15 E.	25	25	"	16.8	17.2	6.2	6.2	.030	19.0	15.2	5.73	P	"	
"	490	"	" 18	17 05	50	7 50 E.	25	25	"	10.0	9.7	2.4	2.8	.037	19.0	15.2	5.73	H	"	
1907.																				
"	524	"	Jan. 9	16 10	60	7 05 E.	20	20	"	8.4	8.0	12.6	12.5	.033	19.0	15.2	5.63	P	"	
"	528	"	" 11	17 45	40	5 40 E.	20	20	"	10.0	10.2	10.1	10.1	.037	19.0	15.2	5.66	P	Fair..	

RECORD OF SPECTROGRAMS.—Continued.

Star.	No. of Negative.	Camera.	Plate.	Date.	Middle of Exposure.		Duration.	Hour Angle at end.	COMPARISON SPECTRUM.		TEMPERATURE.				Slit width in Millimetres.	FOCAL POSITION.			Seeing.	Observer.	Remarks.	
					G. M. T.	h. m.			End.	Beginning.	Kind.	Room.		Prism Box.		Star Focus.	Collimator.	Camera.				
												Begin- ning.	End.	Begin- ning.								End.

α Draconis...	566	Seed 27	Jan. 21	16 15	60	6 20 E.	20	20	Fe. spark.	6.2	4.3	12.5	12.6	.033	19.0	15.2	5.64	P	
"	593	"	" 30	17 02	56	4 55 E.	22	22	"	9.0	7.8	4.6	4.5	.037	19.0	15.2	5.67	H	
"	603	"	Feb. 6	18 02	55	3 30 E.	22	22	"	12.0	12.2	8.6	8.7	.030	20.5	15.2	5.65	P	
"	612	"	" 12	15 00	50	6 15 E.	22	22	"	6.8	7.3	9.0	8.8	.037	20.5	15.2	5.65	H	
"	772	I.L.	"	May 22	20 23	30	5 30 W.	4-8-4	4-8-4	Fe. Vspark	9.5	8.9	13.4	13.5	.025	45.0	10.8	18.3	P	Centigrade used from here on.
"	799	"	"	" 31	16 37	26	2 15 W.	2-2-2-2	2-2-2-2	"	14.6	13.5	18.9	18.9	.025	45.0	10.8	18.58	P	
"	809	"	"	June 8	18 50	30	5 05 W.	3-3-3-3	3-3-3-3	"	13.3	13.0	16.9	16.9	.025	45.0	10.8	18.58	P	
"	815	"	"	" 10	16 47	30	3 10 W.	2-2-2-2	2-2-2-2	"	14.0	13.5	17.8	17.8	.025	45.6	10.8	18.61	P	
"	823	"	"	" 11	14 17	35	0 47 W.	3-3-3-3	3-3-3-3	"	17.6	16.6	19.4	19.4	.025	45.6	10.8	18.61	H	
"	859	"	"	" 20	15 17	35	2 12 W.	3-4-8	3-4-8	"	22.6	21.4	25.7	25.7	.030	45.3	10.8	18.68	H	
"	870	"	"	" 21	16 36	38	3 45 W.	6-7-7-6	6-7-7-6	"	24.3	24.3	29.0	29.0	.025	47.7	10.8	18.63	P	
"	911	"	"	July 4	15 02	48	3 07 W.	4-4-4-4	4-4-4-4	"	21.5	20.0	29.0	29.0	.030	45.0	10.8	18.75	H	
"	916	"	"	" 5	14 50	40	2 55 W.	2-5-3	2-5-3	"	22.6	21.2	26.4	26.4	.032	45.0	10.8	18.64	P	

1906. July 2.
G. M. T. 16^h 20^m

α DRACONIS 321.

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displace- ment.	Velocity.
3	S 70·0713	4549·642
1	70·0979	4549·956	·956	·642	·314	+20·66
3	65·2872	4494·786	·738
3	64·0885	4481·577	557	·400	·157	+10·50
2	63·5923	4476·168	·185
2	S 56·7442	4404·927
2	54·5836	4383·688	·720
$\frac{1}{2}$	52·9892	4368·373	·403	·071	·330	+22·63
3	S 48·4045	4325·939
3	40·8267	4260·615	·640
1	37·4777	4233·490	·498	·328	·170	+12·03

Weighted mean..... +13·73

V_a —9·73

V_d —·10

Curvature..... —·50

Radial velocity..... + 3·4

1906. July 4.
G. M. T. 17^h 45^m.

α DRACONIS 328.

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displace- ment.	Velocity.
2	70·2010	4549·557	·642
3	S 68·4274	4528·798
2	65·4225	4494·732	·738
3	64·2438	4481·734	·764	·400	·264	+24·35
1	63·7297	4476·127	·185
3	S 56·8874	4404·927
2	54·7276	4383·695	·720
2	50·2072	4341·035	·045	·634	·411	+28·35
3	S 48·5469	4325·939
3	46·5476	4308·152	·081

Weighted mean..... +25·95

V_a —9·48

V_d —·15

Curvature..... —·50

Radial velocity..... — 15·8

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α DRACONIS 376.

1906. Aug. 15.
G. M. T. 14^h 10^m.

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
1	70.2400	4549.622642
3	70.2117	4549.287	.247	.642	.395	-26.04
3	S 68.4580	4528.798
2	65.4466	4494.712738
3	64.2008	4481.002	.012	.400	.388	-25.95
2	63.7596	4476.199185
2	62.8784	4466.689727
3	S 56.9013	4404.927
2	54.7324	4383.683720
2	50.1100	4340.087	.127	.634	.507	-34.98
3	S 48.5513	4325.939
3	46.5436	4308.093081

Weighted mean -28.24
V_a -2.69
V_d - .15
Curvature..... - .50
Radial velocity -31.6

α DRACONIS 381.

1906. Aug. 24.
G. M. T. 13^h 50^m

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	72.8078	4583.992018
$\frac{1}{2}$	69.9561	4549.566642
1	69.9851	4549.909	.869	.642	.227	+14.95
3	S 68.1809	4528.798
2	65.1680	4464.695738
3	63.9714	4481.466	.496	.400	.096	+6.42
1	63.4880	4476.196185
2	62.6072	4466.678727
3	S 56.6392	4404.927
2	54.4785	4383.681720
2	49.9290	4.40.739	.719	.634	.085	+6.55
3	48.3088	4325.982939
3	S 46.2960	4308.081

Weighted mean +7.88
V_a -0.92
V_d - .15
Curvature..... - .50
Radial velocity.. +6.3

1906. Sept. 5.
G. M. T. 15^h 30^m

α DRACONIS 385

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	63·7726	4528·806	793
2	S 60·7656	4494·738
1	59·5384	4481·214	200	400	200	-13·38
1	59·0801	4476·218	185
1	58·2004	4466·710	727
2	S 52·2319	4404·927
2	S 43·9037	4325·939

Velocity. -13·33

V +1·53

V_d - 15

Curvature - 50

Radial velocity.. -12·5

NOTE.—Hγ is not set at micrometer reading 50·0000 as usual.

1906. Sept. 6.
G. M. T. 13^h 50^m

α DRACONIS 386

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	S 63·1702	4528·798
1/2	65·1659	4494·731	738
2	63·9349	4481·156	170	400	230	-15·39
1	63·4777	4476·170	185
1	62·5983	4466·658	727
3	S 56·6347	4404·927
3	48·2985	4325·928	939
3	S 46·2942	4308·081

Velocity. -15·39

V_d +1·70

V_d - 16

Curvature - 50

Radial velocity..... -14·3

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α DRACONIS 389.

1906. Sept. 10.
G. M. T. 14^h 15^m

Observed by }
Measured by } W. E. HARPER.

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displace- ment.	Velocity.
2	70·1700	4549·634	·642
2	70·1237	4549·088	·096	·642	·546	—35·98
3	S 68·3865	4528·798
$\frac{1}{2}$	65·3676	4494·630	·738
2	64·1289	4480·995	·000	·400	·400	—27·60
2	62·8095	4466·709	·727
1	57·8697	4415·294	·293
3	S 56·8356	4404·927
2	54·6829	4383·761	·727
2	50·0654	4340·180	·164	·634	·470	—32·43
3	S 48·5000	4325·939
3	46·4936	4308·064	·081

Weighted mean..... — 32·00

V_a + 2·53

V_d — ·15

Curvature..... — ·50

Radial velocity..... — 30·1

α DRACONIS 394.

1906. Sept. 19.
G. M. T. 13^h 35^m

Observed by }
Measured by } W. E. HARPER.

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displace- ment.	Velocity.
2	*70·0925	4549·642	·642
1	*70·0405	4549·017	·017	·642	·625	—41·14
3	S 68·4238	4528·798
2	65·4129	4494·677	·738
3	64·1348	4480·597	·640	·400	·760	—50·84
2	63·7285	4476·169	·185
2	62·8504	4466·681	·727
3	S 56·8823	4404·927
2	54·7233	4383·697	·720
$\frac{1}{2}$	50·1287	4340·337	·344	·634	·290	—20·00
3	48·5450	4325·935	·939
3	S 46·5397	4308·081

Weighted mean..... — 45·26

V_a + 4·27

V_d — ·15

Curvature..... — ·50

Radial velocity..... — 41·6

Not used in first measurement.

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α DRACONIS 398.

1906. Sept. 27.
G. M. T. 13^h 10^m

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displace- ment.	Velocity.
3	S 65.3536	4494.738
2	64.0960	4480.863	800	400	600	-40.14
3	63.6737	4476.254	185
2	62.7994	4466.793	727
3	S 56.8297	4404.927
2	54.6765	4383.720	720
3	50.0377	4339.876	874	634	760	-52.44
3	S 48.5075	4325.939
3	46.5037	4308.065	081

Weighted mean..... - 47.52
V_a..... + 5.75
V_d..... - .15
Curvature..... - .50
Radial velocity..... - 42.4

α DRACONIS 404.

1906. Oct. 3.
G. M. T. 12^h 30^m

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displace- ment.	Velocity.
3	S 68.3345	4528.798
2	65.3395	4494.748	738
2	*70.1183	4549.743	642
1	*70.0688	4549.154	052	642	590	-26.24
3	64.0699	4480.719	670	400	730	-48.83
2	63.6618	4476.260	185
2	62.7866	4466.776	727
3	S 56.8227	4404.927
3	S 48.4953	4325.939

Weighted mean..... - 45.68
V_a..... + 6.84
V_d..... - .15
Curvature..... - .50
Radial velocity..... - 39.5

*Not used in first measurement.

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α DRACONIS 412.

1906. Oct. 18.
G. M. T. 12^h 30^m

Observed by }
Measured by } W. E. HARPER.

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displace- ment.	Velocity.
1	69.-----	4550.011	.011	.642	.369	+24.71
3	S 68.3213	4528.798
2	65.3250	4494.694738
1	64.1560	4481.751	.750	.400	.350	+23.41
1	63.6515	4476.225185
2	62.7761	4466.723727
3	S 56.8302	4404.927
3	S 48.5299	4325.939

Weighted mean..... +23.84
V_a..... + 9.16
V_d..... - .12
Curvature..... - .50
Radial velocity..... +32.4

α DRACONIS 416.

1906. Nov. 1.
G. M. T. 13^h

Observed by }
Measured by } W. E. HARPER.

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displace- ment.	Velocity.
1	69.9793	4549.632642
1½	69.9081	4548.784	.792	.642	.850	-55.93
3	S 68.2089	4528.798
1	65.2177	4494.705738
2½	63.9462	4480.618	.650	.400	.750	-50.18
1	62.6689	4466.703727
3	S 56.7304	4404.927
2	54.5828	4383.703720
1	49.9510	4339.793	.804	.634	.830	-57.27
3	S 48.4341	4325.939
3	46.4368	4308.075081

Weighted mean..... -52.67
V_a..... +10.76
V_d..... - .12
Curvature..... - .61
Radial velocity... .. -42.6

7-8 EDWARD VII., A. 1908

α DRACONIS 417.

1906. Nov. 1.
G. M. T. 14^h

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	70·0521	4549·640	·642
1	69·9868	4548·863	·862	·642	·780	-51·32
3	S 68·2795	4528·798
2	65·2918	4494·768	·738
1	64·0208	4480·694	·680	·400	·720	-48·16
1	63·6034	4476·123	·185
1	62·7386	4466·729	·727
3	S 56·7968	4404·927
2	54·6519	4383·725	·720
1	50·0302	4339·885	·884	·634	·750	-51·75
3	S 48·5042	4325·939
3	46·5071	4308·061	·081

Weighted mean..... - 49·84
V_a..... + 10·76
V_d..... - ·10
Curvature..... - ·50
Radial velocity..... - 39·8

α DRACONIS 418.

1906. Nov. 1.
G. M. T. 15^h 50^m

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	72·8817	4584·215	·018
1	72·1993	4575·801	·702	·512	·810	-52·97
1	71·3441	4565·369	·249	·842	·593	-38·84
2	70·0434	4549·735	·642
2	69·9815	4548·998	·942	·642	·700	-46·06
3	S 68·2644	4528·798
2	65·2796	4494·778	·738
3	64·0156	4480·773	·730	·400	·670	-44·82
1	63·6044	4476·267	·185
2	62·7263	4466·725	·727
1	62·6559	4465·965	·927	·727	·800	-53·68
1	62·5824	4465·172	·135	·975	·840	-56·36
3	S 56·7851	4404·927
2	54·6416	4383·747	·720
1	50·0281	4340·011	·014	·634	·620	-42·78
3	S 48·4871	4325·939
3	46·4865	4308·051	·081

Weighted mean..... - 46·84
V_a..... + 10·79
V_d..... - ·04
Curvature..... - ·50
Radial velocity..... - 36·6

SESSIONAL PAPER No. 25a

α DRACONIS 426.

1906. Nov. 6.
G. M. T. 17^h 05^m

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	69·8776	4549·574	·642
1	68·4960	4533·301	·305	·139	·834	-55·12
3	S 68·1090	4528·798
2	65·1115	4494·687	·738
2	63·8352	4480·568	·590	·400	·810	-54·18
2	63·4353	4476·193	·185
1	62·5608	4466·702	·727
3	S 56·6151	4404·927
1	54·4642	4383·691	·720
½	49·8568	4340·042	·058	·634	·576	-39·74
3	48·3115	4325·936	·939
3	S 46·3141	4308·081

Weighted mean..... - 52·38
V_a..... + 11·21
V_d..... + ·04
Curvature..... - ·50

Radial velocity .. -41·6

NOTE.—Other lines in this spectrum, unidentified as yet.

α DRACONIS 424.

1906. Nov. 6.
G. M. T. 15^h

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	72·7892	4584·099	·018
½	72·6984	4583·978	·938	·018	1·080	-70·41
2	S 69·9459	4549·642
1½	69·8710	4548·752	·752	·642	·890	-58·65
3	68·1710	4528·799	·798
2	65·1727	4494·693	·738
3	63·8975	4480·592	·590	·400	·810	-54·18
2	63·4975	4476·217	·185
2	62·6202	4466·701	·727
2	S 56·6725	4404·927
1	54·5237	4383·719	·720
1	49·8738	4339·691	·694	·634	·940	-64·86
2	S 48·3665	4325·939
2	46·3662	4308·064	·081

Weighted mean..... - 58·43
V_a..... + 11·20
V_d..... - ·04
Curvature..... - ·50

Radial velocity..... 47·8

7-8 EDWARD VIL., A. 1908

α DRACONIS 423.

1906. Nov. 6.
G. M. T. 14^h

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	72.7646	4584.216018
1	72.6892	4583.283	.118	.018	.900	-58.68
2	69.9150	4549.655642
1	69.8431	4548.800	.792	.642	.850	-56.01
3	S 68.1400	4528.798
2	65.1516	4494.787738
3	63.8659	4480.565	.525	.400	.875	-58.53
2	63.4695	4476.227185
1	62.5938	4466.726727
3	S 56.6442	4404.927
2	54.4961	4383.729720
$\frac{1}{2}$	49.8607	4339.848	.854	.634	.780	-53.82
$\frac{3}{2}$	48.3347	4325.931939
3	S 46.3360	4308.081

Weighted mean..... - 57.57
V_a..... + 11.20
V_d..... - .09
Curvature..... - .50
Radial velocity..... - 47.0

α DRACONIS 422.

1906. Nov. 6.
G.M.T. 12^h 45^m

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
3	S 68.2300	4528.798
2	65.2341	4494.700738
2	63.9604	4480.603	.620	.400	.780	-52.18
2	63.5566	4476.190185
2	62.6820	4466.699727
3	S 56.7361	4404.927
3	S 48.4300	4325.939

Velocity..... - 52.18
V_a..... + 11.19
V_d..... - .11
Curvature..... - .50
Radial velocity..... - 41.6

SESSIONAL PAPER No. 25a

α DRACONIS 428.

1906. Nov. 8.
G.M.T. 12^h 40^m

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	69.9312	4549.735		.642		
1	69.8497	4548.766	.692	.642	.950	-62.60
3	S 68.1502	4528.798				
2	65.1595	4494.747		.738		
2	63.8742	4480.523	.510	.400	.890	-59.54
2	63.4790	4476.197		.185		
2	62.6002	4466.659		.729		
3	S 56.6599	4404.927				
2	54.5105	4383.705		.720		
$\frac{1}{2}$	49.8531	4339.598	.604	.634	1.030	-71.07
3	48.3558	4325.937		.939		
3	S 46.3577	4308.081				
$\frac{1}{2}$	46.2693	4307.301	.301	.081	.780	-54.21

Weighted mean..... - 61.09
 V_a + 11.35
 V_d - .11
 Curvature..... - .50
Radial velocity..... - 50.3

α DRACONIS 429.

1906. Nov. 8.
G.M.T. 13^h 40^m

Observed by } W. E. HARPER.
Measured by }

Wt.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displace- ment.	Velocity.
2	70.1414	4549.709		.642		
$\frac{1}{2}$	70.0718	4548.881	.832	.642	.810	-53.37
3	S 68.3617	4528.798				
1	65.3712	4494.765		.738		
2	64.0896	4480.586	.580	.400	.820	-54.85
1	63.6846	4476.155		.185		
2	62.8133	4466.698		.727		
3	S 56.8672	4404.927				
1	*50.1845	4340.752	.750	.530	.780	-53.82
3	S 48.5620	4325.939				

Weighted mean..... - 54.22
 V_a + 11.35
 V_d - .09
 Curvature..... - .50
Radial velocity... - 43.5

* A sharp line showing in the diffuse H γ band.

7-8 EDWARD VII., A. 1908

α DRACONIS 430.

1906. Nov. 8.
G. M. T. 14^h 50^m

Observed by } W. E. HARPER.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	72.8957	4584.126018	2	63.6049	4476.129185
1	72.8267	4583.150	.068	.018	.950	61.94	2	62.7401	4466.739727
1	71.8211	4570.928	.889	.849	.960	- 62.88	3	S 56.7940	4404.927
2	S 70.0539	4549.642	2	54.6446	4383.698720
1	69.9830	4548.798	.782	.642	.860	- 56.42	1	49.9985	4339.681	.680	.640	.960	- 66.24
3	68.2857	4528.850798	3	48.4942	4325.950939
2	65.2942	4494.781738	3	S 46.4954	4308.081
3	64.0222	4480.698	.670	.400	.730	- 48.83							

Weighted Mean..... 56.27
V_a + 11.35
V_d - .04
Curvature..... - .50
Radial Velocity 45.5

α DRACONIS 431.

1906. Nov. 8.
G. M. T. 16^h

Observed by } W. E. HARPER.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	70.0770	4549.657642	1	61.9930	4458.602	.586	.301	.715	- 48.05
1	70.0138	4548.907	.896	.642	.746	- 49.16	3	S 56.7999	4404.927
1	69.2565	4539.964	.958	.772	.814	- 53.72	2	54.6515	4383.723720
1	68.6867	4533.296	.294	.134	.845	55.85	1	50.0245	4339.887	.894	.634	.740	- 51.06
2	S 68.2999	4528.798	3	48.4955	4325.928939
1	65.3091	4494.788738	3	S 46.4995	4308.081
3	64.0232	4480.570	.570	.400	.830	- 55.52	1	37.5207	4232.679	.678	.328	.650	- 46.02
1	63.6216	4476.179185	1	37.9485	4236.101112
1	62.7521	4466.749	..	.727							

Weighted Mean..... - 52.55
V_a + 11.35
V_d00
Curvature .. . - .50
Radial Velocity. - 41.7

SESSIONAL PAPER No. 25a

α DRACONIS 435.

1906. Nov. 16
G. M. T. 15^h 32^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
1	72.7836	4583.895018	1	63.4899	4476.076185
3	72.7171	4583.075	.178	.018	.840	-54.76	2	S 56.6805	4404.927
1	69.9458	4549.544642	1	54.5351	4383.736720
1	69.8785	4548.745	.792	.642	.850	-56.01	1	49.8815	4339.616	.592	.640	1.048	-72.31
2	S 68.1778	4528.798	2	48.3863	4325.952939
1	65.1776	4494.687738	2	S 46.3903	4308.081
3	63.8822	4480.366	.460	.400	.940	-62.88							

Weighted Mean..... -61.63
V_a +11.82
V_d - .04
Curvature..... - .50
Radial Velocity..... -50.3

α DRACONIS 438.

1906. Nov. 19.
G. M. T. 13^h 15^m

Observed by (W. E. HARPER.
Measured by)

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
1	70.0752	4549.633642	1	62.7500	4466.668727
1	69.9975	4548.708	.712	.642	.930	-61.28	3	S 56.8075	4404.927
3	S 68.3017	4528.798	2	54.6582	4383.706720
2	65.3075	4494.722738	2	50.0056	4339.640	.650	.640	.990	-68.31
3	64.0136	4480.414	.440	.400	.960	-64.22	2	48.5047	4325.936939
2	63.6254	4476.166185	2	S 46.5067	4308.081

Weighted Mean..... -65.09
V_a +11.94
V_d - .09
Curvature..... - .50
Radial Velocity..... -53.7

7-8 EDWARD VII., A. 1908

α DRACONIS 447

1906 Dec. 7
G. M. T. 16^h 37^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
1	70·0639	4549·943	·895	·642	·253	+16·67	2	62·7195	4466·677	...	·727
2	70·0427	4549·692	·642	3	S 56·7810	4404·927
3	S 68·2658	4528·798	2	54·6345	4383·718	·720
3	65·2792	4494·783	...	·733	2	50·1348	4341·028	·034	·634	·400	+27·60
3	64·0944	4481·657	630	·400	·230	+15·38	3	48·4832	4325·931	·939
1	63·6035	4476·277	...	·185	3	S 46·4880	4308·081

Weighted Mean.....+19·67
V_a.....+11·94
V_d.....+ ·10
Curvature.....- ·50

Radial velocity.....+31·2

α DRACONIS 457

1906 Dec. 11
G. M. T. 18^h 40^m

Observed by } W. E. HARPER.
Measured by }

Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
1	70·0572	4549·960	·962	·642	·320	+21·08	2	62·6839	4466·704	·727
2	70·0301	4549·638	...	·642	3	S 56·7199	4404·927
3	S 68·2502	4523·798	1	50·0392	4340·933	·934	·634	·300	+20·70
1	66·4960	4508·737	·735	·455	·280	+18·62	3	48·3915	4325·925	·939
2	64·0671	4481·715	·700	·400	·300	+20·07	3	S 46·3900	4308·081
1	63·5640	4476·224	·185							

Weighted Mean.....+20·04
V_a.....+11·78
V_d.....+ ·15
Curvature.....- ·50

Radial velocity+31·5

SESSIONAL PAPER No. 25a

α DRACONIS 457*

1906. Dec. 11.
G. M. T. 18^h 40^m

Observed by W. E. HARPER.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
1	70·0592	4549·615	...	·642	1	63·5910	4476·178	·185
2	70 0839	4549·908	·938	·642	·296	+19·51	2	56·7495	4404·912	·927
2	68·2755	4528·731	·798	1	50·08	4341·030	·038	·634	·404	27·92
4	66·5167	4508·620	·680	·455	·225	14·96	3	48·4238	4325·936	·939
2	65·2717	4494·663	·738	1	48·4486	4326·160	·163	·939	·224	+15·52
2	64·0914	4481·649	·701	·400	·301	20·14	2	46·4179	4308·053	·081

Weighted mean..... +19·20
V_a +11·78
V_d + 15
Curvature.. - 50

Radial velocity..... +30·6

*Check measurement.

α DRACONIS 457.*

1906. Dec. 11.
G. M. T. 18^h 40^m

Observed by W. E. HARPER.
Measured by N. B. McLEAN.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	70·1115	4550·218	·306	·642	·664	+43·76	2	62·7089	4466·623	·727
2	70·0508	4549·546	·642	3	56·7509	4404·912	·927
3	68·2784	4528·748	·798	1	50·0562	4340·791	·819	·634	·185	+12·78
1	66·5236	4508·681	·737	·455	·282	18·75	3	48·4200	4325·889	·939
2	64·0900	4481·610	·658	·400	·258	17·26	3	46·4195	4308·054	·081
1	63·5898	4476·151	...	·185							

Weighted mean..... +27·42
V_a +11·78
V_d + 15
Curvature.. - 50

Radial velocity... +38·5

* Check measurement

α DRACONIS 458.

1906. Dec. 13.
G. M. T. 13^h 27^m

Observed by } W. E. HARPER.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
1	70.1807	4550.120	.129	.642	.487	+32.09	2	62.7824	4466.669727
2	70.1390	4549.628642	3	S 56.8145	4404.927
3	S 68.3582	4528.798	2	54.6584	4383.729720
2	65.3567	4494.790738	1 $\frac{1}{2}$	50.1363	4341.035	.019	.634	.385	+26.91
3	64.1686	4481.695	.685	.400	.285	+19.06	3	48.4796	4325.963939
2	63.6704	4476.263185	3	S 46.4711	4308.081

Weighted mean..... +22.83
V_a +11.68
V_d00
Curvature - .50
Radial velocity +34.0

α DRACONIS 462.

1906. Dec. 17.
G. M. T. 17^h 40^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	70.0536	4549.678642	2	62.7087	4466.666727
3	70.0424	4549.544	.512	.642	.130	-8.56	3	S 56.7532	4404.927
3	S 68.2726	4528.798	3	54.5990	4383.707720
3	65.2756	4494.766738	1 $\frac{1}{2}$	50.0332	4340.529	.514	.634	.120	-8.28
4	64.0368	4481.087	.060	.400	.340	-22.74	2	48.4342	4325.971939
3	63.5915	4476.224185	3	S 46.4282	4308.081

Weighted mean..... -17.94
V_a +11.42
V_d + .15
Curvature..... - .50
Radial velocity..... -6.9

SESSIONAL PAPER No. 25a

α DRACONIS 490.

1906. Dec. 18
G. M. T. 18^h 5^m

Observed by }
Measured by } W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	70.0805	4549.626642	1	62.7312	4466.717727
3	70.0339	4549.075	.092	.642	.550	- 36.24	3	S 56.7629	4404.927
3	S 68.3005	4528.798	2	54.6054	4383.694720
2	65.2975	4494.765738	3	49.9881	4340.083	.084	.634	.550	- 37.95
2	64.0505	4481.019	.020	.400	.380	- 25.42	3	48.4366	4325.959939
2	63.6060	4476.174185	3	S 46.4310	4308.081

Weighted mean.... - 28.34

V_a + 11.35

V_d + .14

Curvature..... - .50

Radial velocity.... - 16.8

α DRACONIS 524.

1907. Jan. 9
G. M. T. 16^h 10^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
1	70.0357	4549.479642	2	62.7148	4466.725727
1	69.9853	4548.881	.992	.642	.650	- 42.83	3	S 56.7590	4404.927
3	S 68.2714	4528.798	2	54.6102	4383.733720
3	65.2685	4494.693	..	.738	2	49.9830	4339.930	.904	.634	.730	- 50.37
3	63.9895	4480.571	.570	.400	.830	- 55.52	3	48.4527	4325.967939
2	63.5907	4476.214185	3	S 46.4514	4308.081

Weighted mean.... - 51.69

V_a + 8.94

V_d + .15

Curvature. - .50

Radial velocity..... - 43.1

7-8 EDWARD VII., A. 1908

α DRACONIS 528.

1907. Jan. 11.
G. M. T. 17^h 45^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
S 3	65.3082	4494.738	1	54.6475	4383.776720
2	64.0390	4480.721	.700	.400	.700	-46.83	2	50.0264	4340.055	.044	.634	.590	-40.71
2	63.6273	4476.224185	3	48.4764	4325.925939
2	62.7546	4466.770727	S 3	46.4778	4308.081
S 2	56.7930	4404.927							

Weighted Mean -43.79
V_a + 8.63
V_d + .16
Curvature.. - .50
Radial Velocity..... -35.5

α DRACONIS 566.

1907. Jan. 21.
G. M. T. 16^h 15^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	70.0520	4549.443642	3	56.7706	4404.972927
1	70.0481	4549.397	.592	.642	.050	-3.29	2	50.0527	4340.613	.434	.634	.200	-13.80
2	65.2711	4494.536738	3	48.4670	4326.154939
2	64.0499	4481.060	.160	.400	.240	-16.05	3	46.4677	4308.335081
2	63.5975	4476.122185							

Weighted Mean -13.17
V_a + 7.03
V_d + .15
Curvature.. - .50
Radial Velocity..... -6.5

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α DRACONIS 593.

1907. Jan. 30.
G. M. T. 17^h 02^m

Observed by
Measured by) W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement	Velocity
2	65.2447	4494.574738	2	50.0820	4341.193	.234	.634	.600	+41.40
3	64.0911	4481.840	.950	.400	.550	+36.79	3	48.4028	4325.896939
3	63.5662	4476.110185	3	46.4055	4308.091081
3	56.7275	4404.863927							

Weighted Mean.....+38.63
V_a.....+ 5.38
V_d.....+ .15
Curvature.....- .50

Radial Velocity.....+43.7

α DRACONIS 603.

1907. Feb. 6.
G. M. T. 18^h 02^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	70.0599	4549.656642	2	54.6360	4383.838720
1	70.0490	4549.527	.512	.642	.130	- 8.56	2	50.0753	4340.652	.554	.634	.080	5.52
1	65.2891	4494.781738	3	48.4740	4326.042939
3	64.0707	4481.314	.275	.400	.125	8.36	3	46.4750	4308.186081
1	63.6050	4476.222185	1	46.4605	4308.058	.961	.081	.120	- 8.35
3	56.7826	4405.012927							

Weighted Mean.....- 7.58
V_a.....+ 4.01
V_d.....+ .14
Curvature.....- .50

Radial Velocity.....- 3.9

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α DRACONIS 612.

1907. Feb. 12.
G. M. T. 15^h

Observed by W. E. HARPER.
Measured by J.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
1	70·5547	4555·551	·622	·202	·580	- 38·17	3	64·0239	4480·801	·740	·400	·660	44·15
2	70·0516	4549·558	·642	2	63·6090	4476·266	·185
1	70·0155	4549·129	·202	·642	·440	28·95	3	56·7835	4405·020	·927
1	68·6833	4533·462	·479	·139	·660	43·62	2	50·0368	4340·298	·154	·634	·480	- 33·12
3	68·2827	4528·805	·798	3	48·4795	4326·092	·939
2	65·2884	4494·774	·738	3	46·4821	4308·250	·081

Weighted mean. - 38·76
V_a + 2·84
V_d + ·15
Curvature - ·50
Radial velocity - 36·0

α DRACONIS 772.

1907. May 22.
G. M. T. 20^h 23^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity	Weight.	Mean of Settings	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	78·6890	4880·625	1·745	1	62·8200	4549·503	9·542	9·642	·100	6·59
2	77·8348	4860·425	1·422	1·527	·105	- 6·48	1½	58·3070	4469·851	9·873
1½	77·4210	4850·750	1·686	2	50·2595	4340·540	0·540	0·634	·094	6·48
1½	75·3228	4802·762	3·072	2	49·2883	4325·930	5·939
S 2	61·6768	4528·798	S 3	54·3854	4404·927
2	59·7517	4494·751	4·738	1¾	47·1338	4294·228	4·233	4·273	·040	- 2·80
2	58·9758	4481·310	1·310	1·400	·090	6·02	S 2	44·7783	4260·640
2	62·8255	4549·602	9·642							

Weighted mean. - 5·67
V_a - 11·73
V_d - ·15
Curvature - ·23
Radial velocity - 17·8

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α DRACONIS 799

1907. May 31.
G. M. T. 16^h 37^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	73.6686	4881.512	1.745	1 $\frac{1}{2}$	54.7212	4494.816	4.738
2	73.2481	4871.441	1.453	2 $\frac{1}{2}$	53.9315	4481.118	1.050	1.400	.350	23.41
2	72.8110	4861.197	1.207	1.527	.320	-19.58	1	53.2791	4469.926	9.871
1 $\frac{1}{2}$	57.7965	4549.772	9.642	2	45.2691	4341.004	1.162
1 $\frac{1}{2}$	57.7735	4549.352	9.222	9.642	.420	27.67	2	45.2025	4339.995	0.154	0.634	.480	-33.12

Weighted Mean... -25.94
V_a -11.75
V_d - .09
Curvature..... - .28
Radial velocity..... - 38.1

α DRACONIS 809

1907. June 8.
G. M. T. 18^h 50^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.	Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity
2	72.9591	2	45.2871
2	72.7925	.7957	.0230	-33.73	2	45.2251	.2217	.0270	28.35
1 $\frac{1}{2}$	72.3971	1 $\frac{1}{2}$	27.4655	.4591	.0374	-39.27
2	54.0231	2	27.3286
1 $\frac{1}{2}$	53.9339	.9267	.0300	34.68					

Weighted Mean -32.63
V_a -11.55
V_d - .15
Curvature.. - .28
Radial velocity..... - 44.6

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α DRACONIS 815

1907. June 10.
G. M. T. 16^h 47^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.	Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.
2	72.9231				2	54.7184			
1½	72.7684	.8054	.0133	-19.34	1½	53.9433	.9471	.0095	10.98
1½	72.3694				1	53.1034			
1½	57.7967				2	45.2925			
1	57.7938	.7950	.0097	11.73	1½	45.2318	.2236	.0251	-26.35

Weighted mean..... -17.59
V_a -11.47
V_d - .12
Curvature - .28
Radial velocity..... -29.5

α DRACONIS 815*

1907. June 10.
G. M. T. 16^h 47^m

Observed by J. S. PLASKETT.
Measured by N. B. McLEAN.

Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.	Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.
2	72.9213				½	53.6484			
1	72.7622	.8043	.0144	-20.92	1	53.0940			
2	72.3575				2	45.2842			
2	54.7145				2	45.2268	.2262	.0225	-23.55
1	54.0019				3	44.2704			
1½	53.9293	.9423	.0143	-16.50					

Weighted mean.... -20.62
V_a -11.47
V_d - .12
Curvature - .28
Radial velocity..... -32.5

*Check measurement ; the mean of the two, 31.0 kms. per sec. was used.

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α DRACONIS 823.

1907. June 11.
G. M. T. 14^h 17^m

Observed by }
Measured by } W. E. HARPER.

Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.	Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.
2	72.9557				1	54.0131			
2	72.7926	.7976	.0211	-30.66	1½	53.9347	.9369	.0197	22.73
1	72.3945				2	45.2836			
1½	57.8036				1½	45.2077	.2077	.0410	-42.92
½	57.7961	.7904	.0143	17.24					

Weighted mean..... - 30.62
V_a - 11.43
V_d - .12
Curvature..... - .28
Radial velocity..... - 42.4

α DRACONIS 859.

1907. June 20.
G. M. T. 15^h 17^m

Observed by }
Measured by } W. E. HARPER.

Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.	Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.
2	72.9723				2	53.9569	.9719	.0021	+2.42
1	72.8087	.8487	.0161	-23.36	1½	53.0967			
1½	73.2661				2	45.2665			
2	54.7300				1½	45.2236	.2307	.0080	-8.37

Weighted mean..... - 9.05
V_a - 10.87
V_d - .12
Curvature..... - .28
Radial velocity..... - 20.3

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α DRACONIS 870.

1907. June 21.
G. M. T. 16^h 36^m

Observed by J. S. PLASKETT.
Measured by N. B. McLEAN.

Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.	Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.
1	73·0191	1	53·9945	·9665	·0033	- 3·80
1	72·8634	·8530	·0118	- 17·12	1	45·3060
1	72·4577	1	45·2782	·2458	·0071	+ 7·41
1	54·0568					

Weighted mean - 6·37
V_a - 10·79
V_d - ·12
Curvature..... - ·28
Radial Velocity... . . . - 17·6

α DRACONIS 870.

1907. June 21.
G. M. T. 16^h 36^m

Observed by J. S. PLASKETT.
Measured by

Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.	Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.
2	72·9975	1½	53·6785
2	72·8430	·8542	·0106	- 15·38	1½	53·1194
2	72·4366	2	45·2851
2	54·7473	3	44·2635
2	54·0370	1	45·2569	·2473	·0086	+ 8·98
3	53·9617	·9533	·0165	- 18·99					

Weighted mean..... - 13·12
Radial Velocity... . . . - 24·3

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α DRACONIS 911.

1907. July 4.
G. M. T. 15^h 02^m

Observed by }
Measured by } W. E. HARPER.

Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.	Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.
2	72·9870				2	54·0187	·0297	·0599	68·94
$\frac{1}{2}$	72·8801	·9021	·0373	+ 54·13	1 $\frac{1}{2}$	53·1057			
1	72·4322				2	45·2628			
2	54·7262				1 $\frac{1}{2}$	45·2895	·3003	·0616	+ 64·85

Weighted mean ... +65·55
V_a - 9·52
V_d - ·12
Curvature ·28
Radial velocity... .. +55·6

α DRACONIS 916.

1907. July 5.
G. M. T. 14^h 50^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.	Weight.	Mean of Settings.	Corrected Setting.	Displace- ment in Rev ^{ns}	Velocity.
2	73·0028				1 $\frac{1}{2}$	53·0129			
2	72·8963	9039	·0391	+ 56·74	2	45·2709			
1	72·4350				2	45·2857	·2884	·0497	+ 51·87
1 $\frac{1}{2}$	54·7364				2	27·2771			
2	53·0186	·0180	·0482	+ 55·47					

Weighted mean..... +54·69
V_a - 9·40
V_d ·09
Curvature ·28
Radial velocity. -44·9

SUMMARY OF VELOCITIES.

Date.			Phase.	Velocity.	Date.			Phase.	Velocity.
1906.					1906.				
July	2.67.....		0.67	+ 3	Dec.	17.75.....		14.61	- 7
"	4.75.....		2.75	+16	"	18.75.....		15.61	-17
					1907.				
Aug.	15.59.....		44.59	-32	Jan.	9.67.....		37.53	- 43
"	24.59.....		2.20	+ 6	"	11.75.....		39.61	-35
Sept.	10.59.....		19.20	-30	"	21.67.....		49.53	- 6
"	19.59.....		28.20	-42	"	30.71.....		7.19	+44
"	27.54.....		36.16	-42	Feb.	6.75.....		14.23	- 4
Oct.	3.5.....		42.12	-39	"	12.62.....		20.10	-36
"	18.5.....		5.74	+32	May	22.64.....		16.57	-18
Nov.	1.6.....		19.76	-40	"	31.68.....		25.41	-38
"	6.6.....		24.77	-44	June	8.78.....		33.50	-45
"	8.6.....		26.76	-45	"	10.7.....		35.42	-31
"	16.67.....		34.91	-50	"	11.6.....		36.32	-42
"	19.54.....		37.78	-54	"	20.64.....		45.36	-20
Dec.	7.71.....		4.57	+31	"	21.69.....		46.41	-21
"	11.8.....		8.65	+31	July	4.63.....		7.96	+56
"	13.54.....		10.40	+34	"	5.62.....		8.96	+45

The above table gives us all the available material for the work in hand so far as our own observations are concerned. However, for an accurate determination of the period it is necessary to have observations extending over a long interval, and we were glad to avail ourselves of some early measures of the Yerkes, Lick and Potsdam observatories extending from 1901 to 1906. A summary of all previous observations known to the writer is given in the table below.

PREVIOUS OBSERVATIONS.

Date.		Phase.	Velocity.	Observatory.
1901 Nov.	20.92.....	11.46	+20	Yerkes.
1902 June	16.6.....	13.82	+ 1	Lick.
1903 Apr.	29.....	22.54	-43	"
" May	4.....	27.54	-42	"
" "	23.....	46.54	-17	Potsdam.
" "	24.....	47.54	-14	"
1904 June	19.....	28.50	-42	Lick.
1905 "	13.....	27.84	-42	"
1906 Jan.	4.....	27.22	-40	"
" "	5.98.....	28.47	-42	Yerkes.
" "	8.9.....	31.43	-55	"
" "	26.89.....	49.41	- 9	"
" "	29.81.....	0.95	+ 1	"
" Feb.	9.93.....	12.07	+24	"

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From all the observations, the velocity curve was plotted beginning with an arbitrary epoch, July 2.0 d, 1906. Previous observations were brought forward by a sufficient number of periods and subsequent ones similarly brought back. A period of 51.38 days brought the observations into the best agreement and owing to the number of periods available, about forty, this can not be in error more than one or two hundredths of a day. From this period, the phases given in the tables of velocities above were computed.

Mr. Harper, in his paper on the orbit of α Draconis, published in the Journal R. A. S. C., which is given below as appendix C, gives a complete discussion of the available data and obtains the velocity curve fig. 1, Appendix C, and elements from an analytical method due to Russell.* I obtained independently from the same data the velocity curve fig. 2, Appendix C, and slightly different elements by the method of Lehmann-Filhes.† In both cases a preliminary curve was drawn from which preliminary elements were obtained. An ephemeris was then computed from these elements and the corresponding curves drawn which deviated from the original curves in several places. An indication was thereby given of the general form of the curve and this was used to indicate the changes to be made in drawing the second curve. These changes did not, however, in any case affect the agreement of the curve with the observations, but in some cases improved it. This was possible on account of the comparatively high residuals which allowed considerable latitude in curve drawing.

A second set of elements was computed from these second curves. In the analytical method, Mr. Harper found it necessary to again change these values of e and ω to bring the curve drawn through the computed points into agreement with the observational curve. In the geometrical method, the curve computed from the second set of elements was found to agree closely with the observations and no further change was deemed necessary.

As the observations are scarcely of sufficient accuracy to permit of applying a least squares method of correction, the method followed above is perhaps as satisfactory as possible, although possibly not carried sufficiently far to give the most accurate values of the elements. As several other geometrical methods of obtaining the elements from the velocity curve have been deduced by Schwarzschild§ and quite recently by Zurhellen‡, it has been deemed of sufficient value and interest to give determinations of the elements of α Draconis by each of these methods. Further, in order to bring together in one place the various methods, and for convenience in our own work, the essential steps of each are summarized below, using, as far as possible, the notations of the original authors. Following each method is given as a numerical example the values of the elements for α Draconis, the velocity curve used being reproduced in fig. 7. This curve is almost identical with fig. 2, Appendix C, of Harper's paper below, but was redrawn from the observation. Very slight differences in the drawing make sometimes considerable variation in some of the elements, and this will explain the slightly different values of e and ω obtained by Lehmann-Filhes method from practically the same curve.

SPECTROSCOPIC BINARY ORBITS.

General Symbols.

a = major semi-axis of the orbit of the bright star around the centre of gravity of the system.

e = eccentricity.

ϕ = eccentricity angle $e = \sin \phi$.

ω = angular distance of periastron from the ascending node.

T = time of periastron passage.

* Astrophysical Journal XV p. 252.

† Astronomische Nachrichten, 3242.

§ Astronomische Nachrichten, No. 3629.

‡ Astronomische Nachrichten, No. 4191.

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i = inclination of the plane of the orbit to the normal plane.

U = periodic time.

μ = mean daily motion = $\frac{2\pi}{U}$

v = true
 M = mean
 E = eccentric } anomalies.

u = angular distance of the star from the ascending node = $v + \omega$.

γ = velocity of the centre of gravity of the system with respect to the sun.

A = maximum velocity in the orbit
 B = minimum velocity in the orbit } Both taken positive.

N_1 = maximum velocity with respect to the sun.

N_2 = minimum velocity with respect to the sun.

Fundamental Equations.

Take the production of the line of sight through the centre of gravity of the system as the z axis, while the x axis lies along the intersection of the normal plane with the orbit plane, positive towards the ascending node. Then

$$\frac{dz}{dt} = g = \gamma + \frac{f}{\sqrt{p}} \sin i (\cos u + e \cos \omega) \quad (1)$$

If we put $D = \frac{f \sin i}{\sqrt{p}}$, $C = \gamma + D e \cos \omega$ then

$$\left. \begin{aligned} \frac{dz}{dt} &= g = \gamma + D \cos \phi. \frac{\cos \phi \cos \omega \cos E - \sin \omega \sin E}{1 - e \cos E} \\ \frac{dz}{dt} &= g = C + D \cos u \end{aligned} \right\} \quad (2)$$

$$\therefore N_1 = C + D.$$

$$N_2 = C - D.$$

$$C = \frac{N_1 + N_2}{2} = \gamma + \frac{A - B}{2}$$

$$D = \frac{N_1 - N_2}{2} = \frac{A + B}{2}$$

Method of Lehmann-Filhes.*

The velocity curve—abscissa, times, ordinates, velocities—is drawn agreeing as closely as possible with the observed points. Considering $\gamma = 0$

$$\frac{dz}{dt} = \frac{f}{\sqrt{p}} \sin i (\cos u + e \cos \omega) \quad (3)$$

and at maximum and minimum

$$A = \frac{f}{\sqrt{p}} \sin i (1 + e \cos \omega)$$

$$B = \frac{f}{\sqrt{p}} \sin i (1 - e \cos \omega)$$

$$\left. \begin{aligned} \frac{f}{\sqrt{p}} \sin i &= \frac{A + B}{2} \\ \frac{f}{\sqrt{p}} \sin i e \cos \omega &= \frac{A - B}{2} \\ e \cos \omega &= \frac{A - B}{A + B} \end{aligned} \right\} \quad (4)$$

$$\frac{dz}{dt} = \frac{A + B}{2} \cos u + \frac{A - B}{2} \quad (5)$$

* Astronomische Nachrichten No. 3242.

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In the curve used as an example fig. 7, A corresponds to transit through the ascending, B through the descending node. $\int \frac{dz}{dt} dt$, the area between curve, axis, and two ordinates is proportional to the difference of the z 's to which these ordinates correspond.

This extended over a whole period, say from ascending node A to next ascending node A' must vanish, hence the areas above and below the axis must be equal. If this condition is not fulfilled, determined easily by counting the squares of cross section paper, the axis must be raised or lowered until the areas are equalized.

Consider the points C and D fig. 7 in which $\frac{dz}{dt} = 0$ then from (3) or (5)

$$\cos u = -e \cos \omega = -\frac{A-B}{A+B} \quad (6)$$

at the corresponding times t_1, t_2 the star is moving parallel to the nodal line. As it moves as far away from the ascending node as towards the descending node, $\int \frac{dz}{dt} dt$ from ascending to descending and also from descending to ascending nodes must be each zero. Therefore the area AC must equal CB and BD equal DA' and the curve drawing, with the positions of the maximum and minimum points, must be conformed to the above conditions.

At C and D from (6)

$$\sin u_1 = \frac{2\sqrt{AB}}{A+B}, \cos u_1 = -\frac{A-B}{A+B}$$

$$\sin u_2 = -\frac{2\sqrt{AB}}{A+B}, \cos u_2 = -\frac{A-B}{A+B}$$

$$\therefore u_2 = 2\pi - u_1$$

If we represent the maximum values of z by z_1 and z_2 (proportional to areas AC and BD respectively) and the corresponding radii vectors by r_1 and r_2 then

$$z_1 = r_1 \sin i \sin u_1$$

$$z_2 = r_2 \sin i \sin u_2 = -r_2 \sin i \sin u_1$$

$$r_1 = \frac{z_1}{\sin i}$$

$$r_2 = \frac{z_2}{\sin i}$$

$$r_1 = \frac{p}{1+e \cos (u_1 - \omega)} \quad r_2 = \frac{p}{1+e \cos (u_1 + \omega)}$$

$$\therefore \frac{z_1}{z_2} = \frac{1+e \cos u_1 \cos \omega - e \sin u_1 \sin \omega}{1+e \cos u_1 \cos \omega + e \sin u_1 \sin \omega} \text{ and from (6)}$$

$$\frac{\sin u_1 - e \sin \omega}{\sin u_1 + e \sin \omega} = \frac{z_1}{z_2}$$

$$e \sin \omega = \frac{z_2 + z_1}{z_2 - z_1} \sin u_1 \quad (7)$$

giving with (6) e and ω .

At periastron T , $u = \omega$ and from (3) and (4)

$$\frac{dz}{dt} = \frac{A+B}{2} (1+e) \cos \omega \quad (8)$$

Also from the known values of u_1 and t_1 of the point C we get

$$\tan \frac{E_1}{2} = \sqrt{\frac{1-e}{1+e}} \cdot \tan \frac{u_1 - \omega}{2}$$

$$T = t_1 - \frac{E_1 - e \sin E_1}{\mu} \quad (9)$$

As the mean daily movement

$$\mu = \frac{2\pi}{U} = \frac{f}{a^{3/2}} \text{ and from (4)}$$

$$\mu = \frac{A+B}{2} \cdot \frac{\sqrt{p}}{\sin i} \cdot \frac{1}{a^{3/2}}$$

$$\therefore a \sin i = \frac{A+B}{2\mu} \sqrt{1-e^2} \text{ and reducing to the same units}$$

$$a \sin i = 43200 \frac{A+B}{\mu} \sqrt{1-e^2} \quad (10)$$

Example.

In the curve the γ axis is 16.93^{kms} below the axis of zero velocity.

$$t_1 \text{ at maximum ordinate} = 8.0 \quad z_1 = 262.5 \quad A = 66.18$$

$$t_2 \text{ at minimum ordinate} = 28.58 \quad z_2 = -362.5 \quad B = 27.52$$

$$\log e \cos \omega = 9.61552$$

$$\log \sin u_1 = 9.95948 \quad \therefore u_1 = 114^\circ$$

$$\log e \sin \omega = 9.16360$$

$$e = 0.438 \quad \omega = 19^\circ 27'$$

$$T = \text{ordinate } 46.85 (1.438) \cos 19^\circ 27' = 62.06 \quad \therefore T = 9.1$$

$$\text{Also } \log \tan \frac{E_1}{2} = 9.83350 \quad E_1 = 1.19642$$

$$E_1 - e \sin E_1 = .78915 \quad \mu = .12229$$

$$\therefore T = 15.6 - 6.45 = 9.15 = \text{July } 11.15$$

$$a \sin i = 29,763,000 \text{ km.}$$

*Method of Schwarzschild.**

Draw in the position C or midway between the maximum and minimum points of the curve a parallel to the axis of abscissa calling the new ordinates ζ . Then

$$\frac{dz}{dt} - C = \zeta = D \cos u$$

It is required to find pairs of points on the curve distant $\frac{U}{2}$ from one another whose ordinates ζ are equal but of opposite sign. If the curve with the ζ axis be traced, be placed on the original after turning it through 180° around the ζ axis, and be moved along this axis the distance $\frac{U}{2}$, the intersections of the two curves will be points fulfilling these conditions. There will in general be four intersecting points P_1, P_2, P_3, P_4 , two pairs, the two points of a pair being easily recognized as their ordinates are $\frac{U}{2}$ apart.

$$\text{The abscissa of the points are } t_1, t_2 = t_1 + \frac{U}{2}, t_3, t_4 = t_3 + \frac{U}{2}$$

$$\text{and their ordinates } \zeta_1, \zeta_2 = -\zeta_1, \zeta_3, \zeta_4 = -\zeta_3$$

$$\frac{\zeta_1}{D} = \cos u_1, \quad \frac{\zeta_2}{D} = \cos u_2 = -\cos u_1$$

$$\therefore u_2 = u_1 + \pi \text{ (a) or } u_1 + u_2 = \pi \text{ (b)}$$

(a) is fulfilled if the points of the pair lie on different branches (b) if they lie on the same branch of the curve. If P_1 and P_2 are on different branches then

$$u_1 = v_1 + \omega \quad u_2 = v_2 + \omega \quad \therefore v_2 = v_1 + \pi$$

Hence P_1 and P_2 must be the positions of periastron and apastron and the former is

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easily distinguished by two conditions. 1. Around periastron the curve is more steeply inclined to the axis of abscissa. 2. The curve extends over a shorter time interval on the side of the ζ axis on which periastron lies.

As in periastron $v = 0$ $u = \omega$ so we obtain from the ordinate of the periastron ζ its longitude ω

$$\frac{\zeta}{D} = \cos \omega \quad (11)$$

Choose any point Q with abscissa t and ordinate ζ nearly midway between periastron and apastron. Find point Q' with abscissa t' whose ordinate $\zeta' = -\zeta$. There are generally two such points, one on the same, one on the other branch of the curve. Choose the point on the other branch then

$$\begin{aligned} \frac{\zeta}{D} &= \cos u & \frac{\zeta'}{D} &= \cos u' = -\cos u \\ u' &= u + \pi \text{ also} \\ u &= v + \omega, & u' &= v' + \omega, & v' &= v + \pi \end{aligned} \quad (12)$$

Let the corresponding eccentric anomalies be E and E'

$$\begin{aligned} E - e \sin E &= 2\pi \frac{t - T}{U} \\ E' - e \sin E' &= 2\pi \frac{t' - T}{U} \\ E' - E - e (\sin E' - \sin E) &= 2\pi \frac{t' - t}{U} \end{aligned} \quad (13)$$

$$\tan \frac{v}{2} = \tan \frac{E}{2} \sqrt{\frac{1+e}{1-e}}, \quad \tan \frac{v'}{2} = \tan \frac{E'}{2} \sqrt{\frac{1+e}{1-e}} \quad (14)$$

$$\tan \frac{v}{2} \tan \frac{v'}{2} = -1$$

$$\therefore \tan \frac{E}{2} \tan \frac{E'}{2} = \frac{e-1}{e+1}$$

$$e = \frac{\cos \frac{E' - E}{2}}{\cos \frac{E' + E}{2}} \quad (15)$$

$$e (\sin E' - \sin E) = \sin (E' - E)$$

$$E' - E - \sin (E' - E) = 2\pi \frac{t' - t}{U} \quad (16)$$

From (14) and (12) we get

$$\sin \frac{E' + E}{2} = \cos v \sin \frac{E' - E}{2} \quad (17)$$

If we place $\frac{E' - E}{2} = \eta$ $\frac{E' + E}{2} = \xi$ equation (16) becomes

$$2\eta - \sin 2\eta = 2\pi \frac{t' - t}{U} \quad (18)$$

The solution of this equation is given in the table below which gives the value of η corresponding to any value of $\frac{t' - t}{U}$, which is in turn obtained from the abscissa t and t' of the points Q and Q' .

From the ordinate ζ of Q we have

$$\begin{aligned} \frac{\zeta}{D} &= \cos u & v &= u - \omega \\ \sin \xi &= \cos v \sin \eta \\ e &= \frac{\cos \eta}{\cos \xi} \end{aligned} \tag{19}$$

$$a \sin i = D \frac{U}{2 \pi} \sqrt{1 - e^2} \tag{20}$$

$$\frac{m^3 \sin^3 i}{(m + m')^2} = \frac{U}{2 \pi k^2} D^3 (\sqrt{1 - e^2})^3 \tag{21}$$

$$\gamma = C - D e \cos \omega \tag{22}$$

If the relative velocity of one star against the other has been observed $\gamma = 0$ and therefore

$$\frac{C}{D} = e \cos \omega \tag{23}$$

giving e when ω has been determined.

SOLUTION OF THE EQUATION $2\eta - \sin 2\eta = 2\pi \frac{t' - t}{U}$

η	$\frac{t' - t}{U}$	η	$\frac{t' - t}{U}$	η	$\frac{t' - t}{U}$	η	$\frac{t' - t}{U}$	η	$\frac{t' - t}{U}$	η	$\frac{t' - t}{U}$
0	0.0000	30	0.0290	60	0.1956	90	0.5000	120	0.8044	150	0.9710
1	0.0000	31	0.0318	61	0.2040	91	0.5111	121	0.8127	151	0.9738
2	0.0000	32	0.0348	62	0.2125	92	0.5222	122	0.8208	152	0.9763
3	0.0000	33	0.0386	63	0.2213	93	0.5333	123	0.8287	153	0.9787
4	0.0001	34	0.0414	64	0.2303	94	0.5443	124	0.8364	154	0.9809
5	0.0001	35	0.0450	65	0.2393	95	0.5554	125	0.8439	155	0.9830
6	0.0002	36	0.0488	66	0.2485	96	0.5665	126	0.8512	156	0.9849
7	0.0004	37	0.0527	67	0.2578	97	0.5774	127	0.8584	157	0.9867
8	0.0006	38	0.0568	68	0.2673	98	0.5883	128	0.8654	158	0.9883
9	0.0008	39	0.0611	69	0.2769	99	0.5992	129	0.8722	159	0.9897
10	0.0011	40	0.0656	70	0.2867	100	0.6100	130	0.8788	160	0.9911
11	0.0015	41	0.0703	71	0.2966	101	0.6207	131	0.8853	161	0.9923
12	0.0020	42	0.0751	72	0.3065	102	0.6313	132	0.8915	162	0.9935
13	0.0025	43	0.0802	73	0.3166	103	0.6419	133	0.8975	163	0.9945
14	0.0031	44	0.0855	74	0.3268	104	0.6525	134	0.9033	164	0.9954
15	0.0038	45	0.0910	75	0.3371	105	0.6629	135	0.9090	165	0.9962
16	0.0046	46	0.0967	76	0.3475	106	0.6732	136	0.9145	166	0.9969
17	0.0055	47	0.1025	77	0.3581	107	0.6834	137	0.9198	167	0.9975
18	0.0065	48	0.1085	78	0.3687	108	0.6935	138	0.9249	168	0.9980
19	0.0077	49	0.1147	79	0.3793	109	0.7034	139	0.9297	169	0.9985
20	0.0089	50	0.1212	80	0.3900	110	0.7133	140	0.9344	170	0.9989
21	0.0103	51	0.1278	81	0.4008	111	0.7231	141	0.9389	171	0.9992
22	0.0117	52	0.1346	82	0.4117	112	0.7327	142	0.9432	172	0.9994
23	0.0133	53	0.1416	83	0.4226	113	0.7422	143	0.9473	173	0.9996
24	0.0151	54	0.1488	84	0.4335	114	0.7515	144	0.9512	174	0.9998
25	0.0170	55	0.1561	85	0.4446	115	0.7607	145	0.9550	175	0.9999
26	0.0191	56	0.1636	86	0.4557	116	0.7697	146	0.9586	176	0.9999
27	0.0213	57	0.1713	87	0.4667	117	0.7787	147	0.9620	177	1.0000
28	0.0237	58	0.1792	88	0.4778	118	0.7875	148	0.9652	178	1.0000
29	0.0262	59	0.1873	89	0.4889	119	0.7960	149	0.9682	179	1.0000

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Example of Schwarzschild's Method.

$$T \text{ of periastron} = 8.95 \quad = \text{July } 10.95.$$

$$\text{apastron} = 34.52$$

$$\zeta_1 = 44.4, \quad \zeta_2 = -45.0, \quad D = 46.85$$

$$\cos \omega = \frac{44.4}{46.85}, \quad \omega = 18^\circ 37'$$

$$\text{Take } t = 20 \quad \zeta = -38.7$$

$$t' = 6.08 \quad \zeta' = +38.7$$

$$\frac{t' - t}{U} = \frac{13.92}{51.38} = .2709 \quad \eta = 69^\circ .375$$

$$\cos u = \frac{-38.7}{46.85} \quad u = 145^\circ .69$$

$$v = 127^\circ .07$$

$$\log \sin \xi = 9.75140, \quad \log \cos \xi = 9.91680, \quad \log \cos \eta = 9.54685$$

$$\log e = 9.63005 \quad e = 0.427$$

$$\log a \sin i = \log 86400 + .91261 + 1.95637 + 1.67071$$

$$a \sin i = 29,936,000^{\text{kms.}}$$

$$\gamma = 2.4 - [1.67071 + 9.63005 + 9.97665]$$

$$= -16.55^{\text{kms.}}$$

*Methods of Zurhellen.**

The integral conditions of Lehmann-Filhes are more simply determined as follows:—

Copy the curve with axis of abscissa on tracing paper, turn the copy in its plane 180° and move it along on the γ axis until the ordinates of a maximum and a minimum fall on one another. Then Lehmann-Filhes conditions are equivalent to saying that the areas between these ordinates, the γ axis, and the quadrants of the curve must be equal. This method works well when e is small, but if e is large the dissimilarity of the portions above and below the γ axis prevents its being advantageously used.

In Zurhellen's methods for determining e and ω

g denotes observed velocity.

ζ denotes velocity with respect to mean axis.

z denotes velocity with respect to γ axis or the velocity in the orbit.

$$\therefore g - \gamma = z = \zeta + D e \cos \omega \quad (24)$$

Zurhellen's First Method.

This differs only from Schwarzschild's by using the difference of the ordinates of periastron and apastron for the determination of ω

$$\cos \omega = \frac{\zeta_P - \zeta_A}{2 D} \quad (25)$$

Example.

$$\zeta_P = 44.4 \quad \zeta_A = -45.0 \quad D = 46.85$$

$$\cos \omega = \frac{89.4}{93.7} \quad \omega = 17^\circ 25'$$

and using changed value of ω as in Schwarzschild's example $e = 0.432$

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Zurhellen's Second Method.

Instead of choosing the points Q and Q' , as in Schwarzschild's method, about midway between periastron and apastron, choose them where $v = \mp \frac{\pi}{2}$, or at the ends of the parameter

$$\begin{aligned} v = -\frac{\pi}{2} \quad g_1 - \gamma &= D \left\{ \cos \left(\omega - \frac{\pi}{2} \right) + e \cos \omega \right\} \\ &= D (\sin \omega + e \cos \omega) \\ v = +\frac{\pi}{2} \quad g_2 - \gamma &= D \left\{ \cos \left(\omega + \frac{\pi}{2} \right) + e \cos \omega \right\} \\ &= D (-\sin \omega + e \cos \omega) \end{aligned}$$

Therefore from (24)

$$\begin{aligned} \zeta_1 &= +D \sin \omega & E_1 &= -E_2 \\ \zeta_2 &= -D \sin \omega & M_1 &= -M_2 \\ t_1 - T &= -(t_2 - T) \end{aligned}$$

The points lie therefore in ζ symmetrical to the ζ axis and in t symmetrical to the periastron.

Lay the tracing on the curve, rotate it in its own plane 180° around the intersection of the periastron ordinate with the ζ axis. It intersects the original curve in two points, which are those required and which must be on different branches of the curve. Now as

$$\begin{aligned} \tan \frac{E}{2} &= \tan \frac{v}{2} \tan \left(\frac{\frac{\pi}{2} - \phi}{2} \right) \text{ and for } \tan \frac{v}{2} = \mp 1 \\ E_1 &= -\left(\frac{\pi}{2} - \phi \right), \quad E_2 = +\left(\frac{\pi}{2} - \phi \right) \\ M_1 &= -\left(\frac{\pi}{2} - \phi \right) + \sin \phi \sin \left(\frac{\pi}{2} - \phi \right) \\ M_2 &= +\left(\frac{\pi}{2} - \phi \right) - \sin \phi \sin \left(\frac{\pi}{2} - \phi \right) \\ \therefore \frac{2\pi}{U} (t_2 - t_1) &= M_2 - M_1 = \pi - 2\phi - \sin (\pi - 2\phi) \end{aligned} \quad (26)$$

$t_2 - t_1$ can be directly read off and then $\frac{\pi}{2} - \phi$ can be taken from Schwarzschild's table above.

The ordinates of the points $v = \mp \frac{\pi}{2}$ give ω

$$\sin \omega = \frac{\zeta_1 - \zeta_2}{2D} \text{ or } \tan \omega = \frac{\zeta_1 - \zeta_2}{\zeta_P - \zeta_A} \quad (27)$$

As a control the lines joining the points $v = -\frac{\pi}{2}$ and $v = +\frac{\pi}{2}$ and also $v = +\frac{\pi}{2}$ and $v = +\frac{3\pi}{2}$ must intersect the ζ axis in the abscissa of periastron and apastron.

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Example.

$$\begin{aligned}
\text{For } v = +\frac{\pi}{2} \quad \zeta_1 &= 14.0 \quad t_1 = 2.95 \\
v = -\frac{\pi}{2} \quad \zeta_2 &= -14.4 \quad t_2 = 15.0 \\
\frac{t_2 - t_1}{U} &= \frac{12.05}{51.38} = .2345 \quad \therefore \frac{\pi}{2} - \phi = 64^\circ.17 \\
\phi &= 25^\circ.53 \quad e = 0.431 \\
\sin \omega &= \frac{14.0 + 14.4}{93.7} = \frac{28.4}{93.7} \quad \tan \omega = \frac{28.4}{89.4} \\
\omega &= \underline{17^\circ \ 39'} \quad \text{or} \quad \underline{17^\circ \ 37'}
\end{aligned}$$

The values of e and ω obtained by these two methods are reliable as ω is small and the ζ 's accurately known.

Zurhellen's Third Method.

Frequently the direction of the curve at periastron and apastron is sharply defined and is changed the less by a movement of these points the nearer they are to the null points (velocity in orbit 0).

$$\begin{aligned}
\frac{dg}{dt} &= \frac{2\pi}{U} \frac{dg}{dM} = \frac{2\pi}{U} \cdot \frac{1}{1 - e \cos E} \cdot \frac{dg}{dE} \\
&= \frac{2\pi}{U} \cdot D \cos \phi \cdot \frac{-\cos \phi \cos \omega \sin E - \sin \omega \cos E + e \sin \omega}{(1 - e \cos E)^3}
\end{aligned}$$

At periastron $E = 0$, and at apastron $E = \pi$

$$\begin{aligned}
\left(\frac{dg}{dt} \right)_P &= - \frac{2\pi}{U} \cdot D \cos \phi \cdot \frac{\sin \omega}{(1 - e)^2} \\
\left(\frac{dg}{dt} \right)_A &= + \frac{2\pi}{U} \cdot D \cos \phi \cdot \frac{\sin \omega}{(1 + e)^2} \\
\left(\frac{dg}{dt} \right)_P : \left(\frac{dg}{dt} \right)_A &= - \left(\frac{1 + e}{1 - e} \right)^2
\end{aligned} \tag{28}$$

If we take this ratio $= -k^2$ and take k positive

$$e = + \frac{k - 1}{k + 1} \tag{29}$$

Calling $\left(\frac{dg}{dt} \right)_P$, p and $\left(\frac{dg}{dt} \right)_A$, a

$$\begin{aligned}
p \cdot a &= -4\pi^2 \left(\frac{D}{U} \right)^2 \frac{\sin^2 \omega}{\cos^2 \phi} \\
\sin \omega &= \frac{U}{2\pi D} \cos \phi \sqrt{-p \cdot a}
\end{aligned} \tag{30}$$

Example.

$$\begin{aligned}
\left(\frac{dg}{dt} \right)_P \text{ or } p &= - \frac{75.0}{52.35} = [-.15714]_n \\
\left(\frac{dg}{dt} \right)_A \text{ or } a &= \frac{24.3}{144.5} = [9.22634] \\
p : a &= [.93080]_n \\
\log k &= .46540 \quad k = 2.920 \\
e &= \frac{1.920}{3.920} = \underline{0.490}
\end{aligned}
\quad
\begin{aligned}
U &= 102.76 \\
D &= 23.425 = [.64214]
\end{aligned}$$

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$$\log p \cdot a = 9.38348 \quad \log \frac{U}{2\pi D} = 9.84396$$

$$\sin \phi = 0.490 \quad \log \cos \phi = 9.94042$$

$$\log \sin \omega = 9.47612 \quad \omega = 17^\circ 25'$$

As, owing to the position of periastron near the peak of the curve, the direction of the tangent is not accurately known, the values for e and ω obtained in this method will not be very reliable.

Zurhellen's Fourth Method.

The γ axis is henceforward taken as the axis of abscissa and the ordinates designated by Z .

According to Schwarzschild.

$$\left. \begin{aligned} e &= \frac{Z(\max) + Z(\min)}{Z_P - Z_A} \\ \omega &= \frac{Z_P - Z_A}{2D} \end{aligned} \right\} \quad (31)$$

Example.

$$e = \frac{66.18 - 27.52}{44.4 + 45.0} = \underline{\underline{0.432}}$$

$$\cos \omega = \frac{89.4}{93.7} \quad \omega = \underline{\underline{17^\circ 25'}}$$

Zurhellen's Fifth Method.

In this method the null points $g = \gamma$ are used. If we represent all these magnitudes with the upper index o and distinguish by the subscripts 1 and 2 the formula (2) gives when $g = \gamma$.

$$\tan E^o = \cot \omega \cos \phi, \quad E_2^o = E_1^o + \pi \quad (32)$$

$$M_2^o + M_1^o = E_2^o + E_1^o - e (\sin E_2^o + \sin E_1^o) = E_2^o + E_1^o$$

$$M_2^o - M_1^o = E_2^o - E_1^o - e (\sin E_2^o - \sin E_1^o) = \pi - 2e \cos \frac{M_2^o + M_1^o}{2}$$

$$\therefore e = \pi \left\{ \frac{1}{2} \frac{t_2^o - t_1^o}{U} \right\} \sec \frac{2\pi}{U} \left\{ \frac{t_2^o + t_1^o}{2} - T \right\} \quad (33)$$

$$\tan \omega = -\cos \phi \tan \frac{2\pi}{U} \left\{ \frac{t_2^o + t_1^o}{2} - T \right\} \quad (34)$$

$$\left. \begin{aligned} E_1^o &= \frac{1}{2} (M_1^o + M_2^o) - + \frac{\pi}{2} \\ E_2^o &= \frac{1}{2} (M_1^o + M_2^o) + - \frac{\pi}{2} \end{aligned} \right\} \quad (35)$$

Example.

$$t_1^o = 15.74 \quad \frac{1}{2} \frac{t_2^o - t_1^o}{U} = -\frac{5.85}{51.38}$$

$$t_2^o = 47.28$$

$$T = 8.95 \quad \frac{t_1^o + t_2^o}{2} - T = 22.56$$

$$\log \pi \left(\frac{1}{2} \frac{t_2^o - t_1^o}{U} \right) = n \ 9.55352$$

$$\frac{2\pi}{U} \left(\frac{t_1^o + t_2^o}{2} - T \right) = 158^\circ.17$$

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$$\begin{aligned}\log \sec 158^\circ \cdot 17 &= n \ 0.03232, \log \tan 158^\circ \cdot 17 = 9.60269 \\ \log e &= n \ 0.03232 + n \ 9.55352 = 9.58584 \\ e &= \underline{\underline{0.358}}\end{aligned}$$

$$\begin{aligned}\log \cos \phi &= 9.96510 \\ \log \tan \omega &= 9.96510 + 9.60269 = 9.56779 \\ \omega &= \underline{\underline{20^\circ \ 17'}}\end{aligned}$$

This method like Lehmann-Filhes alternative method of obtaining e fails to give accordant results and is not in general very reliable.

Zurhellen's Sixth Method.

If the second and the fifth methods have been used, we get a further value of e without difficulty.

If we join the points $v = \mp \frac{\pi}{2}$ and $v = + \frac{\pi}{2}$ and $v = + \frac{3\pi}{2}$ the lines cut a length d from the γ axis of the following property.

$$\begin{aligned}d : U &= (\sin \omega - e \cos \omega) : 2 \sin \omega = \frac{1}{2} - \frac{1}{2} e \cot \omega \\ &= \frac{1}{2} - \frac{1}{2} (\cot \omega \cos \phi) \tan \phi\end{aligned}$$

and from (32) and (35)

$$\begin{aligned}\tan \omega \sec \phi &= -\tan \frac{M_1^0 + M_2^0}{2} \\ \tan \phi &= \frac{2d - U}{U} \cdot \tan \frac{M_1^0 + M_2^0}{2}\end{aligned}\tag{36}$$

Example.

$$\begin{aligned}d &= 17.2 - 8.0 = 9.2 \\ \tan \phi &= \frac{18.4 - 51.38}{51.38} \tan 158^\circ \cdot 17 \\ \log \tan \phi &= n \ 9.80746 + n \ 9.60269 = 9.41015 \\ \phi &= 14.42^\circ \quad e = \underline{\underline{0.249}} \\ \log \tan \omega &= 9.98610 + 9.60269 = 9.58879 \\ \omega &= \underline{\underline{21^\circ \ 12'}}\end{aligned}$$

This is unreliable both on account of the acute intersection of the joining lines and also on account of the unreliability of the fifth method on which it depends.

Zurhellen's Seventh Method.

In the points $E = \mp \frac{\pi}{2}$

$$\begin{aligned}M_1 &= - \frac{\pi}{2} + e \\ M_2 &= + \frac{\pi}{2} - e \\ M_2 - M_1 &= \pi - 2e \\ e &= \pi \left\{ \begin{array}{cc} 1 & t_2 - t_1 \\ 2 & U \end{array} \right\}\end{aligned}\tag{37}$$

For finding the above points the condition required is that the Z 's are of equal magnitude but opposite sign. For from (2)

$$\begin{aligned}Z_1 &= + D \cos \phi \sin \omega \\ Z_2 &= - D \cos \phi \sin \omega\end{aligned}$$

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The tracing is therefore turned 180° around the intersection of the ordinate of periastron with the γ axis. And we also have

$$\tan \omega = \frac{Z_1 - Z_2}{Z_P - Z_A} \sec \phi \quad (38)$$

A control is given if the points $E = \mp \frac{\pi}{2}$ and $E = + \frac{\pi}{2}$ and $E = + \frac{3\pi}{2}$ are joined.

The γ axis must be cut at the abscissa of periastron and apastron. From the ζ axis a length d' of the following properties will be cut out.

$$\begin{aligned} d' : U &= (\sin \omega \cos \phi + e \cos \omega) : -2 \sin \omega \cos \phi \\ &= \frac{1}{2} + \frac{1}{2} e \cot \omega \sec \phi \\ e &= \frac{2d' - U}{U} \tan \omega \cos \phi = \frac{2d' - U}{U} \frac{Z_1 - Z_2}{Z_P - Z_A} \end{aligned} \quad (39)$$

Example.

$$\begin{aligned} t_1 &= -0.50 & t_2 &= 18.40 \\ Z_1 &= 14.3 & Z_2 &= -13.77 \\ Z_P &= 61.33 & Z_A &= -28.07 \\ \log e &= .49715 \div .83187 - 1.71079 = 9.61823 \\ e &= 0.415 \end{aligned}$$

$$\begin{aligned} \log \sec \phi &= 0.04108 \frac{Z_1 - Z_2}{Z_P - Z_A} = \frac{28.07}{89.4} = [9.49690] \\ \omega &= 19^\circ \quad 2' \end{aligned}$$

Control.

$$\begin{aligned} d' &= 61.25 \\ e &= \frac{122.5 - 51.38}{51.38} \cdot \frac{28.07}{89.4} = 0.434 \end{aligned}$$

These methods, though giving fairly accordant results in this case, can not in general be considered so trustworthy as the earlier methods.

Zurhellen's Eighth Method.

In many cases the abscissae of the extreme values are accurately known. The condition equations for these, which are designated by the upper index m , are as follows:—

$$\begin{aligned} \tan v^m &= -\tan \omega \\ \frac{\sin E^m}{\cos E^m - e} &= -\tan \omega \sec \phi \\ \sin E_1^m (\cos E_2^m - e) &= \sin E_2^m (\cos E_1^m - e) \\ \sin (E_2^m - E_1^m) &= e \sin E_2^m - e \sin E_1^m = E_2^m - M_2^m - E_1^m + M_1^m \\ E_2^m - E_1^m - \sin (E_2^m - E_1^m) &= M_2^m - M_1^m \end{aligned}$$

and as $M_2^m - M_1^m = 2\pi \frac{t_2^m - t_1^m}{U}$ we get

$$\frac{1}{2} (E_2^m - E_1^m) \text{ from Schwarzschild's table.}$$

From (32)

$$\begin{aligned} \tan \omega \sec \phi &= +\cot E^0 \\ \cos (E^m - E^0) &= +e \cos E^0 \end{aligned}$$

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An equation that holds for each combination of one extreme with one null point. If we consider everything with respect to one determined null point say 1 then.

$$E_1^m - E_1^o = - (E_2^m - E_1^o) \quad \text{and by (35)}$$

$$\frac{1}{2} (E_1^m + E_2^m) = E_1^o = \frac{1}{2} (M_1^o + M_2^o) - \frac{\pi}{2}$$

$$\cos (E_1^m - E_1^o) + \cos (E_2^m - E_1^o) = +2e \cos E_1^o$$

$$e = \cos \frac{1}{2} (E_2^m - E_1^m) \sec E_1^o$$

$$= \cos \frac{1}{2} (E_2^m - E_1^m) \operatorname{cosec} \frac{1}{2} (M_1^o + M_2^o) \quad (40)$$

$$\left. \begin{aligned} \tan \omega &= -\tan v^m & \therefore \omega &= -v^m \text{ or } \pi - v^m \\ \tan \frac{1}{2} v^m &= \tan \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \tan \frac{1}{2} E^m \end{aligned} \right\} \quad (41)$$

Example.

$$t_1 = 8.0 \quad t_2 = 28.53 \quad \frac{t_2 - t_1}{U} = .3996$$

$$\frac{1}{2} (E_2^m - E_1^m) = 80^\circ.84$$

$$e = \cos 80^\circ.84 \operatorname{cosec} 158^\circ.17 = 0.428 \quad \phi = 25^\circ.35$$

$$E_1^m = 158^\circ.17 - 90^\circ - 80^\circ.84 = -12^\circ.67$$

$$\tan \frac{1}{2} v^m = \tan 57^\circ.68 \quad \tan -6.33^\circ$$

$$\frac{1}{2} v^m = -9^\circ.953 \quad v^m = -19^\circ.906$$

$$\omega = 19^\circ \quad 54'$$

As this depends upon the abscissa of the maximum and minimum velocities, and as the positive maximum is well defined, and the negative maximum also well known from the equalization of areas, these values are reliable.

Russell's Analytical Method.

In this method the observed radial velocity is developed into a trigonometric series, and the elements are found by comparing this series with the corresponding analytical expression for the velocity.

The theory of the method may be presented as follows: The period U , and the corresponding value μ of the 'mean motion,' are given at once by the observed velocity-curve. The radial velocity, being a known periodic function of the time, may be expanded into a Fourier series of the form

$$V = c_0 + c_1 \cos \mu (t - t_0) + c_2 \cos 2\mu (t - t_0) + \dots + s_1 \sin \mu (t - t_0) + s_2 \sin 2\mu (t - t_0) + \dots \quad (1)$$

where t represents the time, and t_0 the initial epoch.

The coefficients of this series may best be obtained as follows: Divide the period into any even number $2n$ of equal parts, beginning at the epoch t_0 . Let $v_0, v_1, \dots, v_{2n-1}$ be the corresponding values of the velocity (v_0 corresponding to t_0).

Then we can get

$$\begin{aligned} c_0 &= \frac{1}{2n} \left[v_0 + v_1 + v_2 + \dots + v_{2n-1} \right] \\ c_1 &= \frac{1}{n} \left[v_0 + v_1 \cos \frac{\pi}{n} + v_2 \cos \frac{2\pi}{n} + \dots + v_{2n-1} \cos (2n-1) \frac{\pi}{n} \right] \\ c_2 &= \frac{1}{n} \left[v_0 + v_1 \cos \frac{2\pi}{n} + v_2 \cos \frac{4\pi}{n} + \dots + v_{2n-1} \cos (2n-1) \frac{2\pi}{n} \right] \\ s_1 &= \frac{1}{n} \left[v_1 \sin \frac{\pi}{n} + v_2 \sin \frac{2\pi}{n} + \dots + v_{2n-1} \sin (2n-1) \frac{\pi}{n} \right] \\ s_2 &= \frac{1}{n} \left[v_1 \sin \frac{2\pi}{n} + v_2 \sin \frac{4\pi}{n} + \dots + v_{2n-1} \sin (2n-1) \frac{2\pi}{n} \right] \end{aligned}$$

and similar expressions for the remaining coefficients.

The number of parts into which the period should be divided, in order to obtain sufficiently accurate values of the coefficients, depends upon the rate of convergence of the series (1), which in turn, depends upon the eccentricity of the orbit. If this is not more than 0.3 a division into twelve or sixteen parts will suffice. For values of e greater than 0.3 the method is not as suitable as some of the geometrical methods.

Series (1) may now be transformed into the form

$$V = a_0 + a_1 \cos [\mu (t - t_0) + a_1] + a_2 \cos [2 \mu (t - t_0) + a_2] + + \dots \tag{2}$$

by setting

$$\begin{aligned} a_1 \cos a_1 &= c_1 & a_2 \cos a_2 &= c_2 \\ a_1 \sin a_1 &= -s_1 & a_2 \sin a_2 &= -s_2 \end{aligned} \tag{3}$$

We have now to find an analytical expression of the form (2), for the velocity, in terms of the elements. Let

ω = longitude of periastron measured from the descending node.

z = projection of radius vector (r) on line of sight.

V = velocity of the bright star

the other symbols used having their usual significance.

Then we must have

$$V = \gamma + \frac{dz}{dt} \dots \dots \dots \tag{4}$$

Now,

$$\begin{aligned} z &= r \sin (v + \omega) \cdot \sin i \\ &= r \cos v \cdot \sin i \cdot \sin \omega + r \sin v \sin i \cos \omega \\ \therefore \frac{dz}{dt} &= \sin i \sin \omega \frac{d}{dt}(r \cos v) + \sin i \cos \omega \cdot \frac{d}{dt}(r \sin v) \dots \dots \tag{5} \end{aligned}$$

For central orbits we have the equations

$$E = M + e \sin E \dots \dots \dots \tag{6}$$

$$r = a (1 - e \cos E) \dots \dots \dots \tag{7}$$

$$\cos v = \frac{\cos E - e}{1 - e \cos E} \dots \dots \dots \tag{8}$$

Hence $r \cos v = a (\cos E - e)$

$$r \sin v = a \sqrt{1 - e^2} \cdot \sin E$$

Using equation (6) to develop $\cos E$ and $\sin E$ in terms of e and M by an application of Lagrange's Theorem we get the following expansions.

$$\begin{aligned} r \cos v &= -\frac{3}{2} a e + a (1 - \frac{3}{8} e^2 + \frac{5}{192} e^4 + \frac{7}{9216} e^6 \dots \dots \dots) \cos M \\ &\quad + \frac{1}{2} a e (1 - \frac{2}{3} e^2 + \frac{1}{8} e^4 + \frac{1}{90} e^6 \dots \dots \dots) \cos 2 M. \\ &\quad + \dots \dots \dots \\ r \sin v &= a (1 - \frac{5}{8} e^2 - \frac{11}{192} e^4 + \frac{119}{9216} e^6 \dots \dots \dots) \sin M \\ &\quad + \frac{1}{2} a e (1 - \frac{5}{6} e^2 + \frac{1}{12} e^4 + \frac{77}{720} e^6 \dots \dots \dots) \sin 2 M \\ &\quad + \dots \dots \dots \end{aligned}$$

Differentiating, remembering that $\frac{dM}{dt} = \mu$, substituting in (5), and, for brevity,

setting

$$\begin{aligned} 1 - \frac{3}{8} e^2 + \dots \dots \dots &= X_1 \\ 1 - \frac{5}{6} e^2 + \dots \dots \dots &= Y_1 \\ 1 - \frac{3}{8} e^2 + \dots \dots \dots &= X_2 \\ 1 - \frac{5}{6} e^2 + \dots \dots \dots &= Y_2, \text{ etc.} \end{aligned}$$

we obtain

$$\begin{aligned} \frac{dz}{dt} &= \mu a \sin i (Y_1 \cos \omega \cos M - X_1 \sin \omega \cdot \sin M) \\ &\quad + \mu e a \sin i (Y_2 \cos \omega \cos 2 M - X_2 \sin \omega \sin 2 M) \\ &\quad + \dots \dots \dots \end{aligned} \tag{9}$$

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If in this we set

$$\begin{aligned} X_1 \sin \omega &= b_1 \sin \beta_1 & X_2 \sin \omega &= b_2 \sin \beta_2 \\ Y_1 \cos \omega &= b_1 \cos \beta_1 & Y_2 \cos \omega &= b_2 \cos \beta_2 \end{aligned} \quad (10)$$

we have

$$\begin{aligned} \frac{dz}{dt} &= b_1 \mu a \sin i \cos (M + \beta_1) \\ &+ b_2 \mu e a \sin i \cos (2M + \beta_2) \\ &+ \dots \end{aligned}$$

Substituting in (4) and remembering that $M = M_0 + \mu (t - t_0)$ we obtain

$$\begin{aligned} V &= \gamma + \mu a \sin i \cdot b_1 \cdot \cos [\mu (t - t_0) + M_0 + \beta_1] \\ &+ \mu e a \sin i \cdot b_2 \cdot \cos [2\mu (t - t_0) + 2M_0 + \beta_2] \\ &+ \dots \end{aligned} \quad (11)$$

This is our desired expression for the velocity in terms of the elements and of the time.

The series (2) and (11), considered as functions of the time, are of the same form. If they are to represent the same quantity, their corresponding coefficients must be equal. That is, we must have

$$\begin{aligned} \gamma &= a_0 \\ b_1 \mu a \sin i &= a_1 & M_0 + \beta_1 &= a_1 \\ b_2 \mu e a \sin i &= a_2 & 2M_0 + \beta_2 &= a_2 \end{aligned} \quad (12)$$

Neglecting terms involving e these reduce to

$$\begin{aligned} a \sin i &= \frac{a_1}{\mu} & M_0 &= a_2 - a_1 \\ e &= \frac{a_2}{a_1} & \omega &= 2a_1 - a_2 \end{aligned} \quad (13)$$

It is clear that equations (13) give accurate values of the elements only when e is very small. But, in any case, they give approximate values of e and ω , by the use of which in the complete equations newer and more accurate values of the elements may be deduced.

Example.

Let the number of parts into which the period is divided be twelve.

We find

$$\begin{aligned} \gamma &= c_0 = -16.9^{\text{kms}} \text{ per sec.} \\ c_1 &= +29.253 & s_1 &= +23.465 \\ c_2 &= -2.692 & s_2 &= +15.055 \end{aligned}$$

and hence

$$\begin{aligned} a_1 &= 37.500 \\ a_2 &= 15.294 \end{aligned}$$

from which the preliminary value of e is 0.4078

$$\text{also } a_1 = 321^\circ 16'$$

$$a_2 = 79^\circ 52'$$

from which the preliminary value of

$$\omega \text{ is } 202^\circ 20'$$

Using the above values of e and ω as first approximations, and solving the complete equations we find

$$e = 0.422$$

$$\omega = 201^\circ 09' \text{ measured from the descending node.}$$

$$a \sin i = 29,291,700^{\text{kms.}}$$

Summary of Values.

	<i>e</i>	<i>ω</i>
Lehmann-Filhes.	0.438	19° 27'
Schwarzschild.	0.427	18° 37'
Zurhellen No. 1.	0.432	17° 25'
“ No. 2.	0.431	17° 38'
“ No. 3.	0.490	17° 25'
“ No. 4.	0.432	17° 25'
“ No. 5.	0.358	20° 17'
“ No. 6.	0.249	21° 12'
“ No. 7.	0.415	19° 2'
“ (Control).	0.434	
“ No. 8.	0.428	19° 54'
Russell.	0.422	21° 09'

If we take the mean of all values we get
e = 0.413 *ω* = 19° 3'

If the values of 3, 5, 6 which are not suitable for this orbit be omitted we get
e = 0.429 *ω* = 18° 50'

which may be considered as very close approximations to their true values.

Similarly taking means of the determinations of time of periastron passage we get it very nearly *T* = 9.0 days = July 11.0. For *a* sin *i*, the mean of the three determinations of 29,763,000, 29,936,000 and 29,292,000 is 29,664,000^{kms}.

The final values for the elements of *α*Draconis may therefore be put as above, in the confidence that they represent the observations very closely.

i ORIONIS.

i Orionis R.A. 5^h 30.5^m, Decl.—5° 59', Magnitude Visual 2.8, Phot. 3.4, Spectrum of Orion type with broad and diffuse lines, has been under observation here from the beginning of December, 1906, until it was too near the sun for satisfactory observation in April of this year. In all 45 spectrograms have been secured, of which 43, all that were suitable for measurement, have been reduced. All the spectra were made with the Brashear spectroscope, but it is proposed to continue the observations with the new single prism instrument as soon as possible. The spectra are of very poor quality for measurement, the lines being very diffuse and of non-symmetrical character, which may possibly be due to a second spectrum. As there are, in general, only two measurable lines on the plates λ4471 and λ4341, the resulting measures are subject to considerable uncertainty, and if it were not for the large range of velocity it would be hopeless to attempt any determination of the elements.

As it is, sufficient observations have not yet been obtained for a good curve, but it has been considered advisable to give those already obtained and to draw a provisional velocity curve, leaving its discussion and correction until further observations have been secured. It is hoped that, with the new single prism instrument, several more lines will be measurable and that the smaller linear dispersion will not, owing to the diffuseness of the lines, appreciably diminish the accuracy of measurement of any single line, while the probable error of the velocity should be considerably reduced, and, with a large number of observations, satisfactory elements obtained.

As with *α*Draconis, the Journal of Observations is followed by the separate measures and this is succeeded by a table containing ours and previous measures with the phase, corresponding to a period of 29.12 days, which seems to agree best with the observations. The resulting velocity curve is given in fig. 8, and shows some large deviations in the measurements. It is especially incomplete in its descending

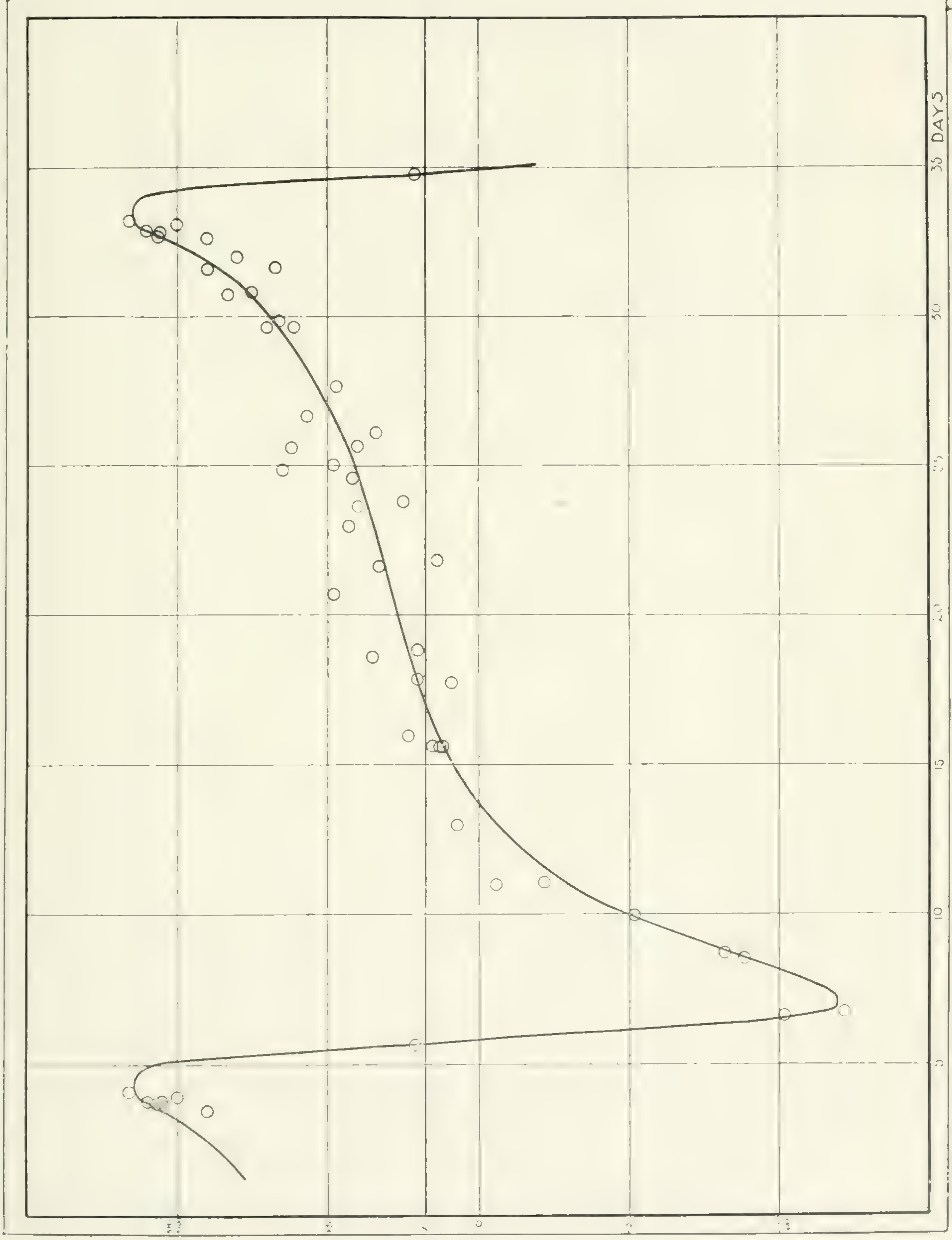


Fig. 8. - Velocity Curve of z Orionis.

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branch as, even after we had discovered its nature and the need of observations at that epoch, the weather was always unfavourable, and we were unable to secure them.

A preliminary determination of the elements by the method of Lehmann-Filhés gave for e about 0.75, and ω about 105° , but no great dependence can be placed on these values owing to the uncertainty in the form of the curve at maximum and minimum.

RECORD OF SPECTROGRAMS.

Star.	Plate.	Date.	Middle of Exposure.		Duration.	Hour Angle at End.	COMPARISON SPECTRUM.		TEMPERATURE.			Slit Width in m.m.	Focal Position.		Seeing.	(Observer.)	Remarks.			
			Number of Negative.	Plate.			Date.	h	m	Beg.	End.		Kind.	ROOM.				PRISM BOX.		
														Beg.				End.	Beg.	End.
1906.																				
Orionis.	Seed 27.	Dec. 11.	15	17	35	1	40 E.	20	20	Fe. Spark.	12.0	11.0	- 3.1	.030	19.0	15.2	5.68	H		
	" 27.	" 18.	14		34	2	30 E.	20	20	"	12.2	12.0	- 1.4	.037	19.0	15.2	5.73	H		
1907.																				
"	27.	Jan. 2.	15	05	30		25 E.	20	20	"	29.5	27.2	2.8	.030	19.0	15.2	5.71	P		
"	27.	" 9.	14	37	25		30 E.	20	20	"	7.8	8.0	- 12.7	.037	19.0	15.2	5.65	P		
"	27.	" 15.	15	20	34		40 W.	25	25	"	5.2	6.0	- 12.8	.037	19.0	15.2	5.65	H		
"	27.	" 16.	15	20	20		30 W.	20	20	"	- 2.0	2.0	- 17.7	.037	19.0	15.2	5.71	H		
"	27.	" 18.	15	40	40	1	10 W.	20	20	"	9.0	7.5	- 8.0	.031	19.0	15.2	5.65	P		
"	27.	" 21.	15	22	15		55 W.	20	20	"	5.8	6.2	- 12.5	.037	17.0	15.2	5.65	P		
"	27.	" 28.	14	50	20	1	00 W.	20	20	"	18.0	18.0	- 7.0	.037	17.0	15.2	5.65	H		
"	27.	" 30.	12	32	25	1	15 E.	20	20	"	16.0	16.2	- 4.2	.037	17.0	15.2	5.67	H		
"	27.	" 30.	15	57	28	2	15 W.	20	20	"	10.0	9.8	- 4.5	.037	17.0	15.2	5.67	H		
"	27.	Feb. 4.	12	15	25	1	15 E.	20	20	"	10.0	10.0	- 6.7	.037	17.0	15.2	5.67	H		
"	27.	" 6.	15	15	20	2	00 W.	20	20	"	14.6	14.0	- 8.6	.031	20.5	15.2	5.65	P		
"	27.	" 7.	12	17	25		57 E.	20	20	"	17.5	17.0	- 2.5	.037	20.5	15.2	5.67	H		
"	27.	" 12.	12	00	30		55 E.	20	20	"	14.0	12.0	- 9.3	.031	20.5	15.2	5.67	H		
"	27.	" 21.	13	30	20	1	10 W.	20	20	"	11.7	11.7	- 5.1	.037	20.3	15.2	5.65	H		
"	27.	" 22.	15	00	20	2	45 W.	20	20	"	2.5	1.8	- 14.8	.031	20.3	15.2	5.65	P		
"	27.	" 25.	15	10	20	3	05 W.	20	20	"	10.4	10.8	- 7.8	.031	20.5	15.2	5.65	P		
"	27.	" 27.	14	29	22	2	30 W.	20	20	"	12.4	-	- 2.8	.031	20.5	15.2	5.68	P		
"	27.	Mar. 6.	12	19	24		50 W.	20	20	"	34.0	29.5	4.3	.037	20.5	15.2	5.70	P		
"	27.	" 6.	15	19	24	3	50 W.	20	20	"	22.5	22.0	3.9	.031	20.5	15.2	5.70	P		
"	27.	" 8.	12	31	20	1	15 W.	20	20	"	34.0	33.5	3.9	.031	20.5	15.2	5.70	P		
"	27.	" 8.	15	15	26	3	55 W.	20	20	"	28.5	28.5	3.1	.031	20.5	15.2	5.70	P		
"	27.	" 11.	12	52	25	1	45 W.	20	20	"	35.5	31.0	7.0	.031	20.5	15.2	5.72	P		
"	27.	" 11.	15	26	28	4	20 W.	20	20	"	32.0	32.0	6.9	.031	20.5	15.2	5.72	P		
"	27.	" 20.	12	32	25	2	00 W.	20	20	"	36.0	35.0	4.7	.037	20.5	15.2	5.70	P		
"	27.	" 20.	13	00	25	2	30 W.	20	20	"	35.0	34.2	4.8	.037	20.5	15.2	5.70	P		
"	27.	" 20.	14	37	30	4	10 W.	20	20	"	32.3	31.5	3.8	.037	20.5	15.2	5.70	P		
"	27.	" 26.	12	26	23	2	18 W.	25	25	"	46.0	45.0	11.0	.037	20.5	15.2	5.70	H		

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"	673	"	27.	"	28..	12	27	25	2	18	W.	25	25	25	"	54.3	53.8	15.0	15.0	15.2	5.76	H	Focus not exact(5.80)
"	678	"	27.	"	30..	12	50	20	3	05	W.	25	25	25	"	—	—	8.8	8.8	15.2	5.76	P	
"	680	"	27.	April	1..	12	55	30	3	10	W.	25	25	25	"	33.7	32.0	4.8	5.0	15.2	5.70	Better	H	
"	686	"	27.	"	3..	12	50	25	3	15	W.	25	25	25	"	48.0	46.3	11.0	11.0	15.2	5.72	P	
"	687	"	27.	"	3..	13	22	35	3	50	W.	25	25	25	"	46.2	42.8	11.2	11.2	15.2	5.72	P	
"	693	"	27.	"	5..	12	30	20	3	00	W.	25	25	25	"	38.0	36.2	6.3	6.2	15.2	5.70	Good	P	
"	695	"	27.	"	5..	13	19	22	3	50	W.	25	25	25	"	35.2	33.8	6.2	6.2	15.2	5.70	"	P	
"	702	"	27.	"	6..	12	26	23	3	00	W.	25	25	25	"	40.8	38.7	7.7	7.9	15.2	5.73	"	H	
"	703	"	27.	"	6..	12	50	20	3	24	W.	25	25	25	"	38.7	39.0	7.9	8.1	15.2	5.73	"	H	
"	705	"	27.	"	11..	12	46	57	3	55	W.	25	25	25	"	46.0	44.0	11.6	11.8	15.2	5.73	Fair	H	Clouds 40m

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ORIONIS 453.

1906. Dec. 11.
G. M. T. 15^h 17^m

Observed by W. E. HARPER.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
3	S 65.3050	4494.738	2	54.6327	4383.751720
2	63.6260	4476.251185	1	50.2460	4342.207	.184	0.634	1.550	+106.95
1	63.3760	4473.550	.516	1.676	1.670	+123.75	3	48.4569	4325.961939
1	62.7450	4466.712727	3	S 46.4497	4308.081
3	S 56.7836	4404.927							

Weighted mean +115.35
V_a + 0.97
V_d + .15
Curvature.... - .50
Radial velocity +116.0

ORIONIS 485.

1906. Dec. 18.
G. M. T. 14^h

Observed by W. E. HARPER.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
3	S 68.3000	4528.798	3	S 56.7645	4404.927
2	65.2950	4494.731738	2	54.6105	4383.731720
2	63.6128	4476.237185	2	50.0173	4340.354	.361	.634	.273	18.84
1	63.1665	4471.400	.376	.676	.300	- 20.48	3	48.4320	4325.918939
2	62.7319	4466.712727	3	S 46.4305	4308.081

Weighted mean 19.38
V_a 2.23
V_d + 0.22
Curvature.... - .50
Radial velocity. -22.0

SESSIONAL PAPER No. 25a

ORIONIS 517.

1907. Jan. 2.
G. M. T. 15^h 05^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
3	S 68·3105	4528·798	3	S 56·7695	4404·927
2	65·3117	4494·795	...	·738	2	54·6066	4383·675	...	·720
2	63·6162	4476·159	...	·185	2	50·1155	4341·312	·340	·634	·706	+48·71
2	63·2542	4472·233	·249	·676	·573	+38·39	3	48·4241	4325·943	...	·939
2	62·7390	4466·679	...	·727	3	S 46·4146	4308·081

Weighted mean..... +43·55
Va. -9·00
Vd..... + ·04
Curvature..... - ·50
Radial velocity..... +34·0

ORIONIS 522.

1907. Jan. 9.
G. M. T. 14^h 37^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	S 65·3802	4494·738	2	54·7117	4383·727	...	·720
2	63·6979	4476·220	...	·185	1	50·3174	4342·086	·084	0·634	1·450	+100·05
1	63·4597	4473·630	·582	1·676	1·906	+127·70	3	48·5500	4325·944	...	·939
1	62·8282	4466·803	...	·727	3	S 46·5506	4308·081
3	S 56·8620	4404·927							

Weighted mean..... +113·87
Va. -11·94
Vd..... + ·04
Curvature..... - ·50
Radial velocity..... +101·0

7-8 EDWARD VII., A. 1908

ORIONIS 535.

1907. Jan. 15.
G. M. T. 15^h 20^m

Observed by }
Measured by } W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65·1450	4494·630	·738	2	54·4781	4383·745	·720
2	63·4580	4476·071	·185	2	49·8657	4340·164	·084	·634	·550	- 37·95
1	62·9973	4471·074	·183	·676	·493	- 33·05	2	48·3160	4326·053	·939
1	62·5816	4466·590	·727	2	46·3146	4308·207	·081
2	56·6277	4404·909	·927							

Weighted mean - 36·32
V_a - 14·34
V_d - ·04
Curvature - ·50
Radia velocity..... - 51·0

ORIONIS 539.

1907. Jan. 16.
G. M. T. 15^h 20^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65·2906	4494·650	...	·738	2	54·6468	4383·786	·720
1	63·6140	4476·171	·185	2	50·1247	4340·946	·834	·634	·200	+ 13·80
2	63·2075	4471·751	·736	·676	·060	+ 4·02	3	48·4957	4326·076	·939
2	62·7477	4466·779	·727	3	46·4970	4308·189	·081
3	56·7975	4405·005	...	·927							

Weighted mean..... + 8·91
V_a - 14·61
V_d - ·04
Curvature - ·50
Radial velocity - 6·0

SESSIONAL PAPER No. 25a

ORIONIS 556

1907. Jan. 18.
G. M. T. 15^h 40^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.2792	4494.716738	3	54.6110	4383.721	..	.720
2	63.6007	4476.235185	2	50.0800	4340.853	.884	.634	.250	+17.25
3	63.2246	4472.149	.066	.676	.390	+26.00	3	48.4410	4325.905939
2	62.7198	4466.996727	3	46.4395	4308.044081
3	56.7650	4404.948927							

Weighted Mean... ..+22.50

V_a -15.50

V_d - .09

Curvature . - .50

Radial velocity..... +6.

ORIONIS 565

1907. Jan. 21.
G. M. T. 15^h 22^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.2580	4494.591738	3	56.7526	4404.992	..	.927
2	63.5737	4476.063185	3	50.0999	4341.248	.117	0.634	.483	+33.35
2	63.2239	4472.265	.326	1.676	.700	+46.80	3	48.4416	4326.138	5.939
1	62.7016	4466.628727							

Weighted Mean... ..+38.73

V_a -16.59

V_d - .09

Curvature.. - .50

Radial velocity..... +21.5

7-8 EDWARD VII., A. 1908

ORIONIS 585

1907. Jan. 28.
G. M. T. 14^h 50^m

Observed by } W. E. HARPER.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65·2890	4494·780	...	·738	3	56·7705	4404·890	·927
2	63·6037	4476·208	·185	3	50·1731	4341·554	·634	0·634	1·000	+69·00
2	63·2590	4472·459	456	1·676	·780	+52·26	3	48·4519	4325·842	·939
1	62·7272	4466·709	·727							

Weighted mean.....+62·30
V_a -18·95
V_d - ·09
Curvature . - ·50

Radial velocity..... +42·8

ORIONIS 587

1907. Jan. 30.
G. M. T. 12^h 32^m

Observed by } W. E. HARPER.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65·2770	4494·734	·738	2	54·6023	4383·728	·720
2	63·5903	4476·173	·185	2½	50·1848	4341·940	·904	0·634	1·270	+ 87·63
1	63·2850	4472·911	·916	1·676	1·240	+82·96	2	48·4324	4325·964	.	·939
2	62·7175	4466·727	·727	2	46·4322	4308·126	·081
3	56·7573	4404·960	·927							

Weighted mean.....+84·83
V_a -19·54
V_d + ·12
Curvature . - ·50

Radial velocity... ..+64·9

SESSIONAL PAPER No. 25a

ORIONIS 592

1907. Jan. 30.
G. M. T. 15^h 57^m

Observed by } W. E. HARPER.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.3162	4494.740738	3	56.7975	4404.931927
2	63.6281	4476.155185	1	50.2020	4341.669	.674	0.634	1.040	+ 71.76
1 ₂	63.3041	4472.633	.656	1.676	.980	+ 65.66	2	48.4745	4325.913939
1	62.7615	4466.770727	3	46.4742	4308.068081

Weighted mean..... + 68.10
V_a..... - 19.54
V_d..... - .16
Curvature.. - .50

Radial Velocity..... + 47.9

ORIONIS 594

1907. Feb. 4.
G. M. T. 12^h 15^m

Observed by } W. E. HARPER.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
1	65.3280	4494.815738	3	56.8030	4404.815927
1	63.6450	4476.258185	2	50.2348	4341.723	.854	0.634	1.220	+ 84.18
2	63.3546	4473.098	.026	1.676	1.350	+ 90.45	3	48.4867	4325.758939
1	62.7750	4466.824727							

Weighted mean..... + 87.31
V_a..... - 20.99
V_d..... + .12
Curvature.. - .50

Radial Velocity... + 65.9

7-8 EDWARD VII., A. 1908

ORIONIS 601.

1907. Feb. 6.
G. M. T. 15^h 15^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65·2910	4494·702	·738	3	56·7782	4404·867	·927
2	63·6068	4476·142	·185	2	50·2537	4342·200	·224	0·634	1·590	+109·70
3	63·4104	4474·000	·056	1·676	1·380	+92·46	3	48·4722	4325·926	·939
1	62·7304	4466·642	·727							

Weighted Mean..... +99·36
V_a..... -21·56
V_d..... -·16
Curvature.... -·50
Radial Velocity..... +77·1

ORIONIS 605.

1907. Feb. 7.
G. M. T. 12^h 17^m

Observed by W. E. HARPER.
Measured by

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
1	65·2772	4494·749	·738	3	56·7609	4404·894	·927
1	63·5953	4476·217	·185	1	50·2386	4342·258	·304	0·634	1·670	+115·23
1	63·3719	4473·785	·786	1·676	2·110	+141·37	3	48·4462	4325·891	·939
1	62·7152	4466·678	...	·727							

Weighted Mean..... +128·30
V_a..... -21·78
V_d..... +·09
Curvature.... -·50
Radial Velocity..... +106·1

SESSIONAL PAPER No. 25a

ORIONIS 609.

1907. Feb. 12.
G. M. T. 12^h

Observed by W. E. HARPER.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.2562	4494.615738	3	56.7552	4404.937927
2	63.5802	4476.152185	2	49.9698	4339.881	.814	0.634	.820	-56.58
2	63.0791	4470.709	.756	1.676	.920	-61.64	3	48.4531	4326.054939
1	62.7036	4466.654727							

Weighted mean..... -59.11
V_a..... -22.96
V_d..... + .09
Curvature..... - .50
Radial velocity - 82.5

ORIONIS 618.

1907. Feb. 21.
G. M. T. 13^h 30^m

Observed by W. E. HARPER.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	68.2651	4528.901798	3	56.7485	4404.971927
2	63.5820	4476.272185	2	50.1098	4341.272	.224	0.634	.590	40.48
2	63.2031	4472.152	.086	1.676	.410	+27.47	3	48.4375	4326.003939
1	62.7051	4466.770727	3	46.4316	4308.104031

Weighted mean.. +33.97
V_a..... -24.65
V_d..... - .09
Curvature.. - .50
Radial velocity + 8.7

7-8 EDWARD VII., A. 1908

ORIONIS 627.

1907. Feb. 22.
G. M. T. 15^h

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	65.2750	4494.803738	3	56.7835	4404.923927
2	63.5942	4476.222185	2	50.1903	4341.443	.494	0.634	.860	+59.34
2	63.2302	4472.252	.206	1.676	.530	+35.51	2	48.4880	4325.859939
2	62.7271	4466.795727							

Weighted Mean +47.42
V_a -24.80
V_d - .19
Curvature..... - .50
Radial Velocity +22.0

ORIONIS 636.

1907. Feb. 25.
G. M. T. 15^h 10^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	65.3028	4494.734738	3	56.7982	4404.967927
2	63.6207	4476.193185	2	50.1525	4341.164	1.064	0.634	.430	+29.67
2	63.2634	4472.307	2.276	1.676	.600	+40.20	2	48.4976	4326.065939
1	62.7528	4466.784727							

Weighted Mean..... +33.93
V_a -25.20
V_d - .22
Curvature..... - .50
Radial Velocity..... +8.0

SESSIONAL PAPER No. 25a

ORIONIS 644.

1907. Feb. 27.
G. M. T. 14^h 30^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.3137	4494.805738	3	56.7962	4404.897927
2	63.6314	4476.260185	2	50.1809	4341.376	.464	.634	.830	+ 57.27
2	63.2898	4472.543	.466	.676	.790	+ 52.93	3	48.4790	4325.838939
1	62.7678	4466.900727							

Weighted Mean..... +55.10
V_a..... - 25.42
V_d..... - .19
Curvature..... - .50
Radial Velocity..... + 29.0

ORIONIS 647.

1907. Mar. 6.
G. M. T. 12^h 19^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.2869	4494.798738	3	56.7577	4404.914927
2	63.6104	4476.342185	2	50.2036	4342.063	.164	0.634	1.530	+105.57
2	63.3630	4473.653	.526	1.675	1.850	+ 123.95	2	48.4270	4325.865939
1	62.7353	4466.867727	3	46.4207	4307.975	...	8.081

Weighted Mean..... +109.23
V_a..... - 25.96
V_d..... - .04
Curvature..... - .50
Radial Velocity..... + 83.0

7-8 EDWARD VII., A. 1908

ORIONIS 650.

1907. Mar. 6.
G. M. T. 15^h 19^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65·2700	4494·725	·738	3	56·7494	4404·932	·927
1	63·5987	4476·314	·185	2	50·1832	4341·976	1·984	0·634	1·350	+93·15
1	63·3615	4473·736	3·746	1·676	2·070	-138·69	3	48·4230	4325·929	·939
1	62·7115	4466·712	·727							

Weighted Mean +102·25
V_a 25·96
V_d - ·25
Curvature.... ·50
Radial velocity. . 75·5

ORIONIS 653.

1907. Mar. 8.
G. M. T. 12^h 32^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65·3455	4494·897	·738	3	56·8136	4404·922	·927
2	63·6619	4476·352	·185	2	50·2928	4342·335	·444	0·634	1·810	+124·89
3	63·4322	4473·854	3·726	1·676	2·050	+137·35	3	48·4805	4325·798	9·39
1	62·7820	4466·822	·727							

Weighted Mean..... +132·36
V_a -26·05
V_d - ·09
Curvature... - ·50
Radial velocity..... +106·0

SESSIONAL PAPER No. 25a

ORIONIS 655.

1907. March 8.
G. M. T. 15^h 15^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.3094	4494.894738	3	56.7705	4404.892927
2	63.6220	4476.317185	2	50.2558	4342.394	.494	0.634	1.860	+128.34
1	63.4127	4474.042	3.946	1.676	2.270	+152.09	3	48.4388	4325.821939
1	62.7398	4466.767727							

Weighted mean... +136.26
V_a -26.05
V_d25
Curvature.... - .50
Radial velocity..... +109.5

ORIONIS 659.

1907. March 11.
G. M. T. 12^h 52^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.3169	4494.928738	3	56.7781	4404.917927
2	63.6437	4476.505185	2	49.9547	4339.574	.714	0.634	.920	-63.48
2	63.0960	4470.560	.41	1.676	1.260	-84.42	3	48.4382	4325.765939
2	62.7562	4466.785727							

Weighted mean..... -73.95
V_a -26.12
V_d12
Curvature50
Radial velocity -101.0

7-8 EDWARD VII., A. 1908

ORIONIS 662.

1907. Mar. 11.
G. M. T. 15^h 26^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
1	65·3272	4494·892	...	·738	3	56·7865	4401·851	...	·927
1	63·6463	4476·383	...	·185	1	49·9173	4339·078	254	0·634	1·380	-95·22
1	63·0936	4470·384	256	1·676	1·420	-95·14	3	48·4512	4325·733	...	·939
1	62·7725	4466·919	·727							

Weighted Mean - 95·18
V_a..... - 26·12
V_d..... - ·28
Curvature..... - ·50
Radial velocity... .. 122·0

ORIONIS 665.

1907. Mar. 20.
G. M. T. 12^h 32^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65·3009	4494·799	...	·738	3	56·7648	4404·835	...	·927
2	63·6134	4476·225	...	·185	3	50·1227	4341·168	323	·634	·694	+47·88
2	63·2358	4472·123	091	1·676	·415	+27·80	3	48·4338	4325·776	·939
2	62·7376	4466·744	·727							

Weighted Mean +39·85
V_a -25·90
V_d..... - ·16
Curvature... - ·50
Radial velocity..... . +13·3

SESSIONAL PAPER No. 25a

ORIONIS 666.

1907. March 20.
G. M. T. 13^h

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.3077	4494.875738	3	56.7690	4404.877927
2	63.6172	4476.266185	2	50.1226	4341.167	.269	0.634	.635	+43.81
2	63.2563	4472.346	.266	1.676	.590	+39.53	3	48.4387	4325.820939
1	62.7410	4466.779727							

Weighted mean..... +41.67
V_a..... -25.90
V_d..... - .19
Curvature.. - .50
Radial velocity + 15.1

ORIONIS 667.

1907. March 20.
G. M. T. 14^h 37^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.3222	4494.837738	3	56.7823	4404.809927
2	63.6331	4476.239185	2	50.1376	4341.105	.234	0.634	.600	+41.40
2	63.2648	4472.237	.176	1.676	.500	+33.50	3	48.4588	4325.801939
2	62.7600	4466.785727							

Weighted mean... .. +37.45
V_a..... -25.90
V_d..... - .28
Curvature.. - .50
Radial velocity + 10.8

7-8 EDWARD VII., A. 1908

ORIONIS 672.

1907. March 26.
G. M. T. 12^h 26^m

Observed by W. E. HARPER.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.
3	S 68.3301	4528.798	3	S 56.7690	4404.927
2	63.6232	4476.133185	2	50.1415	4341.589	.574	.634	.940	+ 64.86
2	63.2755	4472.371	.406	.676	730	- 48.91	3	S 46.4106	4308.081
2	62.7500	4466.713727							

Weighted mean. + 58.98
V_a - 25.41
V_d - .16
Curvative. - .50
Radial velocity..... + 33.0

ORIONIS 673.

1907. March 28.
G. M. T. 12^h 27^m

Observed by W. E. HARPER.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.
3	65.3089	4494.944738	3	56.7476	4404.900927
2	63.6109	4476.294185	3	50.1049	4341.342	.484	0.634	.850	+ 58.65
2	63.2999	4472.922	.806	1.676	1.130	+ 76.71	2	48.3939	4325.776939
2	62.7339	4466.821727							

Weighted mean..... + 65.87
V_a - 25.18
V_d - .19
Curvature. - .50
Radial velocity..... + 40.0

SESSIONAL PAPER No. 25a

ORIONIS 678.

1907, Mar. 30.
G. M. T. 12^h 50^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity	Weight.	Mean of Settings	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	65·3161	4494·729	·738	3	56·7667	4404·916	·927
2	63·6137	4476·051	·185	2	50·1699	4341·853	1·794	0·634	1·160	+80·04
2	63·3314	4472·995	3·096	1·676	1·420	+95·14	3	48·4262	4326·001	5·939
2	62·7415	4466·642	...	·727							

Weighted mean..... +87·59

Va..... -25 00

Vd..... - 22

Curvature..... - 50

Radial velocity +62·0

ORIONIS 680.

1907, April, 1.
G. M. T. 12^h 55^m

Observed by } W. E. HARPER.
Measured by }

Weight.	Mean of Settings	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	65·3222	4494·936	·738	3	56·7902	4404·988	·927
2	63·6281	4476·285	·185	3	50·1877	4341·667	·724	0·634	1·090	+75·21
2	63·3080	4472·806	·686	1·676	1·010	+67·67	3	48·4519	4325·839	·939
2	62·7557	4466·838	·727							

Weighted mean..... +72·20

Va..... -24·64

Vd - 22

Curvature..... - 50

Radial velocity +46·8

7-8 EDWARD VII., A. 1903

ORIONIS 686.

1907. April 3.
G. M. T. 12^h 50^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.3298	4494.727738	3	56.7823	4404.921927
2	63.6424	4476.212185	2	50.2157	4342.124	2.104	0.634	1.470	+101.43
2	63.3472	4473.015	2.996	1.676	1.320	+88.44	3	48.4398	4325.973939
2	62.7645	4466.739727							

Weighted mean..... +94.93
V_a -24.40
V_d - .22
Curvature - .50
Radial velocity..... +69.8

ORIONIS 687.

1907. April 3.
G. M. T. 13^h 22^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	65.3150	4494.667738	3	56.7720	4404.919927
2	63.6337	4476.218185	2	50.1852	4341.944	.924	0.634	1.290	+89.01
2	63.3294	4472.922	.926	1.676	1.250	+83.75	3	48.4285	4325.972939
2	62.7527	4466.712727							

Weighted mean +86.38
V_a -24.40
V_d - .25
Curvature - .50
Radial velocity +61.2

SESSIONAL PAPER No. 25a

ORIONIS 693.

1907. April 5.
G. M. T. 12^h 30^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	65.3042	4494.543738	3	56.7750	4404.948927
2	63.6195	4476.065185	2	50.2099	4342.170	.094	0.634	1.460	+100.74
3	63.3650	4473.308	.426	1.676	1.750	+117.25	2	48.4335	4326.017	5.939
2	62.7427	4466.605727							

Weighted Mean.....+110.65

V_a - 23.99

V_d - .22

Curvature - .50

Radial velocity..... +86.0

ORIONIS 695.

1907. April 5.
G. M. T. 13^h 19^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	65.3224	4494.619738	3	56.7897	4404.907927
2	63.6357	4476.101185	1.5	50.2048	4341.891	1.904	0.634	1.270	+87.63
2	63.3505	4473.010	.086	1.676	1.410	+94.47	3	48.4505	4325.927939
1	62.7617	4466.663727							

Weighted Mean +91.54

V_a - 23.99

V_d - .25

Curvature - .50

Radial velocity..... +66.8

7-8 EDWARD VII., A. 1908

ORIONIS 702.

1907. April 6.
G. M. T. 12^h 26^m

Observed by W. E. HARPER.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	65.3065	4494.643	..	.738	3	56.7705	4404.916927
2	63.6195	4476.125185	2	50.2205	4342.236	.274	0.634	1.640	+113.16
2	63.3617	4473.332	396	1.676	1.720	+115.24	2	48.4238	4325.888939
1	62.7431	4466.664727							

Weighted mean..... +114.20
V_a..... -23.80
V_d..... - .22
Curvature..... - .50
Radial velocity..... + 89.7

ORIONIS 703.

1907. April 6.
G. M. T. 12^h 50^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
1½	65.3325	4494.731738	3	56.7935	4404.944927
1½	63.6472	4476.225185	2	50.2682	4342.475	.454	0.634	1.820	+125.58
2	63.4199	4473.761	.736	1.676	2.060	+138.02	3	48.4560	4325.977939
1	62.7571	4466.614727							

Weighted mean..... +131.80
V_a..... -23.80
V_d..... - .22
Curvature..... - .50
Radial velocity..... +107.3

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ORIONIS 705

1907. April 11.
G. M. T. 12^h 46^m

Observed by W. E. HARPER.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	65.3438	4494.732738	3	56.7920	4401.867927
2	63.6437	4476.072185	3	49.9523	4339.557	.604	0.634	1.030	-71.07
2	63.1484	4470.714	.836	1.676	.840	-56.28	3	48.4466	4325.885939
2	62.7677	4466.585727							

Weighted mean..... -65.15

V_a -22.80

V_d - .25

Curvature..... - .50

Radial velocity..... -88.7

ORIONIS.

Number of Neg.	Date.	G.M.T.	Velocity.	Number of Neg.	Date.	G.M.T.	Velocity.
	1906.	h m			1907.	h m	
453.....	Dec. 11..	15 ..	+116.	644.....	Feb. 27..	14 ..	+ 25.
485.....	" 18..	14 ..	- 22.	647.....	March 6..	12 ..	+ 8?
	1907.			650.....	" 6..	15 ..	+ 75.
517.....	Jan. 2 ..	15 ..	+ 34.	653.....	" 8..	12 ..	+106.
522.....	" 9..	14 ..	+100.	655.....	" 8..	15 ..	+110.
535.....	" 15..	15 ..	- 52.	659.....	" 11..	12 ..	-102.
539.....	" 16..	15 ..	- 6.	662.....	" 11..	15 ..	-122.
556.....	" 18..	15 ..	+ 7.	665.....	" 20..	12 ..	+ 13.
565.....	" 21..	15 ..	+ 23.	666.....	" 20..	13 ..	+ 15.
585.....	" 28..	15 ..	+ 43.	667.....	" 20..	13½ ..	+ 11.
587.....	" 30..	12 ..	+ 65.	672.....	" 26..	12 ..	+ 33.
592.....	" 30..	16 ..	+ 48.	673.....	" 28..	12 ..	+ 40.
594.....	Feb. 4..	12 ..	+ 66.	678.....	" 30..	12 ..	+ 62.
601.....	" 6..	15 ..	+ 80.	680.....	April 1..	13 ..	+ 47.
605.....	" 7..	12 ..	+106.	686.....	" 3..	13 ..	+ 70.
609.....	" 12..	12 ..	- 82.	687.....	" 3..	13½ ..	+ 61.
618.....	" 21..	13 ..	+ 9.	693.....	" 5..	13 ..	+ 85.
627.....	" 22..	15 ..	+ 20.	695.....	" 5..	13½ ..	+ 67.
636.....	" 25..	15 ..	+ 14.	702.....	" 6..	12 ..	+ 90.
				703.....	" 6..	13 ..	+107.
				705.....	" 11..	13 ..	- 89.

PREVIOUS OBSERVATIONS.

1903.	h	m		1903.	h	m	
September 5.....	22	29	+ 21	October 23.....	23	37	+ 42
" 25.....	21	58	+ 40	" 24.....	20	38
" 26.....	22	33	+ 57	" 30.....	20		+ 90
October 17.....	23	19	+ 35				

7-8 EDWARD VII., A. 1908

EFFECT OF SLIT WIDTH ON ERRORS OF SETTING.

The investigation on the character of the star image given by the combination of objective and correcting lens had shown that the effective diameter of such an image, so far as the transmission of light through the slit was concerned, was so large that the exposure required was very nearly inversely proportional to the slit width until this reached about 0.15^{mm} between 5 and 6 seconds of arc. In order, therefore, to make use of the greater part of the starlight, a slit several times wider than is normally used would be required. As the purity of the spectrum diminishes in nearly the same ratio as the slit is opened, it is evident that for accurate work the width of slit is limited, and it was the purpose of this investigation to see how much the slit could be widened without increasing the probable error to a prohibitive degree.

Evidently this will depend partly upon the character of the star spectrum and partly upon the optical properties of the spectrograph. Leaving the latter out of consideration for the present, it is evident in stars of the solar type, where the spectrum is complex and most of the lines (with the small dispersion available) are blends, that any decrease in the purity will make the blends more complex and, besides making the error of setting larger, will increase the difficulty of identification and determination of the true wave lengths. Hence such spectra will not admit of much increase in slit width without great loss in accuracy. In the case of early type stars, however, where the lines are single, errors of identification are not likely to cause trouble and only the accidental errors of setting remain. The more diffuse the lines in the spectrum, the less will the probable error be increased by widening the slit within reasonable limits. As no experiments on this line had ever been undertaken, and as the question could not be decided by a theoretical discussion, it seemed worth while to make a number of spectra of the same star at different slit widths and see how the probable error of the velocity as obtained from a single line increased with the slit width.

The value of such work evidently lies in its bearing upon the range of the equipment, for if it is found that the radial velocity of a star can be obtained nearly as accurately with a slit $.063^{\text{mm}}$ as with a slit $.025^{\text{mm}}$ wide, it is evident that stars a magnitude fainter may be obtained and that all exposures are diminished by about 60 per cent.

The star chosen for the test was β Orionis, a helium star with moderately sharp helium and hydrogen lines and with some metallic lines. Five plates at each of five slit widths were made with the Brashear spectroscope, and the twenty-five plates were then measured by myself under similar conditions, using as far as possible the same lines throughout. Owing to the varying quality of the lines for measurement, they were weighted, and the weighted mean was used for determining the velocity and the weighted residuals for obtaining the probable errors. The measurements of the twenty-five plates follow.

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RECORD OF SPECTROGRAMS.

Star.	No. of Neg.	Plate.	Date.	Middle of Exposure.		Duration.	Hour Angle at End.	COMPARISON SPECTRUM.		TEMPERATURE.				Slit Width. in Millimetres	Focal Position.		Observer.	
				G. M. T.	Exposure.			Beg.	End.	Kind.	Room.		Prism Box.		Star Focus.	Coll'r. Camera.		
											Beg.	End.	Beg.					End.
1906.																		
β Orionis.	463	Seed 27.	Dec. 17.	18	33	h.	m.	25	25	Fe. Spark.	17.0	17.0	6.3	6.3	19.0	15.2	5.73 P.	
"	464	" 27.	" 17.	19	00	2	05 W	25	25	"	17.0	16.8	6.3	6.3	19.0	15.2	5.73 P.	
"	465	" 27.	" 17.	19	07	2	40 W	25	25	"	16.8	16.8	6.3	6.3	19.0	15.2	5.73 P.	
"	466	" 27.	" 17.	19	18	2	51 W	23	23	"	16.7	16.7	6.3	6.3	19.0	15.2	5.73 P.	
"	467	" 27.	" 17.	19	24	2	56 W	23	23	"	16.7	16.6	6.2	6.2	19.0	15.2	5.73 P.	
"	468	" 27.	" 17.	19	31	3	04 W	23	23	"	16.6	16.6	6.1	6.1	19.0	15.2	5.73 P.	
"	469	" 27.	" 17.	19	43	3	16 W	20	20	"	16.5	16.5	5.9	5.9	19.0	15.2	5.73 P.	
"	470	" 27.	" 17.	19	48	3	22 W	20	20	"	16.5	16.5	5.9	5.9	19.0	15.2	5.73 P.	
"	471	" 27.	" 17.	19	56	3	28 W	20	20	"	16.5	16.5	5.9	5.9	19.0	15.2	5.73 P.	
"	472	" 27.	" 17.	20	04	3	36 W	18	18	"	16.5	16.5	5.8	5.8	19.0	15.2	5.73 P.	
"	473	" 27.	" 17.	20	09	2	42 W	18	18	"	16.5	16.5	5.8	5.8	19.0	15.2	5.73 P.	
"	474	" 27.	" 17.	20	13	2	47 W	18	18	"	16.5	16.5	5.8	5.8	19.0	15.2	5.73 P.	
"	494	" 27.	" 19.	15	19	1	00 E	25	25	"	13.4	13.0	7.4	7.4	19.0	15.2	5.70 P.	
"	495	" 27.	" 19.	15	27	0	54 E	25	25	"	13.0	12.0	7.4	7.4	19.0	15.2	5.70 P.	
"	496	" 27.	" 19.	15	40	0	40 E	23	23	"	12.0	11.5	7.4	7.4	19.0	15.2	5.70 P.	
"	497	" 27.	" 19.	15	44	0	33 E	23	23	"	11.5	11.0	7.4	7.4	19.0	15.2	5.70 P.	
"	498	" 27.	" 19.	15	54	0	25 E	20	20	"	11.0	10.8	7.4	7.4	19.0	15.2	5.70 P.	
"	499	" 27.	" 19.	15	59	0	20 E	20	20	"	10.8	10.6	7.4	7.4	19.0	15.2	5.70 P.	
"	500	" 27.	" 19.	16	07	0	12 E	18	18	"	10.4	10.4	7.4	7.4	19.0	15.2	5.70 P.	
"	501	" 27.	" 19.	16	12	0	7 E	18	18	"	10.4	10.3	7.4	7.4	19.0	15.2	5.70 P.	
"	502	" 27.	" 19.	16	20	0	0 W	18	18	"	10.3	10.2	7.4	7.4	19.0	15.2	5.70 P.	
"	503	" 27.	" 19.	16	24	0	4 W	18	18	"	10.2	10.1	7.4	7.4	19.0	19.0	5.70 P.	
"	504	" 27.	" 19.	16	31	0	12 W	18	18	"	10.1	10.0	7.4	7.4	19.0	19.0	5.70 P.	
"	505	" 27.	" 19.	16	37	0	18 W	18	18	"	10.0	10.0	7.4	7.4	19.0	19.0	5.70 P.	
"	506	" 27.	" 19.	16	41	0	22 W	18	18	"	10.0	10.0	7.5	7.5	19.0	19.0	5.70 P.	

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β ORIONIS 463.
Slit .025.

1906. Dec. 17.
G. M. T. 18^h 33^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
3	72.8657	4584.396	4.018	2	62.6806	4467.042	7.727
3	70.0181	4549.960	9.642	3	56.7140	4405.174	5.927
2	65.2394	4495.055	...	5.738	1	55.0437	4388.666	8.416	8.100	316	21.58
3	64.0587	4482.013	1.700	1.400	300	+20.07	2	50.0231	4341.019	0.834	0.634	200	+13.80
2	63.5534	4476.493	6.185	3	48.3893	4326.122	6.939
2	63.1691	4472.319	2.006	1.676	330	22.11	3	46.3865	4308.248	8.081

Weighted mean..... +19.08
V_a - 4.59
V_d - .16
Curvature..... - .50
Radial velocity..... +13.8

β ORIONIS 464.
Slit .025.

1906. Dec. 17.
G. M. T. 19^h

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
3	72.8364	4584.035018	2	63.1336	4471.934	.979	.676	.303	20.30
1	72.8603	4584.330	.313	.018	.295	+19.23	3	56.6786	4404.821927
2	69.9855	4549.574642	$\frac{1}{2}$	54.9962	4388.201	.313	.100	.213	14.54
3	65.2061	4494.684738	$\frac{1}{2}$	49.9969	4310.778	.968	.634	.274	+18.90
3	64.0273	4481.669	.713	.400	.313	20.93	3	48.3557	4325.819939
3	63.5214	4476.145185	3	46.3512	4307.936081

Weighted Mean..... +19.82
V_a - 4.59
V_d - .19
Curvature..... - .50
Radial velocity..... +14.5

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β ORIONIS 465.
Slit .025.

1906. Dec. 17.
G. M. T. 19^h 07^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
3	72.8405	4584.086018	1½	62.6541	4466.757727
1	72.8449	4584.153	.088	.018	.070	S 3	56.6892	4404.927927
S 2	69.9912	4549.642	3	55.0195	4388.429	.430	.100	.330	22.53
2	65.2117	4494.747738	2	50.0140	4340.936	.920	.634	.286	+20.42
3	64.0275	4481.671	.650	.400	.250	+16.72	3	48.3713	4325.960939
2	63.5265	4476.200185	S 3	46.3676	4308.081
2	63.1460	4472.068	.046	.676	.370	25.79							

Weighted Mean.....+20.51

V_a.....-4.59

V_d.....- .19

Curvature..- .50

Radial velocity.....+15.2

β ORIONIS 466.
Slit .0375.

1906. Dec. 17.
G. M. T. 19^h 18^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
3	72.8289	4583.943018	2	62.6451	4466.660727
4	72.8700	4584.449	.518	.018	.500	+32.60	3	56.6795	4404.829927
2	69.9821	4549.534642	1½	55.0233	4388.466	.570	.100	.470	32.10
2	65.1958	4494.570738	1½	49.9981	4340.789	.894	.634	.260	+17.94
3	64.0246	4481.639	.710	.400	.310	20.73	3	48.3570	4325.830939
3	63.5150	4475.114185	3	46.3520	4307.943081
2	63.1400	4472.004	.076	.676	.400	26.80							

Weighted Mean+23.02

V_a.....-4.59

V_d.....- .22

Curvature..- .50

Radial velocity... ..+17.7

7-8 EDWARD VII., A. 1908

β ORIONIS 467.
Slit .0375.

1906. Dec. 17.
G. M. T. 19^h 24^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
3	72.8329	4583.992018	2	63.1466	4472.075	.051	.676	.375	25.12
4	72.8457	4584.150	.178	.018	.160	+10.43	3	56.6872	4404.906927
2	69.9900	4549.627642	1	55.0150	4388.385	.400	.100	.300	20.49
3	65.2085	4494.711738	2	49.9975	4340.784	.796	.634	.162	+ 11.17
2	64.0397	4481.865	.780	.400	.380	26.68	3	48.3670	4325.920939
3	63.5300	4476.238185	3	46.3627	4308.037081

Weighted Mean..... +20.98
Va..... 4.59
Vd..... - .22
Curvature..... .50

Radial velocity..... +15.7

β ORIONIS 468.
Slit .0375.

1906. Dec. 17.
G. M. T. 19^h 31^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	72.8397	4584.076018	1	62.6520	4466.735727
4	72.8672	4584.416	.358	.018	.340	+ 22.16	3	56.6890	4404.925927
2	69.9926	4549.658642	1	55.0233	4388.466	.466	.100	.366	25.00
3	65.2122	4494.752738	1	50.0083	4340.884	.884	.634	.250	+ 17.25
2	64.0378	4481.784	.776	.400	.376	25.15	3	48.3690	4325.939939
2	63.5259	4476.194185	3	46.3665	4308.072081
2	63.1438	4472.014	.036	.676	.360	24.12							

Weighted Mean.... +22.96
Va..... - 4.59
Vd..... - .22
Curvature..... - .50

Radial velocity..... +17.6

SESSIONAL PAPER No. 25a

β ORIONIS 469.
Slit .05.

1906. Dec. 17.
G. M. T. 19^h 43^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	72.8265	4583.913018	1	62.6459	4466.669727
$\frac{1}{4}$	72.8595	4584.320	.425	.018	.407	+ 26.53	3	56.6822	4404.856927
2	69.9871	4549.593642	$\frac{1}{4}$	55.0372	4388.602	.674	.100	.574	39.20
$1\frac{1}{2}$	65.2035	4494.655738	1	49.9881	4340.698	.794	.634	.160	+ 11.04
$2\frac{1}{2}$	64.0160	4481.546	.610	.400	.210	14.04	3	48.3586	4325.845939
2	63.5205	4476.135185	3	46.3551	4307.970081
$1\frac{1}{2}$	63.1329	4471.927	.983	.676	.307	20.76							

Weighted mean..... + 16.98
V_a..... - 4.59
V_d..... - .22
Curvature.. - .50
Radial velocity..... + 11.7

β ORIONIS 470.
Slit .05.

1906. Dec. 17.
G. M. T. 19^h 48^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	72.8250	4583.895018	1	62.6381	4466.585727
$\frac{1}{4}$	72.8520	4584.268	.388	.018	.370	+ 24.12	3	56.6760	4404.795927
2	69.9766	4549.568642	$\frac{1}{4}$	55.0253	4388.487	.620	.100	.520	35.51
2	65.2016	4494.634738	$1\frac{1}{4}$	50.0005	4340.811	.984	.634	.356	+ 24.15
3	64.0228	4481.620	.716	.400	.316	21.14	3	48.3521	4325.787939
$1\frac{1}{2}$	63.5176	4476.103	..	.185	3	46.3482	4307.910081
$1\frac{1}{2}$	63.1297	4471.892	.002	.676	.326	21.84							

Weighted Mean..... + 22.82
V_a..... - 4.59
V_d..... - .22
Curvature.. - .50
Radial velocity.. + 17.5

7-8 EDWARD VII., A. 1908

β ORIONIS 471.
Slit .05.

1906. Dec. 17.
G. M. T. 19^h 56^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
3	72.8246	4583.890018	1	62.6389	4466.594727
1 $\frac{1}{2}$	72.8530	4584.239	.368	.018	.350	+22.82	3	56.6806	4404.840927
1	69.9873	4549.595642	1	49.9974	4340.783	.904	.634	.270	18.63
2	65.2017	4494.636738	1 $\frac{1}{2}$	55.0233	4388.466	.600	.100	.500	+34.15
3	64.0180	4481.567	.670	.400	.270	18.06	3	48.3562	4325.823939
2	63.5168	4476.094185	3	46.3552	4307.972081
2	63.1297	4471.892	.006	.676	.330	22.01							

Weighted mean..... +20.26
V_a..... -4.59
V_d..... - .19
Curvature..... - .50
Radial velocity..... +15.0

β ORIONIS 472.
Slit .075.

1906. Dec. 17.
G. M. T. 20^h 04^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
3	72.8363	4584.034018	1 $\frac{1}{2}$	62.6448	4466.657727
1 $\frac{1}{2}$	69.9915	4549.646642	3	56.6827	4404.862927
2	65.2091	4494.717738	1 $\frac{1}{2}$	50.0209	4341.000	.084	.634	.450	+31.05
2	64.0271	4481.634	.690	.400	.290	+19.40	3	48.3607	4325.864939
1 $\frac{1}{2}$	63.5184	4476.112185	3	46.3538	4307.959081
1	63.1460	4472.285	.349	.676	.673	45.09							

Weighted mean..... +28.99
V_a..... -4.59
V_d..... - .25
Curvature..... - .50
Radial velocity..... +23.6

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β ORIONIS 473.
Slit .075.

1906. Dec. 17.
G. M. T. 20^h 09^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	72.8672	4584.415018	1	62.6725	4466.955727
2	70.0161	4549.936642	3	56.7185	4405.219927
1 $\frac{1}{2}$	65.2359	4495.016738	1 $\frac{1}{4}$	55.0624	4388.849	569	.100	.469	32.03
2	64.0521	4481.941	.685	.400	.285	+19.06	1	50.0216	4341.006	.766	.634	.132	+9.11
1	63.5490	4476.445185	3	48.3930	4326.156939
1	63.1724	4472.355	.110	.676	.334	22.37	3	46.3905	4303.283081

Weighted mean..... +18.26

Va..... - 4.59

Vd..... - .25

Curvature..... - .50

Radial velocity..... +12.9

β ORIONIS 474.
Slit .075.

1906. Dec. 17.
G. M. T. 20^h 13^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
3	72.8345	4584.012018	1 $\frac{1}{2}$	62.6359	4466.562727
1	69.9864	4549.584642	3	56.6780	4404.815927
1	65.1895	4494.500738	1 $\frac{1}{4}$	54.9865	4388.106	.210	.100	.110	7.51
3	64.0260	4481.655	.740	.400	.340	+22.74	2	49.9933	4340.750	.834	.634	.200	+13.80
1	63.5190	4476.119185	3	48.3598	4325.856939
1	63.1262	4471.854	.936	.676	.260	17.42	3	46.3567	4307.985081

Weighted mean.. +18.26

Va..... - 4.59

Vd..... - .19

Curvature... .. - .50

Radial velocity..... +13.0

7-3 EDWARD VII., A. 1908

β ORIONIS 494.
Slit .025.

1906. Dec. 19.
G. M. T. 15^h 19^m

Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	72.8247	4584.180018	2	62.6524	4466.898727
1	72.8590	4584.603	.441	.018	.423	+27.57	3	56.6875	4405.032927
1	69.9854	4549.814642	1	55.0451	4388.799	.680	.109	580	39.61
2	65.2063	4494.868738	2	50.0097	4341.005	.854	.634	.220	+15.18
3	64.0383	4481.961	.850	.400	.450	30.10	2	48.3740	4326.094939
2	63.5177	4476.271185	3	46.3695	4308.209081
1½	63.1623	4472.408	.286	.676	.610	40.87							

Weighted Mean +28.39
V_a -5.42
V_d + .09
Curvature.. - .50
Radial velocity +22.6

* β ORIONIS 494.
Slit .025.

1906. Dec. 19.
G. M. T. 15^h 19^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	72.8210	4584.134018	2	62.6388	4466.751727
1	72.8493	4584.484	.368	.018	.350	+22.82	3	56.6789	4404.946927
2	69.9800	4549.751642	1	55.0326	4388.674	.656	.100	.556	37.97
3	65.1989	4494.785738	2	50.0078	4340.988	.964	.634	.330	+22.77
4	64.0307	4481.877	.833	.400	.433	28.96	3	48.3600	4325.967939
3	63.5150	4476.241185	3	46.3556	4308.087081
3	63.1482	4472.255	.206	.676	.530	35.51							

* Check measurement.
Weighted Mean +29.91
V_a -5.42
V_d + .09
Curvature.. - .50
Radial velocity +24.1

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β ORIONIS 495.
Slit .025.

1906. Dec. 19.
G. M. T. 15^h 27^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
3	72.8437	4584.411		.018			2	62.6598	4466.977		.727		
$\frac{1}{4}$	72.8604	4584.620	.224	.018	.206	+13.43	3	56.6974	4405.131		.927		
2	69.9988	4549.974		.642			$\frac{1}{2}$	55.0340	4388.688	.188	.100	.388	27.18
3	65.2232	4495.056		.738			$1\frac{1}{2}$	50.0181	4341.083	.874	.634	.240	+16.56
3	64.0355	4481.930	.690	.400	.290	19.40	3	48.3806	4326.153		.939		
3	63.5348	4476.457		.185			3	46.3770	4308.275		.081		
$2\frac{1}{2}$	63.1569	4472.349	.086	.676	.410	27.47							

Weighted Mean..... +21.76
Va..... -5.42
Vd..... + .09
Curvature..... - .50

Radial velocity .. . +15.9

β ORIONIS 496.
Slit .0375.

1906. Dec. 19.
G. M. T. 15^h 40^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement	Velocity	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	72.8432	4584.498		.018			$1\frac{1}{2}$	62.6507	4466.879		.727		
$\frac{1}{4}$	72.8715	4584.758	.368	.018	.350	+22.82	3	56.6956	4405.113		.927		
2	69.9928	4549.902		.642			$1\frac{1}{2}$	50.0168	4341.070	.910	.634	.276	+19.04
2	65.2142	4494.956		.738			3	48.3736	4326.090		.939		
$2\frac{1}{2}$	64.0390	4481.968	.770	.400	.370	24.75	3	46.3755	4308.262		.081		
$2\frac{1}{2}$	63.5298	4476.402		.185									

Weighted Mean .. . +22.62
Va..... -5.42
Vd..... + .04
Curvature..... - .50

Radial velocity..... + 16.7

7-8 EDWARD VII., A. 1908

β ORIONIS 497.
Slit .0375.

1906. Dec. 19.
G. M. T. 15^h 44^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	69.9746	4549.686642	1½	62.6392	4466.755727
3	64.0218	4481.780	.760	.400	.360	+ 24.08	2	55.0242	4388.592	.600	.100	.500	34.15
2	63.5123	4476.211185	1	50.0015	4340.930	.924	.634	.290	+ 20.01
1½	63.1347	4472.109	.086	.676	.410	27.47	3	48.3594	4325.962939

Weighted Mean..... + 25.09
V_a - 5.42
V_d + .04
Curvature . - .50
Radial velocity..... + 19.2

β ORIONIS 498.
Slit .05.

1906. Dec. 19.
G. M. T. 15^h 54^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
3	72.8427	4584.402018	1	62.6571	4466.948727
4	72.8702	4584.734	.354	.018	.336	+ 21.90	3	56.7000	4405.157927
2	70.0056	4550.055642	4	55.0338	4388.686	.466	.100	.366	25.00
2	65.2146	4494.961738	1½	50.0112	4341.019	.814	.634	.180	+ 12.42
2½	64.0422	4482.003	.767	.400	.367	25.22	3	48.3775	4326.125939
2	63.5338	4476.446185	3	46.3761	4308.268081
1½	63.1656	4472.444	.216	.676	.540	36.18							

Weighted Mean..... + 24.61
V_a - 5.42
V_d + .04
Curvature . - .50
Radial velocity..... + 18.7

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β ORIONIS 499.
Slit .05.

1906. Dec. 19.
G. M. T. 15^h 59^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
3	69.9969	4549.952642	3	56.7004	4405.161927
3	65.2120	4494.932738	3	55.0456	4388.801	.560	.100	.460	31.41
2 ¹ / ₂	64.0510	4482.100	870	.400	.470	+31.44	1 ¹ / ₂	50.0141	4341.046	.814	.634	.180	+12.42
2	63.5304	4476.403185	3	48.3821	4326.167939
1 ¹ / ₂	63.1693	4472.484	212	.676	.536	35.91	3	46.3785	4308.288081
1	62.6708	4467.095727							

Weighted mean.....+27.64

V_a -5.42

V_d+ .04

Curvature - .50

Radial velocity.....+21.7

β ORIONIS 500.
Slit .0625.

1906. Dec. 19.
G. M. T. 16^h 07^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
3	72.8407	4584.377918	1	62.6531	4466.905727
3	69.9912	4549.884642	3	56.6972	4405.129927
2	65.2176	4494.995738	4	55.0072	4388.426	.216	.100	.116	7.92
1 ¹ / ₂	64.0303	4481.873	600	.400	.200	+13.38	2	50.0033	4341.050	.864	.634	.230	+15.87
3	63.5353	4476.462185	3	48.3771	4326.121939
2	63.1550	4472.329	.081	.676	.405	27.13	3	46.3742	4308.251081

Weighted mean.....+18.79

V_a -5.42

V_d00

Curvature - .50

Radial velocity.....+12.9

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β ORIONIS 501.
Slit .0625.

1906. Dec. 19.
G. M. T. 16^h 12^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	72.8040	4583.924018	1	62.6235	4466.587727
1 $\frac{1}{2}$	69.9726	4549.663642	3	56.6706	4404.833927
1 $\frac{1}{2}$	65.1830	4494.608738	1 $\frac{1}{2}$	50.6022	4340.936	.060	.634	.366	+25.25
2 $\frac{1}{2}$	64.0253	4481.818	.946	.400	.546	+36.52	3 $\frac{1}{2}$	48.3488	4325.866939
1 $\frac{1}{2}$	63.4995	4476.076185	3	46.3462	4308.004081
1 $\frac{1}{2}$	63.1436	4472.205	.333	.676	.657	44.01							

Weighted mean..... +35.49
Va..... -5.42
Vd..... .00
Curvature..... - .50
Radial velocity..... +29.6

β ORIONIS 502.
Slit .0625.

1906. Dec. 19.
G. M. T. 16^h 20^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W.L.	Normal W.L.	Displacement.	Velocity
2	72.8235	4584.165018	1	62.6467	4466.835	..	.727
1 $\frac{1}{2}$	69.9810	4549.762642	3	56.6844	4405.005927
1 $\frac{1}{2}$	65.2012	4494.811738	1 $\frac{1}{2}$	55.0210	4388.560	.480	.100	.380	25.95
2	64.0233	4481.796	.700	.400	.300	+20.07	1 $\frac{1}{2}$	50.0089	4340.990	.910	.634	.276	+19.04
2	63.5206	4476.302185	3	48.3655	4326.016	..	.939
1 $\frac{1}{2}$	63.1392	4472.157	.056	.676	.380	25.46	3	46.3639	4308.160081

Weighted mean..... +21.59
Va..... -5.42
Vd..... .00
Curvature..... - .50
Radial velocity..... +15.7

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β ORIONIS 503.
Slit .0625.

1906. Dec. 19.
G. M. T. 16^h 24^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
3	72.8462	4584.445018	1	62.6597	4466.976727
2	70.0030	4550.024642	3	56.6994	4405.151927
2	65.2219	4495.042738	4	55.0276	4388.625	.445	.100	.345	23.56
2	64.0314	4481.885	.600	.400	.200	+13.38	1	49.9985	4340.902	.753	.634	.124	+8.55
1 $\frac{1}{2}$	63.5377	4476.488185	3	48.3708	4326.065939
1 $\frac{1}{2}$	63.1484	4472.257	.986	.676	.310	20.77	3	46.3699	4308.212081

Weighted Mean.....+15.23
V_a.....-5.42
V_d......00
Curvature...- .50
Radial velocity.....+ 9.3

β ORIONIS 504.
Slit .0625.

1906. Dec. 19.
G. M. T. 16^h 31^m

Observed by } J. S. PLASKETT.
Measured by }

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
3	72.8352	4584.310018	1	62.6457	4466.824727
3	72.8683	4584.718	.428	.018	.310	+20.21	3	56.6857	4405.014927
2	69.9925	4549.899642	4	55.0152	4388.503	.415	.100	.315	21.51
2	65.2057	4494.862738	1	49.0085	4340.994	.898	.634	.264	+18.21
2	64.0387	4481.965	.853	.400	.453	30.30	3	48.3686	4326.045939
1 $\frac{1}{2}$	63.5210	4476.307185	1	46.3657	4308.176081
1	63.1545	4472.323	.219	.676	.543	36.38							

Weighted Mean.....+27.91
V_a.....-5.42
V_d......00
Curvature...- .50
Radial velocity+ 22.0

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β ORIONIS 505.

Slit .075.

1906. Dec. 19.
G. M. T. 16^h 37^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	72.8287	4584.229018	1	62.6439	4466.806727
2	69.9830	4549.786642	3	56.6794	4404.951927
1 ¹ / ₃	65.2003	4494.802738	4	55.0267	4388.617	.600	.100	.500	34.15
1 ¹ / ₃	64.0285	4481.853	.800	.400	.400	+ 26.76	1	49.9852	4340.779	.774	.634	.140	+ 9.66
1	63.5130	4476.219185	3	48.3570	4325.940939
1 ¹ / ₂	63.1214	4471.964	.900	.676	.224	15.00	3	46.3560	4308.090081

Weighted Mean..... + 20.26
V_a - 5.42
V_d00
Curvature . - .50
Radial velocity. ... + 14.3

β ORIONIS 506.

Slit .075.

1906. Dec. 19.
G. M. T. 16^h 41^m

Observed by J. S. PLASKETT.
Measured by J. S. PLASKETT.

Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity.	Weight.	Mean of Settings.	Computed Wave Length.	Corrected W. L.	Normal W. L.	Displacement.	Velocity
2	72.8542	4584.544018	1	62.6575	4466.952727
1 ¹ / ₃	70.0029	4550.023642	3	56.7110	4405.267927
2	65.2305	4495.138738	4	55.0170	4388.521	.185	.100	.085	5.80
2	64.0452	4482.036	.610	.400	.210	+ 14.04	1	50.0155	4341.058	.734	.634	.100	+ 6.90
1	63.5501	4476.624185	3	48.3927	4326.263939
1	63.1521	4472.298	.876	.676	.200	13.40	3	46.3888	4308.379081

Weighted Mean..... + 11.72
V_a - 5.42
V_d - .04
Curvature . - .50
Radial velocity..... + 5.8

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SUMMARY OF VELOCITIES.

Slit .025		Slit .0375		Slit .05		Slit .0625		Slit .075	
Plate.	Velocity.	Plate.	Velocity.	Plate.	Velocity.	Plate.	Velocity.	Plate.	Velocity.
463	+13.8	466	+17.7	469	+11.7	500	+12.9	472	+23.6
464	14.5	467	15.7	470	17.5	501	29.6	473	12.9
465	15.2	468	17.6	471	15.0	502	15.7	474	13.0
494	23.3	496	16.7	498	18.7	503	9.3	505	14.3
495	15.9	497	19.2	499	21.7	504	22.0	506	5.8

Means + 16.5 + 17.4 + 16.9 + 17.9 + 13.9
 r = ±1.17 ± 0.38 ± 1.14 ± 2.42 ± 1.91
 General Mean +16.5 kms. per sec.

It will be noticed that there is not very good accordance in the velocity values of some of the plates; this may be accounted for partly by poor temperature control, partly by increased slit width and possible asymmetric position of the star image and partly by faulty guiding. The star is too bright for the best results in guiding by transmitted light, and it would be quite easy to obtain non uniform illumination of the collimator lens which might easily persist for the whole of an exposure. This ranged from 2^{mins} with slit .075^{mm} to 6^{mins} with slit .025^{mm} wide.

Any systematic displacement produced by any of these causes will not, however, affect the result obtained by discussing the residuals from each plate, and this is what was done in the first case. The residuals in kilometres per second from each line on a plate were grouped together for each slit width, five plates between 20 and 25 lines, and the probable error of the velocity obtained from the measurement from a single line was deduced.

Slit Width.	Probable Error.
.025 ^{mm}	2.82 ^{kms} per second.
.037 ^{mm}	3.19 ^{kms} "
.050 ^{mm}	4.44 ^{kms} "
.0625 ^{mm}	3.96 ^{kms} "
.075 ^{mm}	4.63 ^{kms} "

The probable error, as the table indicates, increases gradually with the slit width although not nearly in the same proportion. If this diminished accuracy were permissible, the range of the instrument would be more than doubled. It is probable in spectra with sharper lines that the ratio of increase of probable error with slit width would be greater, and in spectra with more diffuse lines, less. The increase is undoubtedly due to the greater difficulty in setting accurately on the diffuse lines given when a wide slit is used. If the dispersion of the spectrograph were increased and the focus of the camera correspondingly shortened, this difficulty should be materially lessened, and as soon as a short focus camera lens is obtained for the new spectrograph, this investigation will be continued along the lines indicated, for the purpose of determining the maximum permissible slit width for reasonably accurate determinations, and for obtaining the relative efficiency of two spectrographs,—one with low dispersion and camera of long focus, the other with high dispersion and camera of short focus.

However, although not sufficient plates were measured to give a definite determination of the exact relation between slit width and accuracy, what has already been done seems to show that the slit width can probably be materially widened on early type stars without entailing much loss of accuracy.

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The other three investigations mentioned above are given below as Appendices A, B, C, and complete the report of the astrophysical work done during the past year.

In conclusion, allow me again to express my deep sense of obligation for the many kindnesses you have shown and the help and encouragement you have always given me in my work.

I have the honour to be, sir,

Your obedient servant,

J. S. PLASKETT.

APPENDIX A.

(Reprinted from the *Astrophysical Journal*, Vol. XXV., No. 3.)

'THE CHARACTER OF THE STAR IMAGE IN SPECTROGRAPHIC WORK.

'BY J. S. PLASKETT.

'The object of this paper is to describe some experiments on the size and form of the star image given by the combination of objective and correcting-lens, with an investigation into the causes of the observed effects and suggestions for the improvement of existing conditions.

'The equipment of the Dominion Observatory, Ottawa, for radial-velocity work consists of a 15-inch telescope with a Brashear visual objective and photographic correcting-lens, and a spectroscope of the Universal type, also by Brashear. The objective for visual purposes is excellent, and the spectroscope is admirably adapted for general spectroscopic work, but, as the experience of others as well as myself has shown, is not suitable for the accurate determination of radial velocities. Its design as a universal spectroscope does not give sufficient stability, and, in exposures of any length, flexure will not only ruin the definition, but is liable to introduce systematic errors in the velocities obtained. Pending the construction of a spectrograph specially designed for the required purpose, an attempt was made to render the present instrument capable of giving accurate velocity values. The investigation and removal of the known sources of error led to the discovery of the aberrations to be presently described. A brief description of the steps leading thereto may be of interest.

'Trusses connecting the various parts of the instrument, where flexure could occur, with the supporting tubes were applied to such effect that an initial displacement of the spectral lines, equivalent to a velocity of 30 km per second, occasioned by a movement of telescope and spectroscope through two hours in right ascension, was reduced to $1\frac{1}{2}$ km. The prisms were firmly clamped in place, without inducing strains in the glass, by screws passing through the base of the prism-box and the minimum-deviation linkwork into the prism-cells. The slit-jaws, originally too thick on the edge, were reground, and the occulting diaphragms for star and spark light were removed from the slit-head and placed on an independent frame attached to the supporting tubes. The comparison apparatus was remodeled, the direction of the spark being made transverse to, instead of parallel with, the slit-jaws, and many other smaller details were carefully attended to.

'After all known sources of error in the spectroscope itself had been overcome, and after it had been placed in thorough adjustment, it was found that test spectra of the standard-velocity stars occasionally gave values differing by as much as 3 km per second from those obtained by other observers. As the probable error of the mean of the measured lines did not exceed four-tenths of a kilometre, and as all the other known causes of systematic error had been overcome, it seemed probable that this might be due to unsymmetrical distribution of the star light over the collimator and camera lenses. Evidently such unsymmetrical distribution can cause a displacement of the lines only when the camera is not in exact focus. The camera was always carefully focussed by a modification of Newall's method, which readily detected displacements of the sensitive surface from the focal plane of less than 0.05 mm in a focal length of 375 mm. But as the plates are supported only at the ends of the plate-holders, differences in the curvature of the glass may easily cause differences of

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0.1 mm or more in the position of the center of the sensitive surface, where all measurements are made. In the case of a displacement of 0.1 mm from the focus, a distribution of the star light on the collimator objective so that its center of intensity is 5 mm to one side of the axis, is sufficient to cause a displacement of the spectral line $\frac{5}{375} \times \frac{1}{10} = \frac{1}{750}$ mm equivalent to a velocity of 1.8 km per second.

‘An examination of the illumination pattern on the collimator lens, both visual and photographic showed how easily such or even greater displacements of the center of intensity could occur even with the utmost care in guiding. The illumination could never be made uniform, no matter how the relative positions of slit and correcting-lens were altered. The pattern was either a diametrical bar parallel to the slit of a width about one-third or one-fourth the aperture, or else such a bar with the addition of a peripheral ring; while a very slight movement of the slit-jaws to one side or other was sufficient to cause one side only of the lens to be illuminated, without causing any appreciable change in the appearance of the image in the guiding telescope, guiding being done by means of light coming through the slit. It is easy to see how the center of intensity of the star light could be displaced without the observer being aware of the fact, thus causing a displacement of the star lines unless the plate were in exact focus.

‘The appearance of this pattern and its behaviour for change of slit position indicated spherical aberration of the condensing system. That aberrations of some nature were present was indicated not only by the long exposures required—upward of two hours for a star of the fourth photographic magnitude—but also by the large effective diameter of the image as shown by the wide opening, 0.25 mm, of the slit required to obtain uniform illumination.

‘An examination of the correcting-lens showed that part of the difficulty might arise from the accidental inversion of the diverging element, which had been so placed in the cell that surfaces of unlike curvature were adjacent to each other. On inverting this concave element so that surfaces of like radius of curvature were in contact, the illumination pattern became more uniform, the required exposure time was diminished by 50 per cent. and no errors of a greater magnitude than should be expected with the dispersion employed, appeared in velocity determinations of standard stars. If the diameter of the object-glass, 15 inches, and the linear dispersion of the spectrograph, 18.6 tenth-meters per millimeter at $H\gamma$, be taken into account, the exposures required—less than an hour for stars of the fourth photographic magnitude—compare very favourably with those of other equipments.

‘Notwithstanding the great improvement shown, photographic tests of the star focus for different temperatures indicated that the star spectrum was much wider than could reasonably be accounted for by atmospheric disturbance, and I was led to make thorough tests of the character and diameter of the image.

‘To determine whether a narrower spectrum could be obtained by a change in adjustment, a plate was made for each of six settings of the correcting-lens, above and below its computed position, over a range of four inches. A simple device applied to one of the plate holders enabled ten successive star spectra to be made side by side on each of these plates, at different settings of the slit position in the neighborhood of the star focus; the sixty spectra forming a record of the diameter of the star image under varying conditions. To insure that the spectrum had not been widened by a drift of the star image along the slit, the spectroscope was turned in position angle until the slit-jaws were parallel to an hour circle. By opening the slit 0.2 mm, and by using a bright star, *Vega*, a fully exposed linear spectrum was obtained in eight or ten seconds, evidently with no chance of widening due to drift. The width of the narrowest part of the narrowest spectrum on each plate, presumably where the star was in focus on the slit, was measured, and these widths ranged from 0.085 to 0.115 mm. As the camera and collimator objectives are of the same focal length, and as one second of arc in the focus of the refractor is equivalent to 0.0275 mm, the

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diameter of the star image according to this test must be between $3''$ and $4''.5$. The diameter of the central diffraction disk as given by the formula $d = \frac{1.2197\lambda}{r}$ is, for a 15-inch objective and $H\gamma$ light, about $0''.57$, while the actual effective diameter as obtained from the width of star spectra is five to eight times as great.

'This enlargement of the diffraction image may be due to three causes: (1) aberrations in the spectroscope; (2) atmospheric disturbances; (3) aberrations in the system of objective and correcting-lens.

'1. *Aberrations in the spectroscope.*—It is a simple matter to determine whether the wide star spectra obtained are due to this cause, for by direct photography of the star image no aberrations in the spectroscope can affect the result. A series of star trails was therefore made on ordinary plates by the system of objective and correcting-lens. A small plate, held in guides in the slit-cap of the spectroscope, could be moved in these guides between exposures so as to make a number of trails on each plate. The collimator tube, carrying the plate with it, was moved by the rack and pinion about a quarter of a millimetre between each exposure, to insure having one of the trails within an eighth millimetre of the focus. A plate each was made of six stars ranging from the third to the sixth magnitude, and the width of the narrowest trail on each plate, corresponding to the position where the star was most nearly in focus, was measured. Although the conditions of seeing both for trails and spectra were above the average, about 3 in a scale of 5, the trails were not continuous, but broken and jagged, owing to atmospheric disturbances, and the measurements were made in two ways: first, of the width of narrow short parts of the trails where the seeing had been momentarily steady; and, second, of the average width of a longer strip of trail. In the first series of measurements the widths varied from 0.070 mm in the fainter stars to 0.110 mm in the brighter stars, while the average widths of longer strips were about 20 per cent. greater. Since the widths of spectra were practically the same, it is evident that the cause must be sought in the star image itself, and is not due to aberrations in the spectroscope.

'2. *Atmospheric disturbances.*—Newall, in his paper on the design of spectrographs* has introduced a very useful conception, that of tremor-disks, and he states that atmospheric disturbances enlarge the effective diameter of the star image. Such enlargement may be due either to bodily displacements of the image from its mean position or to the spreading-out of the central image into a more or less expanded disk. He considers that the actual effect, so far as getting light through the slit of a spectrograph is concerned, is the same as if the image consisted of a central core from $1''$ to $2''$ in diameter surrounded by a more or less diffuse and gradually diminishing portion, the whole diameter being in the neighborhood of $4''$ or $5''$. If we accept Newall's estimates as correct, and if we remember that in no case was a sufficiently long exposure given to allow the outlying parts of the tremor-disk to increase the width of spectrum or trail, then the diameter of the image given by the Ottawa objective and correcting-lens, even allowing the extreme limit assigned by Newall for atmospheric disturbances, is nearly twice as great as it should be.

'It is also a simple matter to test this conclusion experimentally. As the objective gives excellent visual definition, it may be safely assumed that the visual star image is of normal diameter. A measurement of the width of spectra and trails produced by the visual image, and a comparison with the widths given by objective and correcting-lens in photographic light, should at once decide whether the observed effect is due to atmospheric tremor. The correcting-lens was therefore removed, the spectroscope was adjusted for yellow light, and spectra were made similarly to the previous ones, though on Cramer Isochromatic plates, which have a pronounced band of sensitiveness almost

* Monthly Notices, 65, 808, 1905.

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identical in wave-length with the turning-point of the color-curve of the objective. The widths of the spectra produced varied between 0.050 and 0.065 mm, about 2", but as the seeing was very unsteady (about 1½ in scale of 5), these widths are doubtless about 25 per cent. greater than would be the case with good seeing. For the star trails the same make of plate was used, light of shorter wave-length than λ 5000 being absorbed by a yellow screen of plane glass placed in contact with the plate. Owing to the insensitiveness of the plate to light of wave-lengths between λ 5000 and λ 5400, and to longer waves than λ 5800, only the light which is effective in forming the visual image can act in producing the trails. As before, the width of the trails varied with the brightness of the stars, ranging from 0.025 mm in faint trails to 0.055 mm in stronger trails, or from 1" to 2", while the average width over a longer strip of trail was about 20 per cent. greater. Notwithstanding the bad seeing, both trails and spectra were much more sharply defined than those made with the correcting-lens in photographic light and of only half the width.

'These experiments conclusively prove that the abnormal width of spectra and trails in photographic light is not due to aberrations in the spectroscope nor to atmospheric disturbances, and clearly point to aberrations in the condensing system as the cause of the observed effects. A short summary of the experimental data will render this more evident. The theoretical diameter of the central disk, or rather of the first dark ring, for visual light λ 5600, is 0".74, for photographic light, λ 4340, is 0".57. The actual width of visual spectra and trails is from 1" to 2", or one and one-half to three times the theoretical diameter. The actual width of photographic spectra and trails is from 3" to 4".5, or five to eight times the theoretical diameter.

'Some further information regarding the size and character of the photographic image may be gained by considering its effective diameter under another aspect, that of the loss of light at the slit. Referring again to Newall's paper, and taking, as he does for an example, a tremor-disk of 5" diameter with a core of 2", we find that a slit 0.025 mm wide will transmit 31 per cent. of the incident star light; a slit 0.037 mm, 44 per cent.; a slit 0.05 mm, 58 per cent.; and so on. I am indebted to a suggestion by Professor Campbell for a method of testing this theoretical result experimentally. A series of star spectra were made at different slit-widths, and the resulting intensities were compared. As it is practically impossible to make a number of wide spectra of uniform intensity throughout their width, photometric measurements cannot be relied upon and recourse must be had to visual estimates. Such estimates can be made more accurately if the exposures are so regulated as to give spectra of equal intensity, and, moreover, within the limits of exposure time and intensity used here, errors due to the characteristics of the plate employed are to a great extent avoided. The spectrum of α *Lyrae*, the star used, is practically continuous except for the *H* series, and is therefore well suited for the estimation of intensities, while its brightness is such that only short exposures are required. Ten different slit-widths between 0.012 and 0.25 mm were used, and ten spectra, one through each slit-opening, were made side by side on the same plate. The exposures were so regulated as to render the resulting spectra as nearly equally intense as possible, and the final estimate is the mean from a number of plates and from spectra of different widths. To render the comparisons more direct, slit-widths will be represented by divisions, a single division corresponding to 0.025 mm, and the relative exposure times will be reduced to a unit of 100 with a slit-width of one division, 0.025 mm, or 0".91, the normal width with the dispersion employed here.

'The following table shows that the exposure required is inversely proportional to the slit-width until this reaches 0.1 mm, leaving out of account widths less than a single division, where diffractive loss within the collimator plays an important part. It also shows that with normal slit-width less than 17 per cent. of the light incident on the slit is transmitted. In Newall's hypothetical case 31 per cent. would be transmitted. The experimental data given above, using Newall's method of calculation, indicate a tremor-disk 8" or 10" in diameter with a core of about 3".5, and, as the

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previous experiments have shown, this is much larger than can be accounted for by atmospheric disturbances.

TABLE I.
LOSS OF LIGHT AT SLIT.

SLIT-WIDTHS.			COMPARATIVE TIMES FOR EQUAL INTENSITY	
Divs.	Mm	Secs.	Experimental.	Computed: $\tau = 5'' \gamma = 2'$
$\frac{1}{2}$	0.012	0.45	300	...
1	.025	0.91	100	100
1 $\frac{1}{2}$.037	1.35	67	70
2 $\frac{1}{2}$.050	1.82	50	54
3	.075	2.73	33	39
4	.100	3.64	28	34
5	.125	4.55	25	31
6	.150	5.45	21.7	31
8	.200	7.27	18.3	31
10	.250	9.07	16.7	31

'The above experiments point conclusively to aberrations in the system of objective and correcting-lens, when used with photographic light, as the cause of the observed effects, but they give no information concerning the nature of these aberrations beyond indicating in a general way, from the appearance of out-of-focus photographs of spectra and trails, that spherical aberration is present. It was decided therefore, to make quantitative tests to ascertain if possible the nature and magnitude of the aberrations and the best means of removing them.

'The most simple and accurate method of determining the zonal errors and axial astigmatism of a telescope objective is Hartmann's method* of extra-focal measurements. The principle of the method and the measurements and reductions necessary are extremely simple, while it gives accurate values with the expenditure of comparatively little time and without the use of any appliances except such as can be readily made by anyone. For the benefit of those who have not the above paper at hand, and in order to render the present article complete, the essential principles of the method will be briefly described.

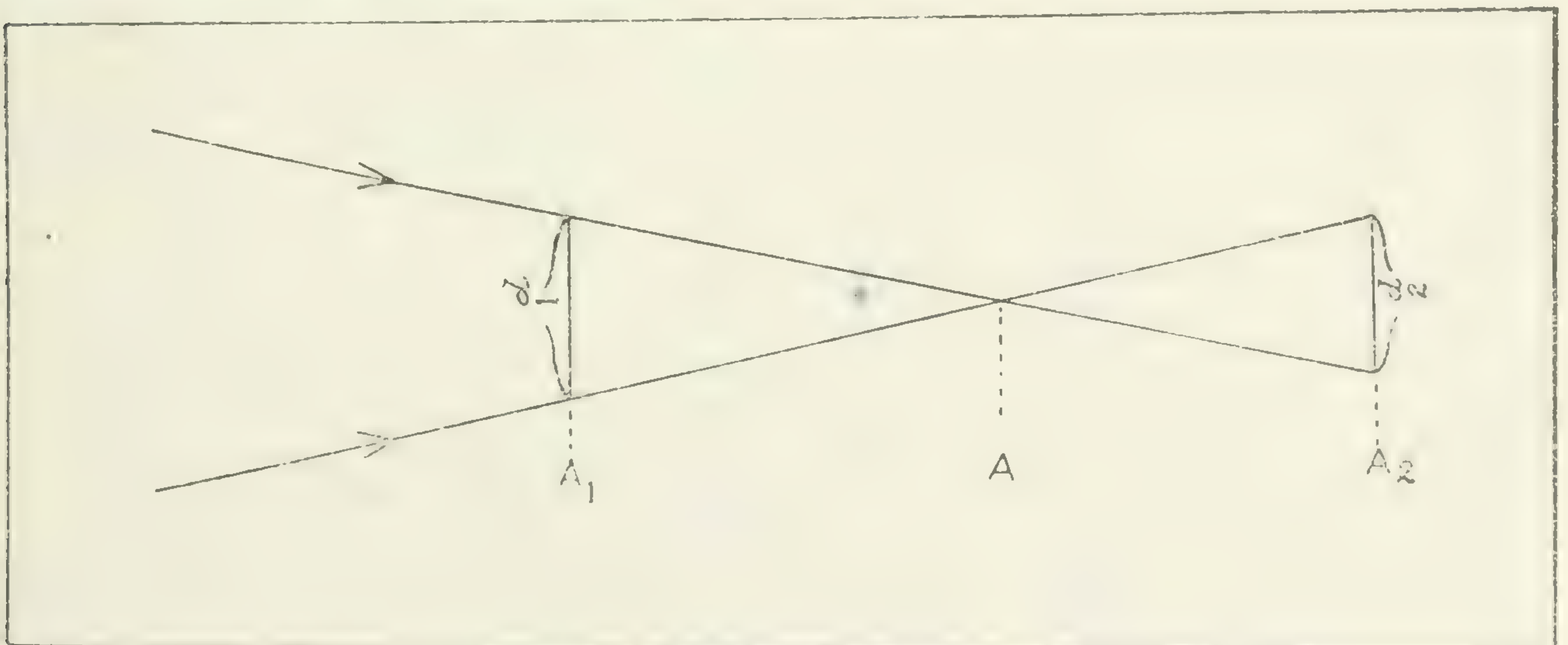


FIG. 1.—Determination of Focus.

'It depends upon the determination of the intersecting point of pencils of light coming from different parts of the objective. Suppose a diaphragm containing two

* Zeitschrift für Instrumentenkunde, 24, 1, 33, 97, 1904.

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small openings, equidistant from the center and along a diameter, be placed over the objective. If the distance between the pencils of light coming from these openings be measured at two points, one within and one without the focus, the point of intersection of the pencils, and consequently the focus for the particular zone in question, can be at once obtained from similar triangles. For let d_1 , Fig. 1, be the distance between the pencils at the scale-reading A_1 within the focus, d_2 the distance at the scale-reading A_2 beyond the focus. Evidently then the scale-reading for the focus

A is $A_1 + \left(\frac{d_1}{d_1 + d_2} \right) (A_2 - A_1)$. The distances d_1 and d_2 may be determined directly by

micrometer measurements on the pencils from a star or distant artificial point-source, or by making exposures on photographic plates in the two positions and measuring the distances between the resulting images by a measuring microscope. The latter method is preferable and was used exclusively, except that the photographic determinations were checked by micrometer measures.

'A zone plate A , Fig. 2, similar to that described by Hartmann, was employed. The apertures, except the four inner ones, were each about 25 mm in diameter, and

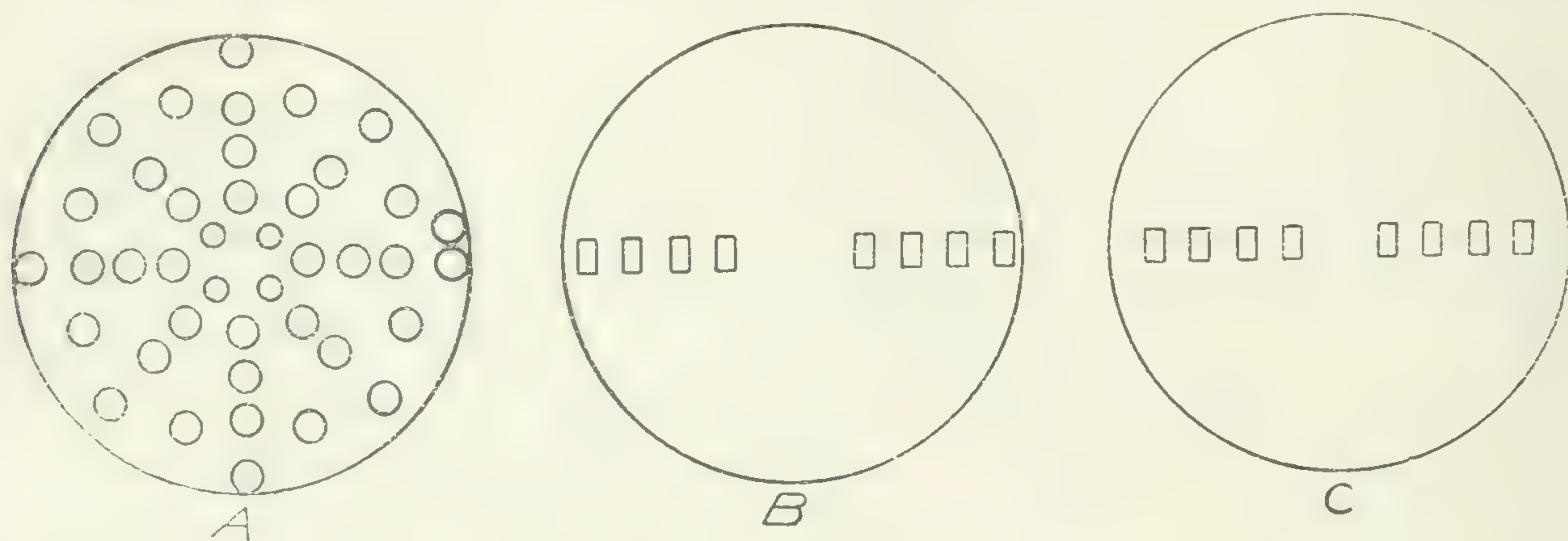


FIG. 2.—Zone Plates.

the radii of the nine zones were respectively 28, 47, 66, 85, 104, 123, 142, 160, and 178 mm. In order to determine the astigmatism along the axis, each pair of openings is duplicated by a second similar pair at right angles, so that the focus of each zone of the objective is determined for two elements perpendicular to each other. In the case of the zone of 142 mm radius the focus can be obtained for four elements 45° apart. Thus an exposure within the focus, and a second one without the focus, give data sufficient to determine the focus of each of nine zones of the objective in two directions perpendicular to each other. These two directions are distinguished from one another in the measurement by making an extra aperture in the zone plate, which, on being reproduced in the negatives, serves to identify the origin and direction of the angle ϕ .

'To determine the zonal errors of objective and correcting-lens, the zone plate was placed in position in front of the objective and a small photographic plate was placed in the guides in the slit-cap of the spectroscop. The spectroscop is supported on two parallel tubes carried by an adapter on the eye-end of the telescope, and can be readily moved up and down through a range of about 20 cm. Experience showed that the images were most sharply defined, and the best measurements could be obtained when the plates were between 6 and 10 cm from the focus. As the photographic focus was to be tested, an ordinary Seed 27 plate was first tried; but it was not found possible to make very accurate settings, as the pencils from the zone plate were spread out into radial spectra owing to the long range of wave-length (λ 5000 to the limit passed by the object-glass, say λ 3600) to which such a plate is sensitive. Several means of overcoming this difficulty were tried. As a yellow screen in front of an ordinary plate did not improve matters, the dispersion of the pencils must evidently be chiefly due to the light around $H\beta$. An ordinary lantern plate, which is sensitive

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from about λ 4600 down, was therefore next tried, and gave good images capable of accurate measurement; while if a yellow screen were used with such a plate the resultant images were again elongated, showing that the prolonged exposure entailed thereby had extended the action on the plate toward the red and reintroduced the first difficulty. A yellow or red star was used in preference to a white or blue, as limiting the action in the violet, shortening the effective range of spectrum, and thus giving images with less spectral dispersion and with no apparent elongation.

‘Four sets of extra-focal plates were made which, on being measured, reduced, and averaged, gave the focal positions of the nine zones as tabulated below (Table II). All four measures are in substantial agreement, which of course is closer for the outer zones where the convergency of the pencils is greater. There the probable error of a single determination of the focus does not exceed 0.1 mm, while near the center it may be as great as 0.5 mm. It will be noticed that the focus for the edge of the objective and correcting-lens is upward of 2 mm longer than the focus near the center, and if astigmatism be taken into account also, the difference is greater than 2.5 mm. The values are plotted graphically in the curve (A) of Fig. 3, the vertical distances being magnified some six or seven times, the appended scale representing millimetres. The horizontal line is drawn in the position of focus 75.34 that gives the smallest circles of confusion, in this case 0.04 mm in diameter. The astigmatism will increase this to some extent, so that probably the diameter will be nearly 2". Unless the slit is set exactly at this mean position, which is not likely, the diameter of the confusion disks will be still further increased, so that we may consider 2" as a moderate estimate. It must be remembered, however, that in speaking of circles of confusion the conceptions of geometrical optics alone are being considered, and no account is taken of diffraction phenomena, which may have some effect on the geometrically calculated dimensions of the star disk resulting from aberrations of the magnitude here present. However, the experiments on the width of spectra and trails showed conclusively that the photographic image was about 2" greater in diameter than the visual image, presumably unaffected by aberrations, and this agrees with the geometrical theory.

To determine where the aberrations arise it is necessary to accurately compare the performance of the objective used visually with the performance of the objective

TABLE II.
ZONAL FOCI OF 15-INCH OBJECTIVE.

Radius of Zone.	ϕ	OBJECTIVE AND CORRECTING-LENS PHOTOGRAPHIC.			OBJECTIVE ALONE VISUAL.		
		Focus.	Mean.	Astigmatism	Focus.	Mean.	Astigmatism.
28	45	73.54		-0.20	106.43		-0.05
	135	73.94	73.74	+ .20	106.54	106.48	+ .06
47	0	74.19		+ .08	108.35		+ .42
	90	74.03	74.11	- .08	107.51	107.93	- .42
66	45	73.54		- .30	106.67		- .13
	135	74.14	73.84	+ .30	106.93	106.80	+ .13
85	0	74.15		+ .11	106.42		+ .26
	90	73.94	74.04	- .10	105.91	106.16	- .25
104	45	74.65		- .23	106.15		- .08
	135	75.11	74.88	+ .23	106.31	106.23	+ .08
123	0	75.68		+ .22	106.20		+ .09
	90	75.25	75.46	- .21	106.02	106.11	- .09
142	22.5	75.93		+ .24	106.08		+ .20
	67.5	75.32		- .37	105.77		- .11
	112.5	75.67		- .02	105.82		- .05
	157.5	75.83	75.69	+ .14	105.83	105.88	- .05
160	45	75.58		- .15	105.91		- .04
	135	75.88	75.73	- .15	105.83	105.87	- .04
178	0	76.11		+ .21	105.93		- .01
	90	75.69	75.90	- .21	105.95	105.94	- .01
Mean focus.....		75.34			106.01		

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and correcting-lens in the photographic part of the spectrum. Zonal tests were therefore made of the objective alone. For this purpose the wave-length of the light used must be limited to λ 5400– λ 5800, the range to which the eye is most sensitive, which is the most luminous in the spectrum, and which coincides with the turning-point of the color-curve of the objective. Fortunately, as the band of color-sensitiveness of Cramer Isochromatic plates almost exactly coincides with the same region, all that is necessary in order to obtain photographic test plates is to absorb the blue and violet light by a suitable screen, and thus confine the action to the visual part of the spectrum. A deep yellow screen with plane parallel surfaces was used in contact with the plate. Although the pencils from the zone plate are displaced slightly on passing through this screen, these displacements are proportional, and the only effect will be to lengthen the focus for all the zones by the same amount, about one-third the thickness of the screen, without in the least altering the relative positions of the pencils. An exposure of about a minute on *Capella*, through the screen, with the plate from 60 to 100 mm from the focus, gives a negative of good intensity in which the images of the pencils are quite round and free from any noticeable spectral elongation, thus allowing accurate measurement.

‘Five sets of extra-focal exposures were made in the visual part of the spectrum, and the mean values resulting from the measurement and reduction of these plates are given in Table II and plotted graphically in curve *F* of Fig. 3. An examination of this curve shows that no point or focus is at a greater distance than 0.2 mm from the position of mean focus, shown by the horizontal line, except a small region near the center of the objective, which has a longer focus. The effect of this region on the performance of the objective must, however, be exceedingly small, owing to its small area, less than one-tenth of the objective, and to the weak convergency of the pencils proceeding from it. In fact if Hartmann’s criterion T^* as to the quality of an objective be computed from the above mean values, it is found to be 0.141. According to this classification an objective is moderately (“mässig”) good when T is greater than 1.5, good when T is between 0.5 and 1.5, and exceedingly (“hervorragend”) good when T is less than 0.5. In the ideal, absolutely zoneless objective T is 0.

‘Evidently the objective when used visually is of the very first quality, and the aberrations appear only when it is used in conjunction with an auxiliary corrector for spectrographic work. Whether the aberrations there present are due to the correcting-lens, or to the objective when used in the photographic part of the spectrum, remains to be determined. For this purpose a further application of Hartmann’s method was necessary to find the color-curves of the objective alone, and of the system of objective and correcting-lens for a number of zones. It was hoped that such observations would throw light on the cause of the aberrations and suggest a possible remedy. They would also serve as a check upon the zone-plate determinations, as, in this case, no spectral dispersion of the pencils could affect the accuracy of setting. To find such color-curves, the pencils of light coming from a zone plate fall on the spectroscope slit, and the distance between the resulting spectra taken with the slit within and beyond the focus gives a measure, calculated in the same way as before, of the focal position of any desired wave-length for any particular zone.

‘It was decided to determine the color-curves of eight zones of 38, 57, 76, 95, 114, 133, 152, 171 mm radius; and, to prevent the spectra from merging into one another, two zone plates were required, one (*B*), Fig. 2, of the four zones of 57, 95, 133, and 171 mm radius, and the other (*C*), Fig. 2, of the remaining four. The central openings were each 20 mm square, and the outer 20 by 25 mm. The zone plates were so placed on the objective that the row of openings was parallel to an hour circle, and the spectroscope was turned in position angle until the slit was parallel to the openings, in order that irregularities in driving would not widen the spectra. To diminish the exposures as much as possible, bright stars, *Vega* and *Sirius*, were used and the

* Zeitschrift für Instrumentenkunde, 24, 46, 1904.

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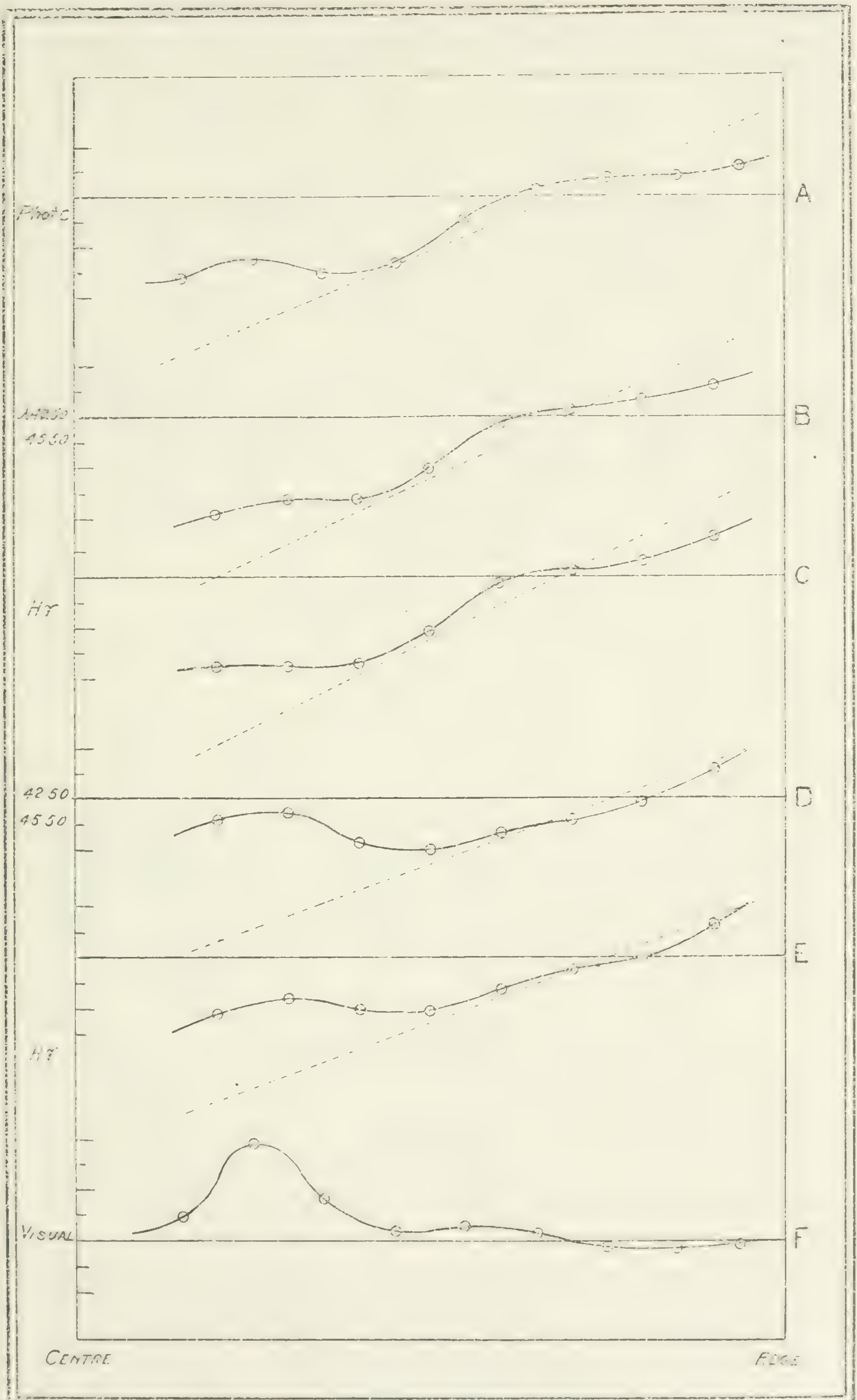


FIG. 3.—Zonal Differences of Focus.

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slit was widely opened, as no inaccuracy would be thereby introduced in the distance between the spectra. The exposures were made on a night when the temperature was nearly stationary, and were arranged in the following order:

Plate 1; Zone Plate (B) Fig. 2; slit about 50 mm within the focus.					
2;	(C)	"	50	"	"
3;	(C)	"	40	beyond	"
4;	(B)	"	40	"	"

‘This procedure was followed to avoid as far as possible any relative displacement of the focal determinations of the two sets, due to slight changes of temperature of the objective. That no measurable displacement has occurred is shown by the continuity of the zonal curves of Fig. 3 drawn from the combination of the two separate determinations, and by their agreement with those made by the regular zone-plate method.

‘Each of these plates contains eight spectra side by side, one from each light pencil transmitted by the zone plate, and the position of the focus for each zone and for any desired wave-length in the range on the plate can be determined in exactly the same way as before. The hydrogen lines, in the first type stars used, serve as datum marks for the identification of wave-lengths, and measurements were made at eleven positions between λ 3970 and λ 5030. The corresponding focal points, as calculated from these measurements, are given in Table III for eight zones of the

TABLE III.
COLOUR-CURVES OF OBJECTIVE ALONE.

Radius of Zone.	WAVE-LENGTHS.										
	5030	H β 4861	4680	4550	4440	H γ 4340	4250	4175	H δ 4102	4035	H ϵ 3970
38	85.57	86.87	89.64	92.02	94.28	96.30	99.78	102.48	105.82	109.59	110.96
57	85.30	86.30	88.95	92.00	94.28	96.60	100.25	102.74	105.95	108.75	111.61
76	83.84	85.78	88.76	91.09	93.67	96.39	99.50	102.34	105.31	108.69	112.31
95	84.67	85.42	88.41	90.82	93.56	96.34	99.37	102.61	105.68	109.11	112.12
114	84.38	85.78	88.68	91.16	93.87	96.77	99.58	103.06	106.19	109.63	112.65
133	84.71	85.93	88.68	91.08	93.91	97.16	100.21	103.16	106.72	110.11	113.08
152	85.06	86.29	89.18	91.49	94.41	97.42	100.53	103.71	106.79	110.10	113.38
171	85.41	86.87	89.65	92.03	95.02	98.04	101.29	104.62	107.81	111.10	114.53

TABLE IV.
COLOUR-CURVES OF OBJECTIVE AND CORRECTING-LENS.

Radius of Zone.	WAVE-LENGTHS.										
	5030	H β 4861	4680	4550	4440	H γ 4340	4250	4175	H δ 4102	4035	H ϵ 3970
38	55.12	54.75	53.11	51.18	50.65	50.92	50.92	51.17	51.36	51.68	51.91
57	53.38	53.89	52.55	51.98	51.21	51.04	50.90	50.91	50.95	51.30	51.82
76	54.51	53.67	52.51	51.60	51.19	51.14	51.16	51.11	51.20	51.46	51.26
95	55.57	54.37	53.16	52.46	51.95	51.70	51.60	51.74	51.96	52.30	52.66
114	55.45	54.82	53.62	53.12	52.79	52.59	52.65	52.79	53.03	53.24	53.31
133	55.91	55.10	53.88	53.33	52.06	52.89	52.93	53.05	53.31	53.60	53.73
152	55.81	55.13	54.05	53.54	53.26	53.07	53.25	53.38	53.49	53.62	53.64
171	56.05	55.39	54.38	53.90	53.60	53.53	53.56	53.97	54.15	54.27	54.34

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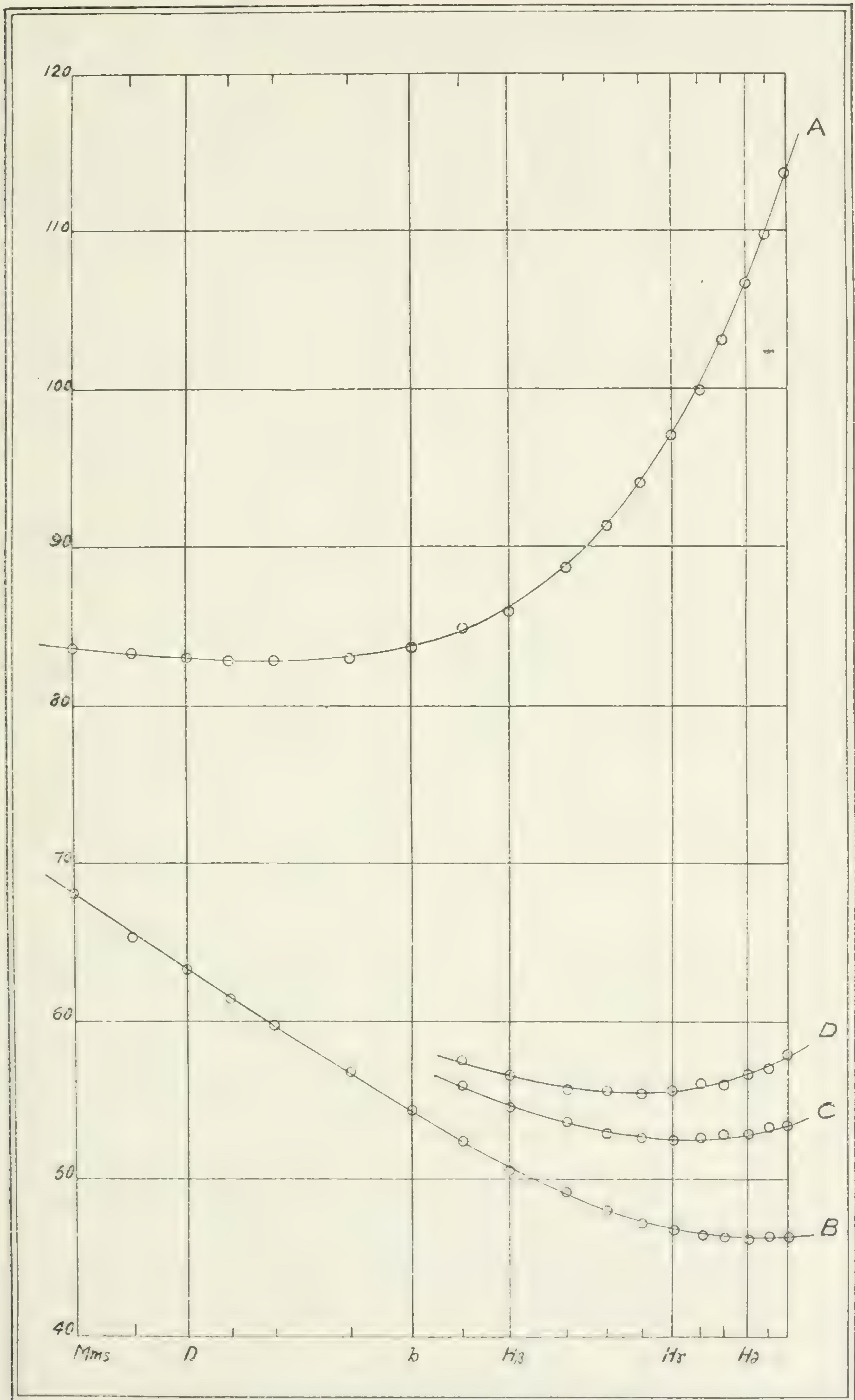


FIG. 4. Colour Curves for a Median Zone.

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objective alone, and in Table IV for the same eight zones of the objective with correcting-lens, the latter being about 40 mm nearer the focus than its computed position.

'The reason for using the correcting-lens below its computed position at once appears on inspection of Fig. 4, which represents, in their correct relative positions, the color-curves of a median zone of 108 mm radius, determined in exactly the same way as above. Curve *A* (Fig. 4) is the color-curve of the visual objective between the limits λ 6250 and λ 3970, which shows that the minimum focus is at about λ 5600, exactly in its computed position. Curve *B* is the color-curve of the system of objective and correcting-lens between λ 6250 and λ 3970, which shows that the minimum focus is at about $H\delta$, instead of $H\gamma$, its computed position. When the correcting-lens is moved down, away from the objective, some 40 mm we get curve *C*, and at 70 mm, curve *D*. In curve *C* the minimum focus is nearly at $H\gamma$, and in *D* at λ 4460. Evidently the lowering of the correcting-lens some 40 mm effects considerable improvement in the color-correction without, as the earlier experiments showed, appreciably enlarging the image, and the lens has been used in this position almost from the first.

'Although all the data in regard to the complete color-curves are given in Tables III and IV, still the actual curves drawn from these figures show all the conditions at a glance, and are hence worth giving. To prevent too great a confusion of lines, the curves for four zones only (zone plate (*B*), Fig. 1), of 57, 95, 133, 171 mm radius, are shown here in Fig. 5, the upper curves being of objective alone, the lower of objective and corrector. These curves show at a glance that, in the photographic part of the spectrum, the focus for the edge of the objective is longer, than the focus for the center, that it has negative spherical aberration. This chromatic difference of spherical aberration is inherent in two-part objectives of the ordinary glasses, and the only remedy is to compensate for it by introducing the correct amount of positive aberration by the correcting-lens. However, the lower curves show that, instead of compensating for this chromatic difference, the correcting-lens has, on the contrary, increased it somewhat, and the focus for marginal rays is upward of 2 mm longer than the focus for central rays. This agrees almost exactly with the previous determination of the zonal foci of objective and corrector, and is good evidence of the substantial accuracy of the determinations. Before leaving these curves it may be pointed out that the crossing of the curve from the 57 mm zone over the others in passing from short to long waves is due to the longer focus of the central zones in the visual part and is further evidence in favour of the accuracy of the determinations.

'To obtain a still more striking comparison of the cause and magnitude of the aberrations present in the system, the color-curves can be presented in another form, that of zonal foci curves like *A* and *F*, Fig. 3, previously determined. We have the color-curves, or the positions of focus, of the whole photographic region for eight zones of the objective in Tables III and IV, and these can be readily plotted in the same way and on the same scale as *A* and *F*, Fig. 3. If such curves were plotted for every wave-length in these tables, they would show a striking agreement in form, but I have satisfied myself with representing the positions of the focus of eight zones for $H\gamma$, the wave-length for which the system was computed, and for the mean of λ 4250, 4340, 4440, and 4550, the range of spectrum used here in velocity determinations. *E*, Fig. 3, is the curve for $H\gamma$ of the objective alone; *C* is the curve for $H\gamma$ of objective and corrector. *D* is the curve for λ 4250 to λ 4550 of the objective alone; *B* is the curve for λ 4250 to λ 4550 of the objective and corrector.

'A comparison of curves *D* and *E* with *F* shows in a striking manner the chromatic differences of spherical aberration in the objective when used with photographic light. If we leave out of account or allow for the deviations in the central zones, we see that the focus of the outer is about 1.8 mm longer than the focus for the central zones, a figure that agrees almost exactly with the computed difference as furnished me by Professor Hastings. A comparison of curves *A*, *B*, and *C* with *D* and *E* shows that this difference, instead of being removed or diminished by the introduction of

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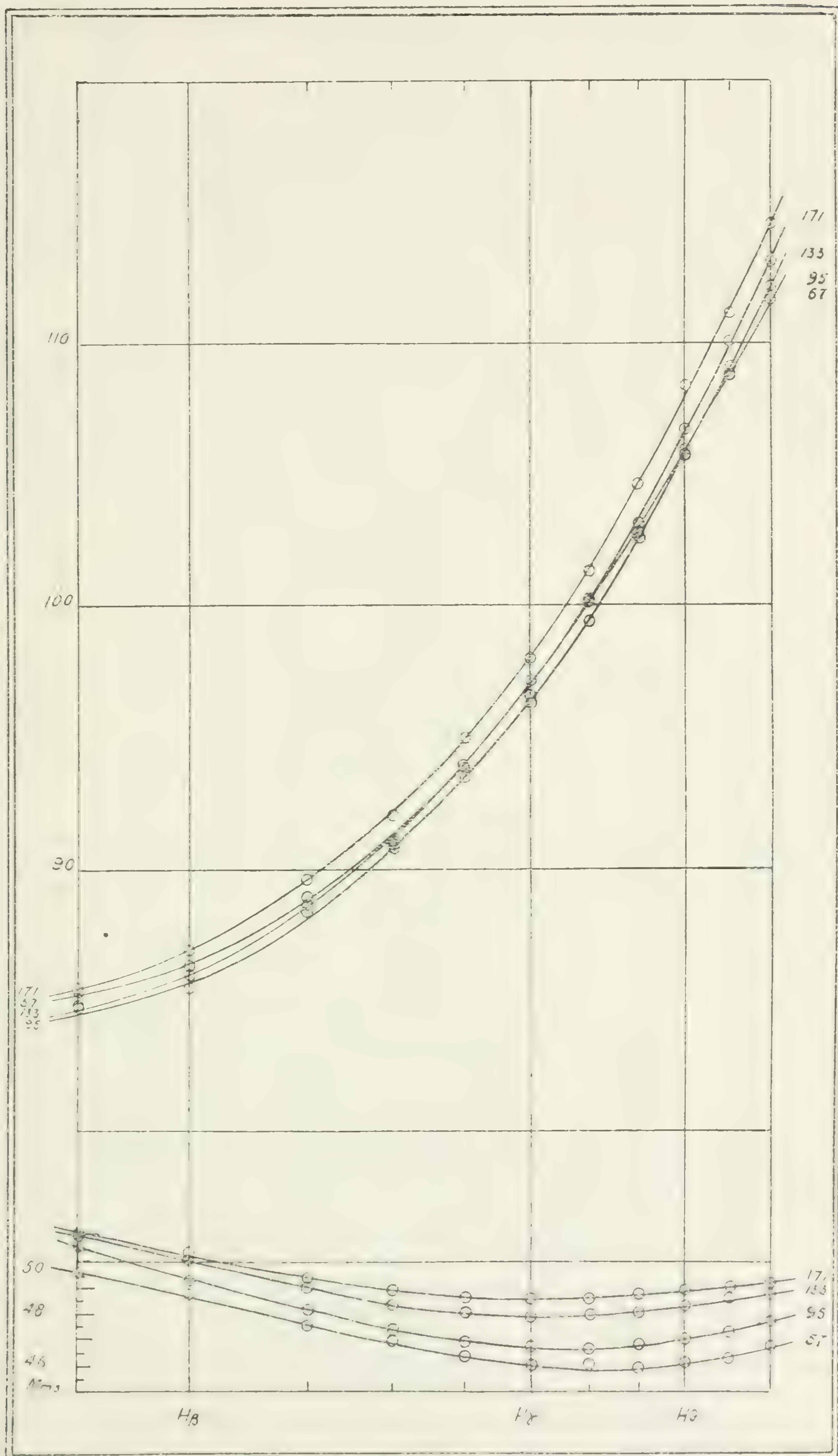


FIG. 5.—Colour-Curves of four Zones of Objective and of Objective with Corrector.

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the correcting-lens has on the contrary been increased by about 0.6 mm, so that the difference in focus between outer and central zones is now about 2.5 mm, which, as before stated, will give a confusion disk nearly 2" in diameter. I wish to point out, before leaving these curves, how the form of the curve is maintained throughout from *F* up to *A* except that the axis of the curve is inclined downward by the chromatic differences in the photographic region, and further tilted by the introduction of the correcting-lens. To show this I have dotted in the approximate positions of such axes in the curves *E* to *A* to correspond with the horizontal axis in *F*. It will be noticed that the irregularities in the visual curve are continued throughout, but in an intensified form, as is to be expected when it is considered that the objective was computed and figured for visual work, and its use in the photographic region with an auxiliary corrector was only a secondary consideration.

‘I see no reason to doubt, however, if sufficient positive aberration were left in the correcting-lens to compensate for the negative aberration introduced by the chromatic differences, that the performance of the system could be much improved, although it is not likely, from the magnifying of the unavoidable zonal aberrations, that it would equal its visual quality. If the curve *A*, Fig. 3, representing the present condition of the system, could be tilted through the angle between the horizontal and dotted lines, by such a change in the correcting-lens, the resulting confusion disk would certainly have a diameter less than half its present magnitude, while the percentage of the incident star light transmitted by the slit would be considerably increased, probably doubled, with a proportionate diminution of the required exposure times for stellar spectra.

‘Such an improvement would be well worth considerable effort, and I have been in communication with the Brashear Company and with Professor Hastings to that end. With their well-known willingness, I may even say anxiety, to produce the highest quality of optical work and to make any improvements that may be suggested to them, the Brashear Company are undertaking to make a new correcting-lens to computations by Professor Hastings, to whom I am very much indebted for criticisms and suggestions on the present paper. I may say that Professor Hastings finds a very marked agreement between his computed data of the objective, color-curves, and chromatic differences, and my observations. He explains the failure of the correcting-lens to compensate for the chromatic differences of focus, which it was computed to do, by the fact that this lens has to correct the errors of an objective of nearly fifty times the area, that the small departures of the wave-surfaces from a true sphere have grown enormously when these surfaces have contracted to one-fiftieth their original area, and that a very perfect correction by spherical surfaces can hardly be hoped for. He thinks, however, that considerable improvement can be effected, and I have no doubt myself that he and the Brashear Company can do much better than he says when they have quantitative values of the existing aberrations.

‘The reason for publishing this paper in its present incomplete form, before the new correcting-lens is ready, is to bring before stellar spectroscopists the important matter of the size and character of the star image given by their telescopes. I have gone fully into the details of the investigation and explained the difficulties that arose with the means of overcoming them, in order to smooth the way for similar investigations into the character of the star image given by other systems of objective and correcting-lens. It seems to me extremely probable that, in the major part if not all of the telescopes employed in spectrographic work, aberrations of the same or a similar nature are present. If a correcting-lens computed to compensate for the chromatic difference fails in one case, it is possible, even probable, that it may fail in others. Another basis for this belief is a comparison of the relative exposure times required for different installations taking into account size of object-glass, slit-width, and dispersion of the spectrograph. I am well aware that such a comparison must necessarily be incomplete, and the results reached subject to an uncertainty, say, of

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25 per cent., owing to the difficulty of comparing different installations under different conditions of seeing, etc. We have already seen how important a part is played by atmospheric disturbances in enlarging the star image so that the linear diameter of the image increases nearly in proportion with the focal length, and therefore approximately, as the ratio of aperture to focal length does not vary much in large instruments, with the diameter of the object-glass. Consequently, the effective value of increase of aperture is not proportional to the increase of area, but more nearly to the increase of diameter, which was accordingly used in the comparison. So far as regards the relative dispersion of different instruments, the exposure time was taken as directly proportional to the linear dispersion, presuming the same height of spectrum in each case. No account was taken of the difference in the loss due to absorption and reflection in the prism-train, although this may be quite important in some cases. The exposure time required was taken as inversely proportional to the slit-width, and this, as one of the experiments detailed above shows, is probably nearly in accordance with the facts. In the following Table V, data of the various equipments which are and have been used in radial velocity work, so far as they were available to the writer, appear, but these data are incomplete and may in some cases be in error, although probably not to a marked degree.

TABLE V.
COMPARISON OF EFFICIENCIES OF INSTALLATIONS.

Equipment.	Diameter of Objective, inches.	Ratio of Diameters.	Ratio of Areas.	Linear Dispersion, Tenths, per mm.	Slit-Width, mm.	Theoretical Exposure.	Actual Exposure Required.		
							β Ophiuchi.	γ Aquilae.	α Bootis.
Ottawa.	15	1	1	18.6	0.025	1	50m	60m	6m
Yerkes	40	2.67	7.1	10.8	.038	0.42	75	115	15
Lick.	36	2.4	5.76	12.5	.025	0.62	25 ?	25 ?	4 ?
Lowell	24	1.6	2.56	11.4	.025	1.02	120	120	20 ?
Newall ...	25	1.67	2.78	14.6	.025	0.76	70	75	15
Bonn	12	0.8	0.64	15.2	.020	1.91	75	75	15
Pulkowa . . .	30	2.0	4.0	13.0	.020	0.89	65 ?	65	15
Lord.....	12½	0.83	0.69	18.6	.025	1.20	60 ?	60 ?	4

‘The above comparison shows that the Lick, Bonn, and Lord equipments in *practice* approach more nearly the theoretical efficiency than the Ottawa, but the Yerkes, Lowell, Newall, and Pulkowa depart farther from it.

‘There seems therefore reasonable ground for believing that considerable improvement in the efficiency, and considerable increase in the range of the majority of spectrographic equipments can be attained by looking into the character of the star image given by the condensing system. Although the exact effect of atmospheric disturbances on the effective diameter of the star image is difficult of determination, I feel satisfied, if I can obtain a correcting-lens that will give a star image reasonably free from aberration, that the exposure times required here can be very materially reduced, I hope by 50 per cent., and I see no reason why a similar or even greater improvement could not be effected in some of the other equipments.

‘I acknowledge with pleasure my indebtedness to Dr. W. F. King, the Director of the Observatory, for help and encouragement in the prosecution of the work, and to Mr. W. E. Harper for making duplicate measures for comparison purposes on some of the test plates.

‘DOMINION OBSERVATORY, OTTAWA.

‘January, 1907.’

APPENDIX B.

(Reprinted from the Journal of the Royal Astronomical Society of Canada, Vol. 1, No. 1.)

'THE SPECTRUM OF MIRA CETI.

'BY J. S. PLASKETT.

'The spectrum of *o Ceti* has been photographed at the Dominion Observatory 18 times on 11 nights during the months of December, 1906, and January, 1907. The number of observing nights during these two months has been very limited, the weather having been unusually cloudy, and no more spectra of this interesting variable could be obtained.

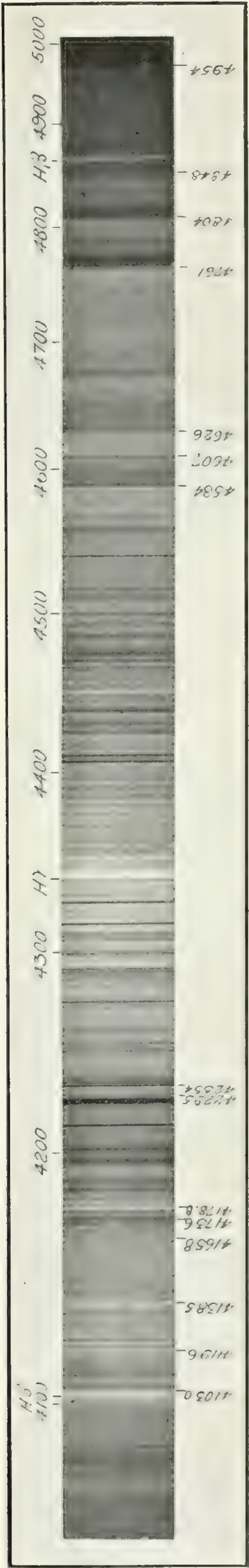
'The spectrograph at present in use is an adapted Brashear Universal Spectroscope, having collimator and camera lenses of $1\frac{1}{4}$ inches aperture and 15 inches focus, and a train of three dense flint prisms, index for $H\gamma$ about 1.64, giving a linear dispersion at $H\gamma$ of 18.6 tenth-meters per millimeter, with a resolving power of 40,000. A spectrum about 55 mms. long is obtained, of which, however, owing to curvature of field of the triplet camera lens, only about 15 mms. in the centre is in the best focus. The balance of the spectrum becomes more and more diffuse towards the ends of its range, which extends between $\lambda 3950$ and $\lambda 5100$. The extreme limits measured for radial velocities lie between $\lambda 4200$ and $\lambda 4584$, but it is possible to obtain fairly accurate values of the wave lengths, within one tenth of a tenth-metre, between $H\beta$ and $H\delta$.

'The spectrum of *Mira*, observed at this maximum, differs in some essential particulars from previously recorded observations.

'The star has been much brighter than for several previous maxima and it is natural enough, if we consider its variability to be due to changes in its internal condition, to expect a change in its spectrum. These changes appear both in the absorption and the emission spectrum, and will be treated in greater detail later on.

'Probably the most striking change is in the character of $H\beta$, which had been previously recorded as either dark, or as only faintly bright. Sidgreaves (*M.N.* LVIII., p. 344) did not consider he had certainly seen $H\beta$ bright. Miss. Maury (*H.C.O. Annals* XXVIII., p. 45) saw it bright on some Harvard plates. Campbell (*Astrophysical Journal* IX., p. 31) could not see it visually, while Stebbins, (*Ibid* XVIII., p. 341) in his exhaustive paper, was successful in recording both it and $H\epsilon$ on some plates, but with much less relative intensity than at this maximum. In every spectrum made here, even those with only two minutes exposure, $H\beta$ is distinctly and certainly bright, and there is not the slightest doubt of its emissive character. No trace whatever has been seen of $H\epsilon$ on any of our plates and it is apparently not present. It also had never been seen bright until Stebbins recorded it.

'A number of comparative exposures from one minute to twenty minutes were made to determine, among other things, the relative intensity of the emission and absorption spectra. As an estimate from these plates,—no attempt was made to accurately determine intensities,—I would say that the bright $H\beta$ had an intensity about 15 times that of the continuous spectrum in that region, $H\gamma$ about 25 times and $H\delta$ at least 50 times. These estimates apply to the plates of January 23 and 26, when the star was considerably past maximum. In December no comparative tests were made, but the ratio would not be very much different, so far as can be judged, from the over-exposed emission lines.



Spectrum of O Ceti (Mira).

Photographed by J. S. Plaskett at the Dominion Observatory, Ottawa, 1906, Dec. 18th, 14 h. 32 m. G.M.T.
Length, 4 ; width, 70 times the original.

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'Before discussing the character of the spectrum it will be preferable to give the record of observations and the measures of the wave-lengths of the lines and bands obtained from the most suitably exposed spectrum, No. 486. Although 486, 515 and 521 are the best of the plates, the first eight are all measurable, and of these 486 and 515 have been reduced for the wave lengths of the absorption lines and bands, and for the determination of the radial velocity. All the plates, with the exception of 575 to 578, in which the camera was accidentally not in good focus, have been measured for the velocities due to the $H\gamma$ emission, and they show, as will be seen, fair agreement with one another and with Professor Campbell's previously determined values. The velocities obtained from the absorption part of the spectrum in the two plates measured agreed so closely with one another, and at the same time were nearly the same as Professor Campbell's and Mr. Stebbins' values, that it was not thought necessary to measure more plates.

RECORD OF OBSERVATIONS.

Plate No.	Date.	G.M.T.	Exp.	Prism temp	Seeing.	Observer.	Remarks.
452	Dec. 11	14 29	18 m.	- 3.5	Good	H	Absorption spectrum underexposed.
486	" 18	14 32	19 m.	- 1.6	"	H	Good spectrum.
493	" 19	14 50	20 m.	- 7.4	Fair	P	Underexposed.
515	" 27	15 55	30 m.	+ 2.1	Poor	P	Fair spectrum.
521	Jan. 9	13 45	30 m.	12.8	Fair	P	Good spectrum.
534	" 15	14 35	40 m.	- 12.8	Poor	H	Underexposed.
555	" 18	14 30	60 m.	- 8.0	Poor	P	"
563	" 21	13 55	20 m.	12.3	Good	P	"
569	" 22	15 16	05 m.	8.9	Fair	H	For emission lines only.
575	" 23	13 43	20 m.	18.8	"	P	For emission lines.
576	" 23	14 07	10 m.	18.8	"	P	" "
577	" 23	14 17	05 m.	- 18.8	"	P	" "
578	" 23	14 23	02 m.	18.8	"	P	" "
579	" 26	12 15	20 m.	10.0	Good	P	" "
580	" 26	12 32	10 m.	- 10.0	"	P	" "
581	" 26	12 41	05 m.	- 9.9	"	P	" "
582	" 26	12 46	01 m.	- 9.8	"	P	" "
583	" 26	12 50	02 m.	- 9.8	"	P	" "

'In the above measures, the wave lengths of the star lines are determined in the usual way, from the linear positions of the star and comparison lines on the plate, by Hartmann's interpolation formula. The displacement of the lines in tenth-metres due to the motion of the star is known, when the velocity is known from the formula

$$\delta \lambda = \frac{v \lambda}{299,860}$$

'The velocity is obtained from the mean of the velocities due to 25 lines near the middle of the plate, which had been identified as far as possible with known terrestrial or solar wave lengths. This velocity, on being transferred back into displacement by the above formula, gives the correction to be applied to the measured wave-lengths of the absorption lines, emission lines, and bands at the ends of the plate, to reduce them to normal wave lengths.

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o CETI, No. 486.

1906. Dec. 18.
G. M. T., 14^h 32^m

Observed by W. E. HARPER.
Measured by J. S. PLASKETT.

Measured Wave Length.	Normal Wave Length.	Displacement.	Velocity.	Remarks.
4955.520	4.04	1.350	[+82.75]	Red Edge of Bright Band
4862.877	1.527			H β Emission
4848.948	7.55			R. Edge of Band
4805.639	5.24			Line near edge of band
4805.866	4.46			R. Edge of Band
4763.309	1.91			Mn line near edge of band
4762.766	1.36			R. Edge of Band
4657.795	6.39			Ti Cr
4627.889	6.49			Cr Mn Line at R. Edge of Band
4608.688	7.28	1.393	+91.10	Sr 7.51 Line at V. Edge of Band
4595.652	4.27			V 4.30
4585.917	4.53			Fe Line at R. Edge of Band
4581.841	0.46			V Cr 0.59, 0.23
4578.749	7.356			V
4537.372	5.965			Ti Cr
4528.920	7.490			Ti
4524.335	2.974			Ti
4519.698	8.198			Ti
4472.804				Ti
4463.437				Fe Mn V
4454.705	3.505			Ti Mn
4436.630	5.439			Ca Ca
4428.730	7.420			Ti Fe
4406.211	4.951			Fe
4402.065	0.738			V
4396.746	5.286			Ti V
4386.213	4.873			V
4385.076	3.720			Fe
4380.616	9.396			V
4369.560	8.26			Ti Fe
4354.312	3.038			Fe V
4345.977	4.597			Ti Cr
4341.734	0.634			H γ Emission Line
4334.204	2.988			V
4331.409	0.189			V
4316.224	5.018			Ti Fe
4307.438	6.078			Ti
4297.334	5.914			Ti Cr
4292.800	1.50			Ti Fe
4276.252	4.922			Cr Ti
4259.747	8.477			Fe
4248.296	6.996			Y
4234.618	3.36			Fe Emission
4231.277	0.00			Fe 9.93
4230.768	9.51			Fe Emission?
4228.131	6.904			Ca
4208.190	6.862			Fe
4180.090	8.84			V Ce Emission?
4180.937	9.68			V 9.54
4179.069	7.82			Fe 7.70
4175.473	4.22			Fe 4.09
4174.826	3.58			Ti Fe Emission?
4167.096	5.84			Ce Cr Fe Emission
4139.778	8.53			V Ce Mo Emission
4135.897	4.49			Fe V 4.49, 4.59
4126.867	9.56			Fe V Ce Mo Emission?
4104.185	2.95			Mn Emission?
4103.030	2.000	1.030	[+75.29]	H δ Emission

Mean of Absorption Lines +90.43

$\epsilon = \pm 5.2$
 $\epsilon = \pm 1.0$

V_d - 24.20
 V_d - 0.09
Curvature correction - 0.50

Radial velocity = 65.6

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o CETI, No. 515.

1906, Dec. 27.
G.M.T., 15^h 55^m.Observed by J. S. PLASKETT.
Measured by W. E. HARPER.

Measured Wave Length.	Normal Wave Length.	Displacement.	Velocity.	Remarks.
4572.705	1.275	1.430	+93.66	Mg
4550.222	8.938	1.284	85.27	Ti
4546.234	1.845	1.389	91.70	Cr Ti
4541.976	0.776	1.200	79.32	Cr
4537.385	5.965	1.420	94.00	Ti Cr
4528.780	7.490	1.290	85.40	Ti
4524.360	2.974	1.386	91.96	Ti
4497.975	6.57			Ti Mn Cr
4490.902	9.60			Cr
4463.517	2.21			Fe Mn
4461.557	0.20			V Mn
4459.016	7.656	1.360	91.39	Ti V Mn
4454.885	3.505	1.380	92.87	Ti Mn
4439.396	8.996	1.390	93.82	V
4428.780	7.420	1.360	92.07	Ti Fe
4427.351	6.201	1.350	91.39	Ti
4406.331	4.951	1.380	93.82	Fe
4402.076	0.738	1.338	91.18	V
4396.696	5.286	1.410	95.88	Ti V
4389.806	9.396	1.410	96.58	V
4369.560	8.071	1.489	102.06	Fe
4354.348	3.038	1.310	90.00	Fe V
4353.316	2.006	1.310	90.00	Cr Mg
4341.784	0.634	1.150	[79.35]	H γ Emission
4334.308	2.988	1.320	91.10	V
4320.247	8.817	1.430	99.24	Ca Mn
4307.558	6.078	1.480	103.00	Ti
4302.203	0.945	1.258	87.68	Ti
4297.244	5.914	1.330	92.83	Cr Ti
4290.617	9.237	1.389	+96.32	Ti

Mean of Absorption Lines + 92.48

$$\epsilon = \pm 4.7$$

$$\epsilon_0 = \pm 0.9$$

$$V_a = -26.45$$

$$V_d = -0.22$$

$$\text{Curvature correction} = -0.50$$

$$\text{Radial velocity} = +65.3$$

'In the tables above, of plates 486 and 515, the first column contains the wave lengths computed from the linear measures by Hartmann's formula. The second column contains the normal wave-lengths determined, in the cases where there are no entries in the two succeeding columns, by the process outlined above, and in the other cases where the lines have been identified, by taking the corresponding wave-lengths from Rowland's table.

'These identifications have been made as consistently as possible, using only those elements which it was considered probable from the similarity of o *Ceti* to third type stars, would be present in the star. The third column contains the displacement of the line in tenth-metres from its normal position due to motion, and is obtained by subtraction of the second column from the first. The fourth column contains the velocity corresponding to this displacement, obtained by multiplying by 299,860/ λ .

'Let us consider in the first place the radial velocity of o *Ceti* as determined from the displacements of the absorption and emission lines. The mean velocity from the absorption lines in No. 486 is + 90.43 kms. per second, which, on applying the correction for the orbital and diurnal movement of the earth, and for the curvature of the spectral lines, reduces to 65.61 kms. per sec., recession, compared with the sun.

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For plate 515 the velocity is + 65.3 kms., in good agreement with the first. Professor Campbell,* from his determinations in 1897 and 1898, obtained a mean velocity of + 62.3 kms., and Stebbins in 1902, of 66 kms. This shows that the motion of the star is constant, as the variation between the Lick and Ottawa determinations can readily be accounted for by the uncertainty in the identification of the lines, and in the intensity to be assigned to them in the blends, in a star so different from the sun in its absorption. Campbell's value of the velocity is probably more nearly the true one on account of the greater dispersion and resolving power of the Mills spectrograph, which admits of the resolution of lines much closer together than is possible with the Ottawa instrument.

'The errors in identification and blending are plainly shown by the very high mean error $\epsilon = \pm 5.2$ of the determination from a single line. In the case of stars like β *Geminorum* and α *Boötis*, where their similarity to the Sun allows of satisfactory identifications and blends, the mean error is only one-third of the above, while the mean error of setting on the lines of α *Ceti* which are of good quality for measurement, is not materially greater than with solar stars. It is evident, therefore, from the satisfactory agreement of the velocities obtained at two epochs nine years apart, that the star's velocity, so far as it is determined from displacements of the absorption lines, is constant, and, as Professor Campbell has already said, its variability is probably not dependent upon or connected with any orbital motion.

A comparison of the displacements of the bright hydrogen lines on the two plates already measured, and their corresponding velocities, with the mean velocity from the absorption lines, shows that the former is about 15 kms. smaller, that, if the displacement could be explained by velocity changes only, the emissive layer is lagging behind the absorptive layer at the rate of 15 kms. per sec. It is of course more likely that the difference is due to some unknown condition in the atmosphere of the star which may displace the spectral lines. To obtain all the information possible in regard to the character and displacement of the hydrogen emission lines, a number of plates were made with varying exposure, from 1 minute to 20 minutes, and these were carefully compared with one another and with the previously exposed more intense plates to determine the form of the emission lines. No trace could be found of Campbell's triple formation in any of the plates, although the earlier ones, when the star was near maximum, were not suitably exposed to exhibit such an effect. The lines were, however, in the majority of the plates, unsymmetrically broadened with respect to the actual centre of intensity determined from the tips of the emission lines. These tips were nearer to the violet side of the bands, showing that the radiation was not symmetrical, and this asymmetry became more evident, the more intense became the line. This is indicated in two ways in the table of the velocities due to the bright hydrogen lines, first by the actual measure of the positions of the red edge of the tips, and of the violet edge of the bright $H\gamma$ lines, and second, by the smaller velocities given by the short exposure, less intense plates, as compared with the plates exposed for the absorption spectrum, in which the emission lines were much over-exposed.

* *Astrophysical Journal*, IX., p. 31.

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RADIAL VELOCITIES, *o CETI*.

From *H* γ Emission line.—Reduced to the Sun.

Number of Plate.	Exposure Time.	Observer.	Red Edge of <i>H</i> γ to Tips Revs.	Tips of <i>H</i> γ to Violet Edge Revs.	Rad. Vel.	Remarks.
452	18 min.	H	.086	.073	+48.5	
486	19 "	H	.099	.083	51.1	
493	20 "	P	.088	.076	46.7	
515	30 "	P	.083	.056	52.2	
521	30 "	P	.091	.078	48.8	
534	40 "	H	.075	.070	51.1	
555	60 "	P	.102	.046	37.9	Abnormal? (Poor night and change of temp.)
563	20 "	P	.067	.068	43.0	
569	05 "	H	44.0	
579	20 "	P	.057	.055	45.4	
580	10 "	P	.057	.038	46.8	
581	05 "	P	.046	.047	44.3	
582	02 "	P	45.7	
583	01 "	P	40.1	

Mean of 14 plates = + 46.1
Mean of plates exposed for absorption spectrum = + 48.0
" " emission spectrum only = + 44.2

‘These measures show a fair agreement among themselves, but this accordance is considerably increased when they are divided into two sets—of the strongly and moderately exposed plates,—and when plate No. 555 is omitted. It is abnormal in the marked asymmetry of the bright line, as shown by the measure in columns 4 and 5, and its low velocity may be due to the long exposure on a poor night, where an instrumental displacement might have occurred through change of temperature.

‘The mean of the first six, exposed for the absorption spectrum, is 49.9, and the mean of the last seven, exposed for the emission spectrum, is 44.2. This difference may be due to two causes, either an actual change in the position of the centre of intensity of the bright *H* γ , or an apparent change due to an unsymmetrical broadening of the line on the plate, caused by the over exposure of a bright line whose curve of intensity is not similar on each side of the centre. In the case of the first six plates, in which the emission lines are over exposed, the velocity obtained is greater, indicating that the setting of the microscope wire had been further to the red than in the case of the last seven. Mr. Harper, to whom I am indebted for the measurement of these plates, tells me that in each case he set the wire as nearly as possible on the centre of the broad black line and no attention was paid to the tips. This would indicate that the emission line was slightly asymmetric towards the red, thus shifting the setting towards the red with increased exposure, and the displacement is not likely due to an actual change in the position of *H* γ itself.

‘There is a remarkable agreement between the mean velocity 44.2 kms. obtained from the last 7 plates, and the mean velocity 44.4 kms. found by Prof. Campbell from 6 plates made by him in November 1898, when, as he says, the lines appeared nearly monochromatic, with a faint broadening or companion to the red side, practically of the same character as observed here. This would tend to show that the conditions in the star under which the bright *H* γ lines are produced, tend to repeat themselves at different maxima, so far, at any rate, as the displacement is concerned, although the relative intensity of the different members of the *H* series is widely different.

‘No trace can be found, however, in these spectra of the bright *Fe* lines at 4308.081 and 4376.107, recorded by Profs. Campbell and Stebbins, but there are no fewer than 8 lines between *H* δ and λ 4235 which have every appearance of emission lines. They stand out as isolated narrow bright lines in a fairly uniform strip of absorption

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spectrum, with an intensity at least twice as great as the back ground of spectrum in which they lie, and are even shown prominently in the widened reproduction of plate 486. It seems hardly possible that they can be narrow strips of continuous spectrum left unabsorbed, as their width is generally less than half a tenth-metre. It may be said on the contrary, however, that they have not been identified with any one element, and that the nearest identifications, are of elements which have the most pronounced lines in the absorption spectrum. There is an exception to this statement in the case of four of the lines which fall reasonably close to four lines in the spectrum of Cerium.

‘The wave-lengths, and the nearest metallic lines are as follows:—

BRIGHT LINES IN THE SPECTRUM OF *o* CETI.

NORMAL W.-L.	NEAREST METALLIC LINES.			
4233·36	4233·76 Fe,	4233·33 Mn Fe		
4229·51	4229·61 Fe,	4229·87 V		
4178·84	4178·54 V,	4179·45 Ce		
4173·58	4173·71 Ti,	4174·00 Fe,	4173·39 Fe	
* 4165·84	4165·78 Ce,	4165·71 Cr,	4165·60 Fe	
* 4138·53	4138·27 V,	4138·51 Ce	4138·70 Mo	
4119·56	4119·62 V,	4119·99 Ce,	4119·77 Mo	4119·55 Fe
4102·95	4103·14 Mn,			

‘The three lines marked with a star (*), are those which appear the most sharply defined and separated from the absorption spectrum, and which seem to be almost certainly emissive in character.

‘The normal wave-lengths were obtained from the measured wave-lengths by subtracting the displacement equivalent to the velocity of the absorption lines. If the mean value of the velocity due to the bright *H* lines were applied to the normal wave lengths above given, they would be increased by 0·25 tenth-metres. Owing to the distance from the centre of the spectrum and the consequent poor focus, the wave-lengths above given may be uncertain to the extent of one tenth of a tenth-metre, possibly more, although the identifications of the absorption lines measured in that region agree to the same limit with the values in Rowland’s table. It seems, therefore, impossible to certainly identify any of these lines with the metallic emission lines, though their appearance and their isolated positions in the general absorption in that region scarcely admit of any other interpretation of their character than the emissive one. A further evidence in this regard is their appearance in some of the other early spectra, in which the exposure was insufficient to show any but the faintest trace of absorption spectrum in the given region. Stebbins, in his paper found only one of the above lines as bright, λ 4233·36, but did not attempt any identification. He also finds λ 4178·84 as apparently bright, but considers it to be only a bright place between two absorption lines. He gives no record of the other lines registered as bright here, and evidently they were not visible in his spectra. Professor Campbell, in his observations says there is good reason to believe in a bright line at λ 4102·8, evidently the same as the one observed here at λ 4102·95. He also mentions one or two more as probably present on the violet side of *H* δ , but no such lines can be seen in our spectra.

‘The absorption spectrum of *o Ceti* is of the banded type, Secchi’s third, Miss Maury’s XX., and has scarcely any recognizable similarity to the solar type. It is considerably different from *α Orionis* and even further advanced than *Herculis*. Its character is well shown by the identifications in the tables of measures of plates 486 and 515. The only absorbing elements present in the strong and best defined lines, those which were measured, are Ti, V, Fe, Mn, Cr, Ca also is present, and a stray *Mg* line appears in No. 515, which is undoubtedly the same line seen as distinctly bright by Stebbins. The first specified are those which are most strongly affected in the spectra of sun spots, and which, as Professor Hale and Mr. Adams have shown,*

* *Contributions of the Solar Observatory*, Nos. 8 and 18

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are much intensified in the spectrum of Arcturus and still more so in α *Orionis* as compared with their intensity in the sun. Apparently, they are even more prominent in α *Ceti* than in α *Orionis*, as our measures have disclosed no other elements as certainly present in its spectrum. Stebbins doubts the presence of *Ti*, but the number of positive identifications in Nos. 486 and 515, and its analogy with the other sun spot elements, seem to offer conclusive evidence in its favour.

‘As the spectrum does not extend much below λ 5000, only the bands in the blue-green are shown, but they are distinctly marked, sharply limited towards the red if considered as bright bands. They are brighter than the neighbouring bands, and fade off gradually towards the violet. There is one exception to this last statement however, the band beginning at λ 4626.0, which is of quite uniform intensity and sharply limited toward the violet at λ 4607.3. As the measures of plate 486 show, when the band was very distinctly and sharply limited, its edge was measured, and generally also the centre of intensity of the absorption line to the red side of the edge, but where not very sharply limited the absorption line at the red edge was measured. Taking these measures and estimating the distance of the edges from the measured positions, we get the following approximate wave-lengths:—

4954.0	Red Edge of Band
4847.5	“ “
4804.5	“ “
4761.4	“ “
4626.2	“ “
4607.6	Violet Edge of above band
4584.2	Red Edge of Band

‘These measures are only given to the nearest tenth of a tenth-metre, as, owing to the poor focus in this region, they are not trustworthy beyond that limit.

‘The spectrum of α *Ceti* is very interesting, and will well repay a more extended study than has yet been given to it. Sufficient has been learned about it, however, to say that it is not necessarily identical at successive maxima, and this is very well shown by the behaviour of the $H\beta$ and $H\epsilon$ lines. It may be considered as well established now that it has a constant velocity of recession with respect to the sun of about 64 kms. per second and that the velocity determined from the bright hydrogen lines is some 15 kms. per second less. This difference of velocity is probably not real, the corresponding shift of the bright lines being produced by some other cause such as abnormal conditions of pressure, temperature, or electrical state in the atmosphere of the star.

‘The difference in the spectrum of α *Ceti* as observed here and at previous maxima may be summarized as follows:—

‘1. Absorption Spectrum.

‘Titanium, whose presence has been considered doubtful by Stebbins, is now very prominent as at least one fourth of the identifications of the prominent absorption lines measured in the two spectra appear to be due to this element.

‘The magnesium line at λ 4571, which was undoubtedly bright in 1902, is now, quite as undoubtedly represented by an absorption line, which was measured in plate 515, and gives a velocity displacement in close agreement with the mean.

‘The bands seem to end towards the violet at λ 4584, as in none of the negatives obtained here could any banded appearance be recognized below that limit. This is also clearly shown in the reproduction. The position of the bands in the blue green, however, agrees with Stebbins’ values.

‘2. Emission Spectra.

‘ $H\beta$ which at previous maxima had either been invisible or faint, is now of a decidedly emissive character, apparently over half as intense as $H\gamma$.

‘ $H\epsilon$ recorded by Stebbins as bright in 1902, but previously invisible, cannot be seen in any plate made here.

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‘There is no trace of the triple character of $H\gamma$ and $H\delta$ observed by Campbell, but no plates were made here at as early a date in the period as those obtained by him. $H\beta$, $H\gamma$, $H\delta$ are slightly asymmetric, more intense to the red side of the true emission line, similar to the later plates obtained by Campbell.

‘No evidence, whatever, can be seen of the bright iron $\lambda 4308$ and $\lambda 4376$, observed by Campbell and Stebbins. The magnesium $\lambda 4571$ observed as distinctly bright by Stebbins is now represented by an absorption line. The bright lines $\lambda 4202$, $\lambda 4216$ and $\lambda 4373$, observed bright by Stebbins are not now present.

‘Eight other bright lines are present in some of the negatives obtained here at $\lambda 4233.4$, $\lambda 4229.5$, $\lambda 4178.8$, $\lambda 4173.6$, $\lambda 4165.8$, $\lambda 4138.5$, $\lambda 4119.6$ and $\lambda 4102.9$. Of these the first and last have been seen bright by Stebbins and Campbell, respectively, and the third Stebbins considers as a bright space between absorption lines. There is no doubt in my mind that the first, fifth and sixth are emissive, but of the others I do not feel so certain.

‘I acknowledge with thanks my indebtedness to Dr. W. F. King, the Director of the Observatory, for his interest and encouragement in the work, and to my assistant, Mr. W. E. Harper, who has very efficiently performed the greater part of the measurement and reduction, as well as assisting in the observing.’

APPENDIX C.

(Reprinted from the Journal of the Royal Astronomical Society of Canada, Vol. 1, No. 3).

'THE SPECTROSCOPIC BINARY α DRACONIS.

'W. E. HARPER.

'The star α Draconis R. A. = $14^h 1.7^m$, $\delta = +64^\circ 51'$. Visual Mag. 3.6, Phot. Mag. 4.0 has been under observation here intermittently since July, 1906. Up to Feb. 12 of this year 37 spectrograms in all had been secured. The radial velocities obtained from the measurement of these were used to obtain provisional values for the elements of the star's orbit. These provisional elements were announced in a previous number of this JOURNAL.

'In drawing the original curve more or less difficulty was experienced from the fact that there were certain intervals for which, owing to unfavourable weather, there had not been any corresponding observation. This was particularly the case at the maxima and minima. Furthermore since the measurements for velocity on the spectrograms already obtained were for the most part dependent on three lines; a faint *Fe* λ 4549, a sharp *Mg* λ 4481 and a broad diffuse *H* γ λ 4340, the resulting velocity was liable to be in error to the extent of say 5 kms. per sec. Three of the plates gave residuals from the computed velocity curve of upwards of 10 kms. but owing to the few lines in the spectrum this large discrepancy may probably be ascribed to accidental distortions of the photographic film, or it may be that the character of the spectrum of the star may have had something to do with the large residuals, as at times an apparent doubling of some of the lines was noticed. At any rate when the new spectrograph was put into regular use about the middle of May the time seemed opportune for securing more spectrograms of this star. This spectrograph with the single-prism attachment gives a flat field from about λ 3600 through the whole range of the visible spectrum. Although its linear dispersion is much less than that of the old spectrograph, the number of additional lines that can be measured *e.g.* *H* β λ 4861, *H* δ λ 4102 and *H* ϵ λ 3970 renders the resulting velocity much less liable to error than would be the case with the former spectrograph. Nine more spectrograms have thus been secured, most of which fall in the gaps already alluded to.

'With the exception of the more recent negatives, for which a newer and shorter method has been evolved, the spectrograms have all been reduced by means of the Hartmann interpolation formula

$$\lambda = \lambda_0 + \frac{c}{s_0 - s}$$

where λ is the apparent wave-length of the line measured, s is the micrometer reading, and λ_0 , c , s_0 are constants determined from known standard comparison lines with their corresponding micrometer readings. The difference between the measured wave-length of the stellar line and its normal wave-length gives us the displacement $d\lambda$ due to the approach or recession of the star. This displacement is easily converted into velocity by means of the simple formula

$$V_s = \frac{299,860}{\lambda} \cdot d\lambda$$

where V_s is the required radial velocity. To this velocity is then added a correction V_a due to the orbital motion of the earth and another V_d due to its axial rotation.

No allowance is made for the motion of the solar system through space; the resulting velocities are therefore relative to the sun.

‘For the sake of brevity the Journal of Observations is omitted. The exposure time required in fair seeing was about 30^m with the single-prism, and about 55^m with the three-prism instrument, the slit-width being usually .025 mm. Several plates were made on each of the nights November 1, November 6 and November 8. I suspected that the star might have a very short period, but the measures showed no rapid change of motion such as might be looked for in a short period binary and accordingly the mean of these measurements for each night was used. In the summary of velocities which follows the phase is given with each velocity. This is the time-interval after some initial epoch selected arbitrarily. I have taken the initial epoch when the computed velocity is zero and becoming positive, *i.e.*, $T_0 = 1906$, July 2, G.M.T., 0^h.

SUMMARY OF VELOCITIES.

Date.	Phase.	Velocity.	Date.	Phase.	Velocity.
1906					
July 2.67	0.67	+ 3	Dec. 17.75	14.61	- 7
" 4.75	2.75	+16	" 18.75	15.61	-17
			1907		
Aug. 15.59	44.59	-32	Jan. 9.67	37.53	-43
" 24.59	2.20	+ 6	" 11.75	39.61	-35
Sep. 10.59	19.20	-30	" 21.67	49.53	- 6
" 19.59	28.20	-42	" 30.71	7.19	+44
" 27.54	36.16	-42	Feb. 6.75	14.23	- 4
Oct. 3.5	42.12	-39	" 12.62	20.10	-36
" 18.5	5.74	+32	May 22.64	16.57	-18
Nov. 1.6	19.76	-40	" 31.68	25.41	-38
" 6.6	24.77	-44	June 8.78	33.50	-45
" 8.6	26.76	-45	" 10.7	35.42	-31
" 16.67	34.91	-50	" 11.6	36.32	-42
" 19.54	37.78	-54	" 20.64	45.36	-20
Dec. 7.71	4.57	+31	" 21.69	46.41	-21
" 11.8	8.65	+31	July 4.63	7.96	+56
" 13.54	10.40	+34	" 5.62	8.96	+45

‘These values when plotted give us a period of about 51 days. To determine the period with greater accuracy it is necessary to take a series of observations extending over a long time and to divide the interval by the number of periods. For this purpose, of course, the longer the star is under observation the more accurately can the period be determined. Our observations extend over only seven periods, whereas observations made at other observatories taken in conjunction with our own give a range of over forty periods. The following are the only other observations made on this star, which are known to the writer.

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PREVIOUS OBSERVATIONS.

Date.		Phase.	Velocity.	Observatory
1901.				
November, 20	92	11 46	+ 20	Yerkes.
1902.				
June	16	13 82	+ 1	Lick.
1903.				
April	29	22 54	- 43	"
May	4	27 54	- 42	"
"	23	46 54	- 17	Potsdam.
"	24	47 54	- 14	"
1904.				
June	19	28 50	- 42	Lick.
1905.				
June	13	27 84	- 42	"
1906.				
January	4	27 22	- 40	"
"	5 98	28 47	- 42	Yerkes.
"	8 9	31 43	- 55	"
"	26 89	49 41	- 9	"
"	29 81	0 95	+ 1	"
February	9 93	12 07	+ 24	"

‘Reducing all the observations within the same period, we find

$$P = 51^d.38$$

which is likely not much in error.

‘It now remains to determine the remaining elements of the orbit from the curve shown. There are two methods; the geometrical, in which the elements are obtained from a consideration of the curve itself, its maximum and minimum points especially, and the areas enclosed by certain portions; the analytical, in which we have recourse to a Fourier series. The former method is that of Lehmann-Filhés; the latter is due to Russell. For orbits of small eccentricity the latter is the preferable I fancy, but in other cases the geometrical is more suitable. I have used the analytical method, and I shall briefly summarize it: Mr. J. S. Plaskett has computed the elements by the other method, and a comparison of the two will prove interesting.

‘The velocity being a known periodic function of the time can be expressed in the form of a series of sines and cosines. Thus we may write

$$v = c_0 + c_1 \cos. \mu (t - t_0) + c_2 \cos. 2 \mu (t - t_0) + + \\ + s_1 \sin. \mu (t - t_0) + s_2 \sin. 2 \mu (t - t_0) + +$$

where $c_0, c_1, \dots, s_1, s_2, \dots$ are constants determined from the curve, t the time at which the velocity is v , and t_0 the initial epoch. With a slight transformation this series can be put in the form

$$v = a_0 + a_1 \cos. [\mu (t - t_0) + a_1] + a_2 \cos. [2 \mu (t - t_0) + a_2] + \dots \quad (1)$$

in which the a 's and α 's are determinable from the c 's.

‘To get an analytical expression for the velocity let our fixed plane of reference be the one perpendicular to the line of sight, let

a = semi-major axis of the orbit

e = eccentricity

i = inclination of plane of orbit to our plane of reference

ω = longitude of periastron measured from the descending node

T = time of periastron passage

θ = true anomaly

M_o = mean anomaly at time t_o
 r = radius vector
 V_o = velocity of system of the whole.

We can then get the following expression for the velocity

$$v = V_o + \mu a \sin. i b_1 \cos. [\mu (t - t_o) + \beta_1 + M_o] + \mu a e \sin. i b_2 \cos. [2 \mu (t - t_o) + \beta_2 + 2 M_o] + + \tag{2}$$

‘The series (1) and (2) considered as functions of the time are of the same form. If they are to represent the same quantity their corresponding coefficients must be equal. Considering terms of e no higher than the first we immediately get preliminary values of V_o , e , ω , $a \sin. i$ and M_o from which by a series of approximations newer and more accurate values can be obtained.

‘From the first curve arbitrarily drawn were obtained the elements

$e = 0.40$
 $\omega = 197^\circ 16.8$
 $V_o = -16.9$ kms.
 $T = 1906$ July 10.69

‘These elements in turn were used to compute an ephemeris by using the two following equations:—

$$t = \frac{P}{2 \pi} . \left[2 \tan.^{-1} \sqrt{\frac{1 - e}{1 + e}} \tan. \frac{\theta}{2} - \frac{e \sqrt{1 - e^2} \sin. \theta}{1 + e \cos. \theta} . \right]$$
$$v = \frac{A + B}{2} \cos. (\theta + \omega) + \frac{A - B}{2} .$$

where t is the time required to describe an angle θ from periastron, v the corresponding velocity, A and B being velocities as designated in the Lehmann-Filhés method. By using values of θ differing by 10° thirty-six points were obtained through which the computed velocity curve was drawn. An examination showed that the original observed velocity curve could be brought into better agreement with the computed curve and still be in as good, if not better, accord with the observations. This was done and a new set of elements computed from this curve. These elements, differing slightly from the previous ones, were in turn used to obtain a second computed velocity curve. The agreement between these latter curves was much better than formerly but was still unsatisfactory. An increase in ω and T would improve matters and when this was done the agreement, though still imperfect, was fairly satisfactory.

‘The residuals of all our observations from this second computed velocity curve were now taken and the probable error of a single observation $\left[r = \pm .6745 \sqrt{\frac{\sum v^2}{n - 1}} \right]$ was computed. Our own observations gave $r = \pm 3.4$ kms. but if the three discordant observations mentioned previously were omitted this was reduced to 2.6 kms. For the previous observations recorded $r = \pm 3.6$ kms. but if we omit a discordant one of Frost’s this would be lessened to 2.9 kms. For a star of this type this may be considered quite satisfactory.

‘Following out the geometrical method Mr. Plaskett used his provisional elements to correct his original curve, from which corrected curve he determined his second and final set of elements. The agreement between his observed and computed velocity curves on this second approximation was sufficient without any further changes and I believe that for an orbit of eccentricity as great as this one under consideration the geometrical method is much preferable. The table of elements as determined by each follows:—

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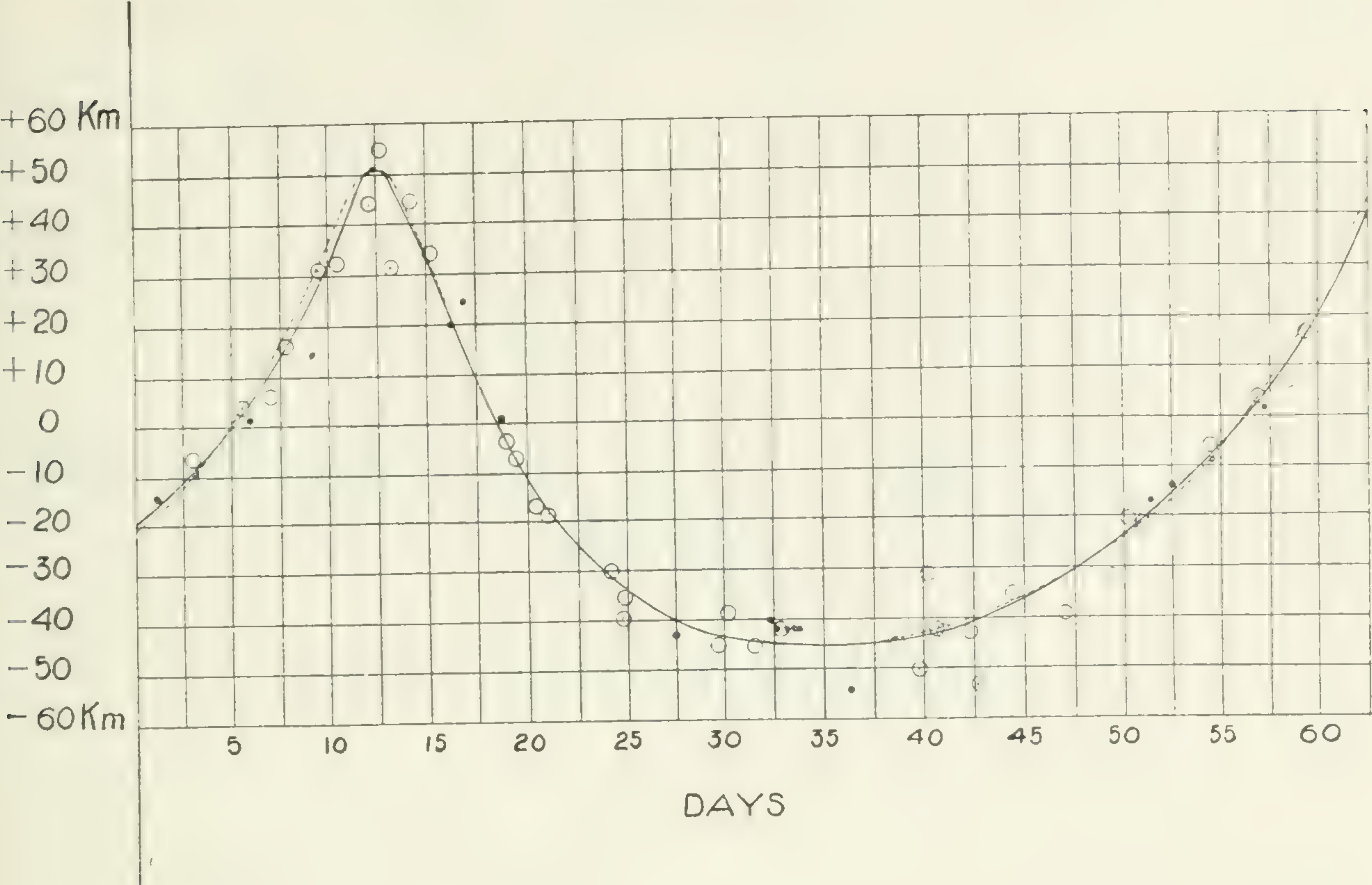


FIG. 1.—Velocity Curve of a Draconis.

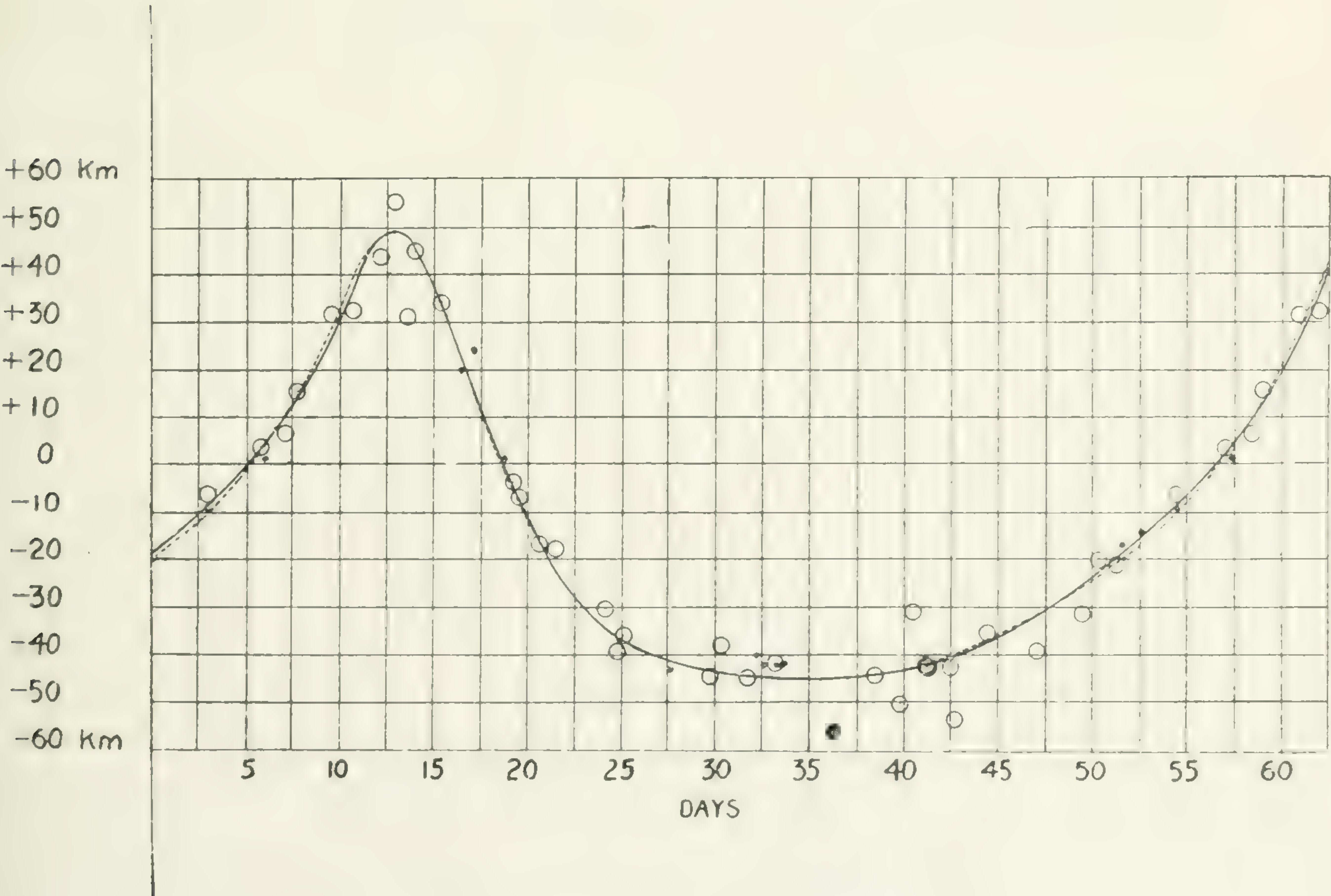


FIG. II.—Velocity Curve of a Draconis.

TABLE OF ELEMENTS.

Elements.	Harper.	Plaskett
P	51 ^d 38	51 ^d 38
V	16.7 kms.	- 17.0 kms.
e	0.42.....	0.44
ω	198 (from descending node).....	20° 15' (from ascending node).
T	1906, July 11 ^d 0 ^h ..	1906, July 11 ^d 0 ^h ..
$a \sin. i$	30,057,900 kms	29,683,000 kms.

‘Both sets of curves are shown; my own in Fig. I. and Mr. Plaskett’s in Fig. II. The heavy line is the observed velocity curve while the dotted is the computed one. The small circles represent our own observations while the dots are those of other observers. A graph of the orbit is also shown.

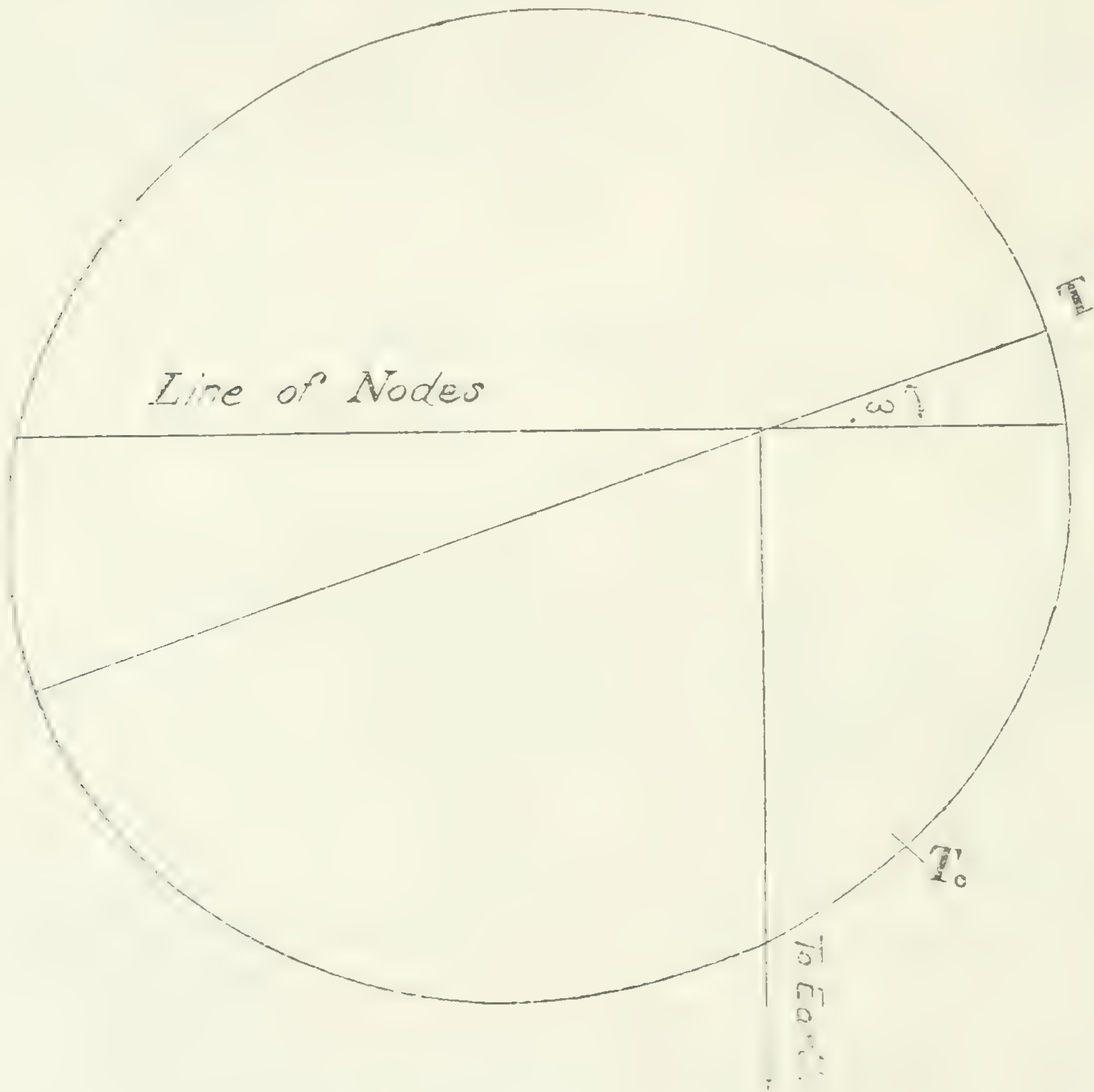


FIG. III.—Orbit of α Draconis.

‘No attempt has been made to correct the elements by the method of least squares as the observations were not considered to be of sufficient accuracy to warrant such a procedure.
‘I acknowledge with thanks my indebtedness to Mr. J. S. Plaskett, who throughout has given me much valuable advice and assistance; and I am glad also of this opportunity of expressing my appreciation of the kindly interest which the Chief Astronomer, Dr. W. F. King, has shown in the prosecution of this work.

‘DOMINION OBSERVATORY,
OTTAWA, CANADA.

APPENDIX No. 4.

REPORT OF THE CHIEF ASTRONOMER, 1907.

TIME SERVICE SYSTEM.

BY

R. M. Stewart, M. A.

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APPENDIX No. 4.

REPORT OF R. M. STEWART, M.A., ON THE TIME SERVICE.

OTTAWA, ONT., March 30, 1907.

W. F. KING, Esq., B.A., LL.D.,
Chief Astronomer,
Department of the Interior,
Ottawa.

SIR,—I have the honour to report as follows on the work carried out under my charge during the past fiscal year.

The ordinary work in connection with the Time Service, in addition to the necessary observations and daily routine work, has consisted mainly in the various extensions and improvements which seemed called for, or became possible, from time to time. The temperature control of the Standard clock has been greatly improved by the installation of the Callendar Recorder described below, and by the erection of the outer case in which it is now enclosed. The automatic arrangement for sending out time-signals has been completed, and has been extended from the telegraph lines to the telephone. The time service to the Government Buildings in the city has been continued, and extensions to other departments provided for. Some additional experimental work, such as time would permit of, has also been carried on. Experiments were made on the time of transmission of telegraphic signals through repeaters, and a few on the time-constants of relays; this work finds its application in longitude determinations and in general meridian observations. An attempt was also made to compare the relative accuracy of transit observations with the observing key and with the travelling-wire micrometer, which led to some interesting and rather unexpected results as described below.

CLOCK ROOM AND APPARATUS.

As stated in my last report, the method of temperature control in use in the clock room at that time had not proved very satisfactory. This was owing to irregular variations of the zero point of the brass-ebonite thermostat which controlled the heating circuit, due to changes in length of the ebonite section, corresponding to fluctuations in the amount of humidity in the air of the room. A Callendar Recording Thermometer with a special attachment was ordered to overcome this difficulty, and was received in December last.

The recorder is essentially a self-balancing slide-wire bridge, in one arm of which a platinum resistance thermometer is inserted; the resistance of the latter serves as a measure of the temperature, while the record is made by a pen attached to the sliding contact. Fig. 1 is from a photograph of the complete instrument, while the circuits are shown diagrammatically in Fig. 2. One side of the bridge-circuit consists of two fixed resistances of ten ohms each; the other comprises several adjusting coils, the slide-wire, and the thermometer; the galvanometer is connected across in the usual way. The self-balancing feature of the instrument depends on the fact that the galvanometer, which is of the D'Arsonval type, acts as an extremely sensitive relay; attached to the moveable coil is a long light arm consisting of two insulated wires which terminate in the two prongs of a fork, enclosing a small contact wheel. The wires lead respectively to two 'motor release magnets,' each of which, when energized, serves to lift a brake off the corresponding one of two motor-clocks and

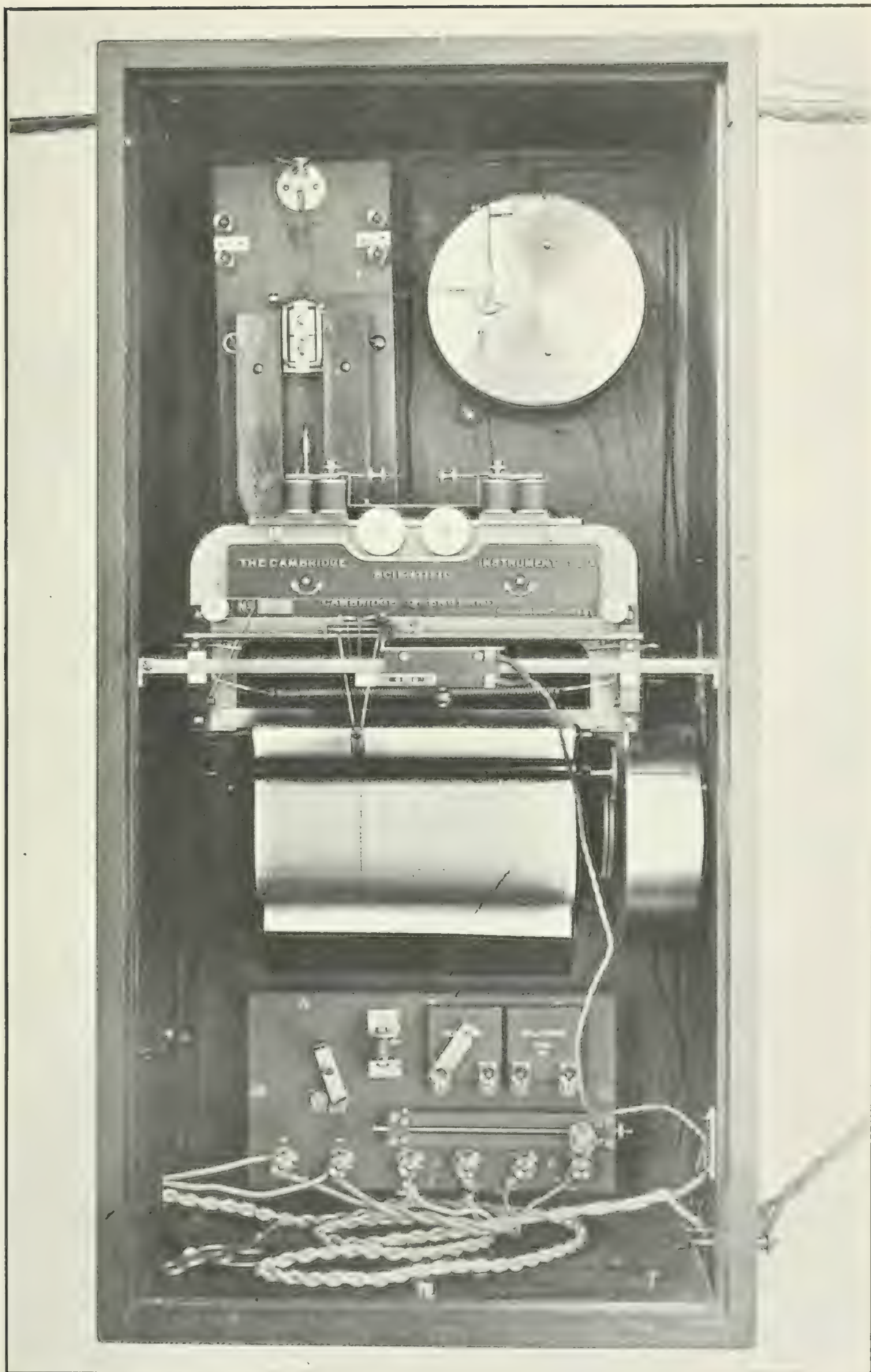


FIG. 1.—Callendar Electric Recorder.

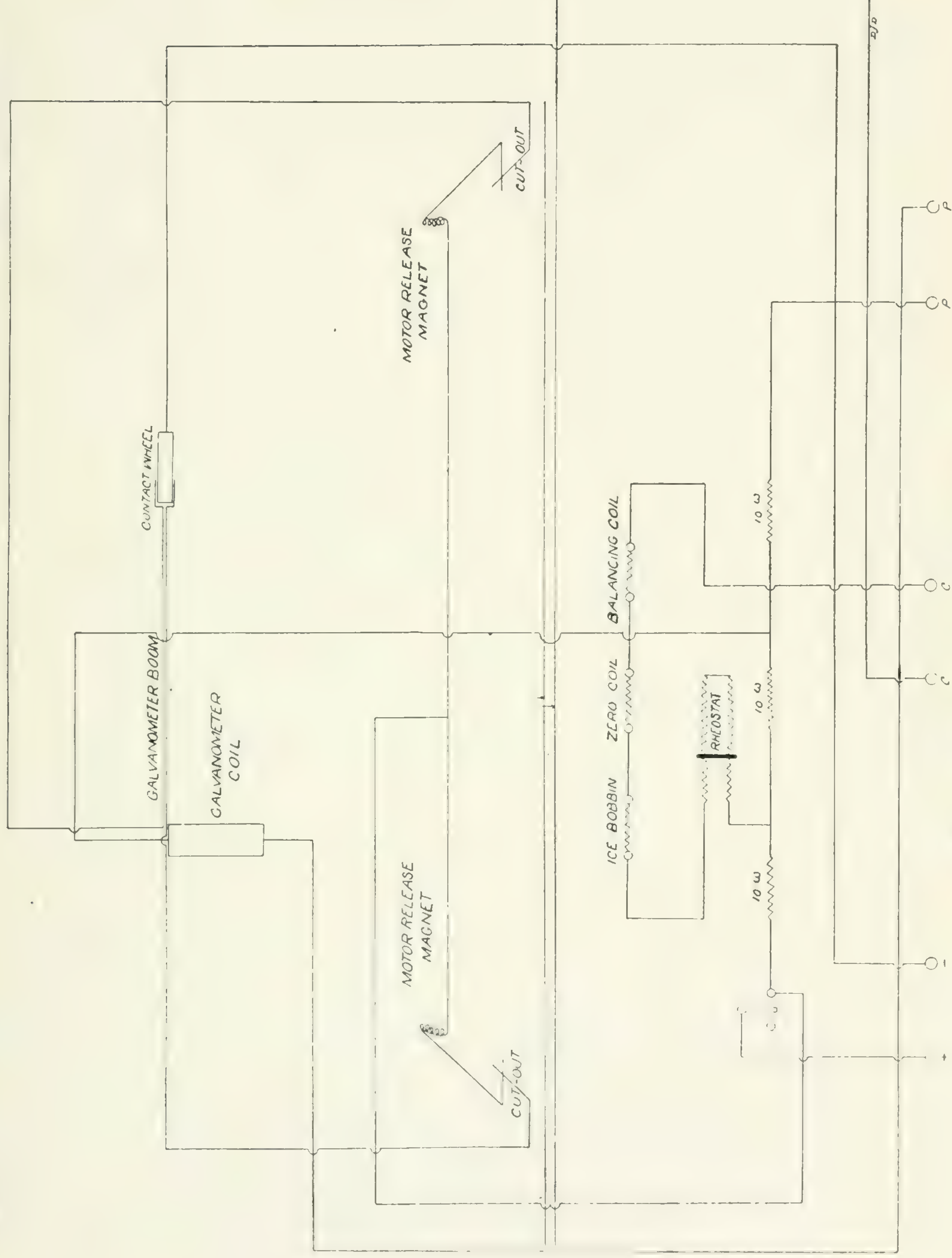


FIG. 2.—Electric Circuits of Callendar Recorder.

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The thermometer consists of about ten feet of open-wound platinum wire, mounted on a frame about six inches square; the open winding is advantageous in that it renders the thermometer particularly quick to take up the temperature of the surrounding air; it is a type introduced recently for meteorological purposes.

These instruments are used usually merely for measuring and recording temperatures; as it was in this instance required to control the temperature as well, an additional attachment was necessary. A brass rod is fixed parallel to the bridge wire, and on it slides a brass frame capable of being clamped in any desired position; this carries an electric contact which is closed by the pen carriage when the temperature drops to the corresponding value; the contact operates the relay which controls the heating circuit.

The chief advantages of the instrument are:—

1. Constancy of the zero point.
2. The temperature recorded is free from the influence of all mechanical errors, such as friction of the pen on the paper, backlash, &c.; this follows from the fact that the balance depends only on the position of the pen-carriage with respect to the bridge-wire.
3. Only the platinum thermometer need be in the position whose temperature is to be measured or controlled; the rest of the instrument may be situated at any distance or in any position convenient.

With regard to the last point, it is necessary to consider the effect of a difference in the temperatures of the thermometer and recorder, or rather, in this case, since the temperature of the thermometer is practically uniform, of variations in the temperature of the recorder. If the sliding contact be placed at a distance x from the lower end of the scale, the whole length of the scale being considered unity, and if the actual temperature at which the thermometer is thereby kept be T , we will have, using the same notation as before,

$$I + B + Z + x W = (1 - x) W + P_o (1 + a T) \quad \dots \dots \dots (6)$$

Now the temperature coefficient of resistance of manganin wire is practically zero;* hence without sensible error we may put $I = P_o$. Also, since the wires B and W are at the same temperature, $B = W$ for all temperatures (provided they are of the same material).

Thus (6) reduces to

$$Z + (1 + x) W = (1 - x) W + P_o a T$$

$$\text{or } T = \frac{Z + 2 x W}{P_o a} \quad \dots \dots \dots (7)$$

Again, if t be the temperature of the recorder,

$$\frac{dT}{dt} = \frac{Z a' + 2 x W a'}{P_o a} = T a' \quad \dots \dots \dots (8)$$

where a' is the coefficient for the wires Z and W , supposed of the same material. Ordinarily T is about 25°C , and if we put $a' = 3.6 \times 10^{-4}$ (the approximate value for German silver), we arrive at the result that a fluctuation of 1°C in the temperature of the recorder will entail a variation of about $.01^\circ \text{C}$ in the controlled temperature. If the variations in t were very large this might be objectionable; in that case the effect might be reduced by removing the coils B and Z and inserting them in the compensating leads beside the thermometer. In that case we should have in (6), $I = P_o$, $B + Z = \text{a constant} = R$, say, and the equation reduces to

$$R + x W = (1 - x) W + P_o a T$$

$$\text{or } T = \frac{R}{P_o a} + \frac{(2 x - 1) W}{P_o a} \quad \dots \dots \dots (9)$$

$$\text{Hence } \frac{dT}{dt} = \frac{(2 x - 1) W a'}{P_o a} \quad \dots \dots \dots (10)$$

* Smithsonian Physical Tables. Coeff. for manganin at $20^\circ\text{--}30^\circ\text{C} = 1.4 \times 10^{-5}$.

Substituting the value of W from (5), and noting that

$$x = \frac{T - T_1}{T_2 - T_1} \text{ very nearly, (10) becomes}$$

$$\frac{dT}{dt} = \left[T - \frac{T_1 + T_2}{2} \right] a' \dots \dots \dots (11)$$

In this case the variation per degree varies from zero at the centre of the scale to about $\cdot 005^\circ \text{ C}$ at either end; for $T = 25^\circ \text{ C}$ it amounts to only $\cdot 001^\circ \text{ C}$.

The instrument was set up in the time room on its arrival, and connected by the compensating leads with the thermometer, which was hung near the centre of the clock room. It was discovered that the manganin ice-bobbin was missing, and pending the arrival of a new one it was replaced for testing purposes by a coil of German silver wire of the proper resistance. After a test, it seemed that there would be no necessity of removing the coils B and Z to the clock room, and they have been left up to the present in their places in the recorder. After connection with the heating circuit, the temperature in the immediate vicinity of the thermometer was kept easily within $\cdot 1^\circ \text{ C}$, which is probably quite close enough for practical purposes. After a time, however, it developed, as might indeed have been expected, that at some distance from the thermometer the variations were considerably in excess of this value, amounting sometimes to perhaps half a degree. In view of the fact that the temperature error of the Riefler clock is somewhat larger than might be desired, it seemed advisable to take some further precautions against variation. While perfectly aware that by many experts it is considered useless to attempt any extreme refinement in temperature control, it has always been my impression that, with this clock at least, temperature is the largest source of error; this feeling was strengthened by its improved performance after the installation of the recorder, and it seemed well worth while to attempt still further improvement. This was deemed advisable not so much for the purposes of the time service as such, but because of the great advantage to be derived in refined meridian work from the highest possible uniformity in clock rate.

The plan adopted was to inclose the clock in an outside heat-proof case whose temperature should be controlled by the recorder, and to keep the less important clocks and the room at large at a fairly uniform temperature by other means. The plan of the case in section is shown in Fig. 3. The outside walls consist of layers of different substances, with an air-space, to prevent loss of heat, and two doors are provided at one side to allow access when necessary. The main part of the case is filled by the pier and clock; behind this a partition extending nearly to the top incloses a space for the electric heater; in the small chamber at the back is situated an electric fan which keeps up a constant circulation of air through the heater and around the clock as shown by the arrows in the figure. The platinum thermometer connected to the recorder is fixed directly above the clock. Two windows of double glass are provided at the level of the clock movement, one in front, the other at the side; for convenience in reading the thermometer and barometer in the clock, a mirror is fixed in a suitable position inside the case. This arrangement was completed and installed in February, and the temperature control is now quite satisfactory; its sensitiveness is exhibited by the fact that the heating circuit is turned on and off on an average about once a minute, while there appears to be no measurable change in the temperature.

The air-tight case of the Riefler was last exhausted and sealed on June 29th, 1906, after replacing the damaged bushing by an improved one; from that time up to the date of this report the leakage, if any, has amounted to less than a millimetre. Some data as to the performance of the clock are contained in Table V. below, and will be referred to later; during the period considered the outside case had not been installed, but the temperature was controlled by the Callendar recorder.

Some time ago a clock was sent to the Chief Astronomer by Louis Fontaine, D.L.S., of Lévis, acting for the estate of the late D. C. Morency, to be tested, with a

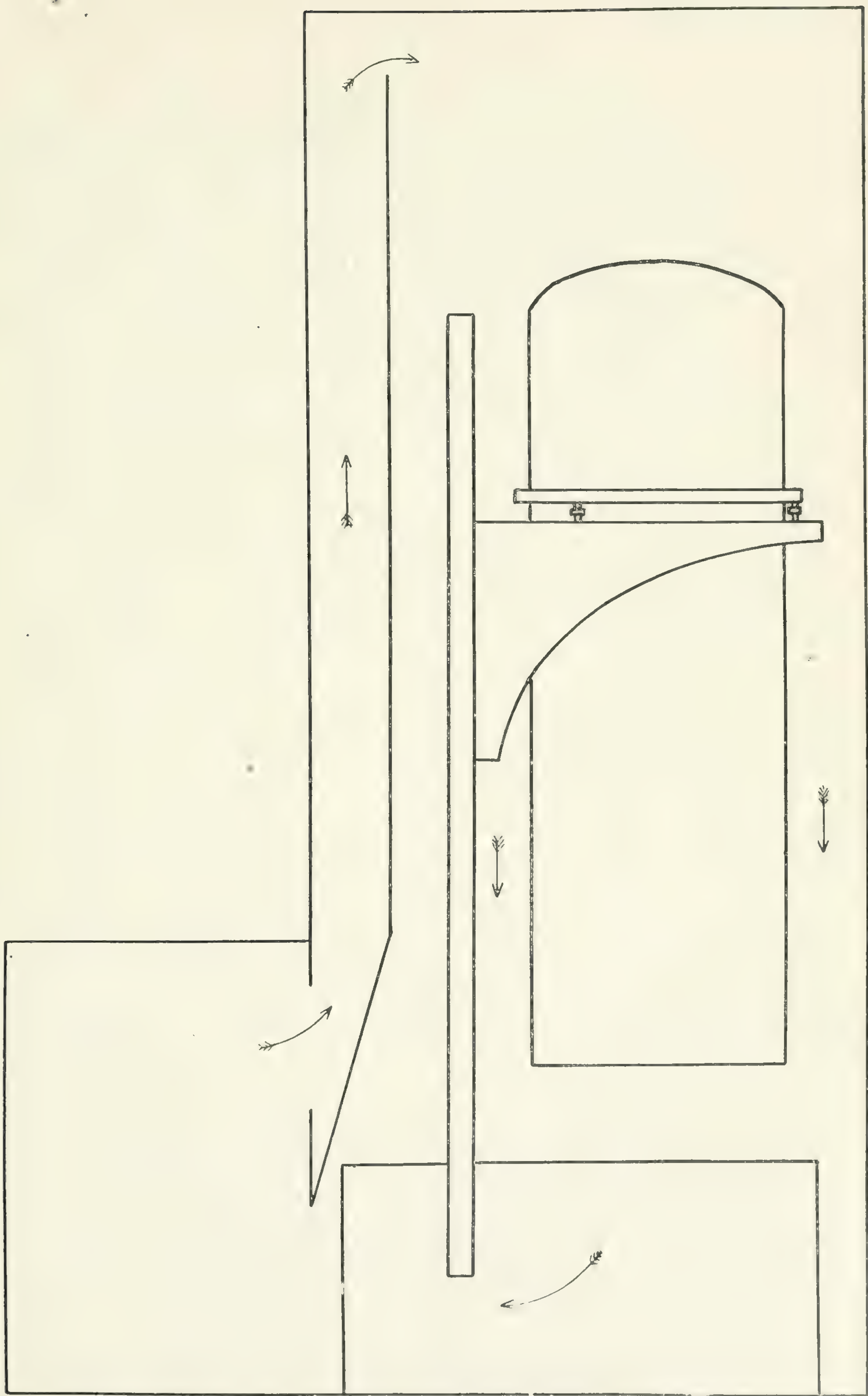


FIG. 3.—Plan of Outside Case of Riefler Clock.

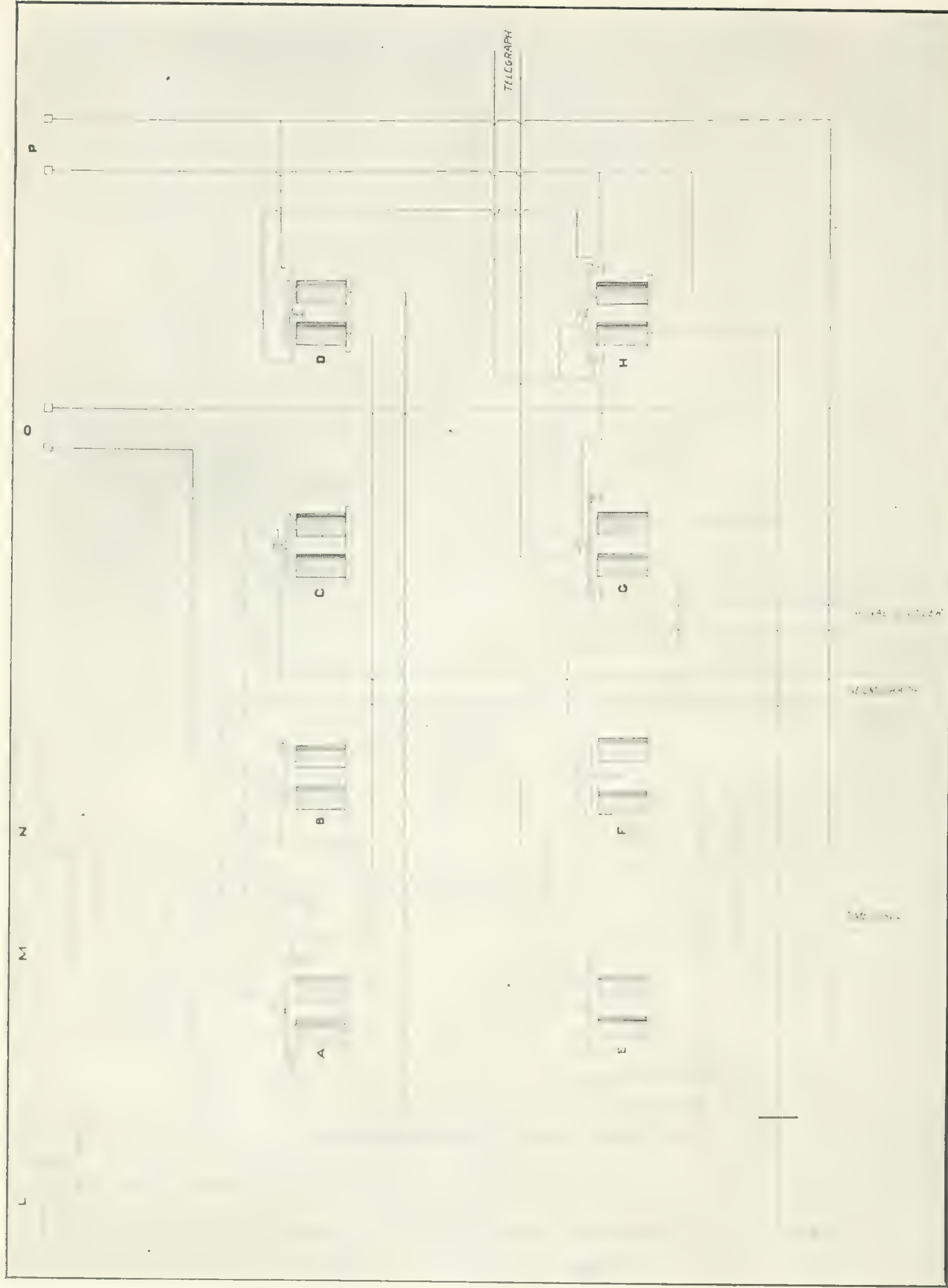


FIG. 4. Time Signal Circuits.

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view to purchase if found satisfactory. The compensation is a mercury one, the pendulum being supported directly from the movement; the escapement is of the dead-beat type. Some preliminary tests of its performance were made in the time room, without any very firm mounting, with the result that its variations of rate were very considerable. However, examination showed that a large part of its irregularity was due to imperfect compensation. The addition of a sufficient quantity of mercury to remedy this defect raised the centre of gravity to such an extent that it could no longer be regulated to keep mean time; accordingly it was rated to sidereal time and a second test made in the clock room. To facilitate comparison, a temporary seconds-contact was attached by arranging a drop of mercury beneath the lower extremity of the pendulum rod. The result of this test showed such a decided improvement as to warrant the conclusion that with some alterations the clock would perform quite creditably, and it was accordingly purchased. The chief alterations required are a firmer suspension of the pendulum and a more rigid attachment of the mercury cup to the pendulum rod, with the addition of the necessary electric contacts. Pending these changes, the clock is at present not in use.

TIME SIGNAL CIRCUITS.

As intimated in my last report, the operation of the circuit in use at that time for sending out time signals was not very satisfactory. The contact controlling the time-signal relay was operated by a wheel in the signal clock which contained 54 teeth and six blank spaces, the latter corresponding to the beats omitted in sending the signals; in this way the signals corresponded in time to the audible beats of the clock. The difficulty encountered was that these beats were not absolutely uniform at different parts of the revolution; the explanation lies in a lack of trueness of the escapement, which is of the pin-wheel type, and so particularly liable to this defect. The remedy evidently lay in making the contact depend, not on the escapement, but on the pendulum. For this purpose the original seconds-contact of the clock, designed for operating seconds dials, was made use of. It consists of two springs, fixed one on each side of the pendulum rod near its upper extremity, and connected together electrically, which make contact alternately with two adjustable screws fixed to the clock-case. This circuit, among others, is shown in Fig. 4, and its working is described below. It was preferred to the arrangement of a mercury drop beneath the pendulum because of the greater firmness and certainty of its action, and also for the sake of uniformity, as the beats can be adjusted to occur sensibly at the same time as the audible beats of the clock, as is the case with electrically driven minute dials and seconds dials.

The arrival of the program clock made it possible to arrange for the automatic action of the signal circuit, and this made necessary a re-arrangement of the circuits controlled by the signal clock. The program clock is of the minute-dial type ordinarily used for ringing bells, and controls two separate circuits. A graduated paper ribbon of a length corresponding to twelve hours is driven by a drum actuated every minute by the impulses of the master-clock; two springs press against the ribbon, being insulated by it from the drum; perforations in the ribbon serve to close the respective circuits at any required time or times. There is also a wheel revolving once a week which may be utilized if required for cutting out either circuit or both at nights and on Sundays or holidays.

In arranging the signal circuit the requirements were that the signal relay should beat seconds, omitting, however, the 29th second and the last five seconds of every minute as well as the last ten seconds of every fifth minute; in addition, it was to give a single beat of one second duration exactly at the even hour, and to remain quiescent for the next ten or fifteen seconds. The telegraph line was to be switched over the points of this relay at 11.55 a.m., and to remain so till a few seconds after noon. This could not be done by the program clock alone, as, being of the minute-

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dial type, it can control a circuit for only an integral number of minutes. The somewhat complexly interconnected system of relays found necessary for this and the other circuits controlled by the clock is shown in Fig. 4. *L* is the pendulum contact mentioned above which is closed for about half a second as the pendulum swings to either side; *M* is a contact controlled by a wheel revolving once a minute, which closes its circuit during the 29th second, and the last five seconds, of every minute; *N* is the contact controlled by the five-minute wheel previously used, closing the circuit from the 50th to the 58th second, of every fifth minute. *O* is the hourly contact, closed from about half a minute before the even hour till a few seconds after; *P* is the contact in the program clock, set to close from 11.55 a.m. to 12 noon. The other contacts and their circuits are omitted for simplicity; one closes its circuit for the first second of every minute, another is closed from the 59th to the 60th second (for operating minute dials), and the third is continuously closed except from the 58th to the 60th second.

The relay *E* is connected in series with the first of these and the hourly contact *O* (circuit not shown); it is actuated for one second exactly at the even hour; from its right-hand pair of points is controlled the circuit running to the time-ball on Parliament Hill. *B* and *C* are worked by the circuit which is open only from the 58th to the 60th second, so that the circuits through their points are closed (see figure) during these two seconds. *A* is operated by a combination of the hourly contact *O* and the relay *B*; the circuit passes from battery through the coils of *A*, thence in multiple through the right hand points *c* of *A* and the points *d* of *B*, thence to *O* and so back to battery. *O* is closed half a minute before the hour, but the circuit still remains open at *c* and *d*; it is, however, closed at *d* at two seconds before the hour, which energizes *A* and draws the armature down, closing *c* and thus leaving the circuit completed even after *d* opens at the 60th second; finally the circuit is opened at *O* after the lapse of a few seconds. In this way *A* is energized from two seconds before the even hour till ten or fifteen seconds after. The circuit for recording on the seismograph passes in series through the points of *C* and the middle points of *A*, thus operating the shutter every minute except that corresponding to the even hour.

F is a differentially wound, neutrally adjusted polar relay; its two pairs of coils are connected one with each side of the pendulum contact *L*; consequently, since it is neutrally adjusted, its action consists of a motion of the tongue *t* to right or left once a second, corresponding to the instants at which the pendulum makes contact at either side. The points *a* and *b* are connected, and a circuit passing between them and *t*, and also through the centre points of *E*, operates the signal relay *G*, which is held closed except at the instant when *t* is passing from *a* to *b* or from *b* to *a*. Consequently the relay *G* beats seconds; the single beat of a second duration every hour is obtained by the action of *E* already described. The omission of the required beats every minute and every five minutes respectively arises from the connection of the contacts *M* and *N* in multiple with *a* *b* and *t*. The condition that the relay shall remain quiescent for a few seconds after the hour is fulfilled by also connecting *a* *b* and *t* with the left-hand points of *A*.

The relay *H* is operated by a combination of the program clock contact *P* and the relay *D*, which works in unison with *A*. The circuit is closed at *P* at 11^h 55^m a.m., and remains so till 12^h 00^m; at 11^h 59^m 58^s, however, an alternative circuit is afforded by *D* through the right-hand points of *H*, which remains effective till opened by *D*; the result is that *H* remains energized from 11^h 55^m till a few seconds after noon. During this period the telegraph line is by it passed over the points of the signal relay *G*, and the clock-beats are thereby transmitted over the line.

An arrangement has also been installed by which the same system of clock-beats can be transmitted by telephone at any time. A telephone transmitter connected with the desk telephone is fixed under my desk in the time room, and immediately in front of it is mounted a telegraph sounder with a switch conveniently situated, so

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that it can at will be connected with the signal relay. The beats are sharp and clear, and of sufficient intensity to be heard even over the long-distance telephone lines; the arrangement has been in use since about the beginning of November and has been in frequent demand by jewellers, surveyors and other parties who require exact time. Methods of time transmission similar to the above have been in use for some time in different localities in Europe, notably by Dr. S. Riefler of Munich.*

UP-TOWN SERVICE.

The time service to the Government Buildings in the city has been continued practically unchanged; a few additional clocks have been installed in some offices, and clocks have been moved from one office to another in several cases as required. The new contacts installed in the master-clocks have given every satisfaction, and have materially improved the service. Some trouble was experienced with the tower clock at the Observatory during the past winter, and was traced to an occasional failure of good contact between the brushes and commutator of the motor; it was remedied by the addition of a small 'trailer' brush consisting of a thin strip of brass bearing on the commutator slightly behind the main brush; the chance of the two contacts failing together has proved negligible. The necessary attention to the up-town circuits has continued under the charge of Mr. D. Robertson, of the Observatory staff; it affords me pleasure to take this opportunity of stating that Mr. Robertson's care and attention to detail in this connection leave nothing to be desired.

An extension of the service has been projected during the past year, designed to include the city Post Office, the Printing Bureau, the Mint and the Archives Building. The plan contemplated involves the connection of the last three by underground wires, to be served by one master-clock. It is also proposed to install an electric tower clock in the post office, to replace the one formerly in use there. Under the instructions of the Chief Astronomer, the buildings in question with the exception of the Mint, were visited by me and an estimate made of the number of dials required for efficient service. In the case of the Mint, the estimate was made from the plans of the uncompleted building, and may require modification. The apparatus required, including master-clocks, dials, switch-boards and switch-board apparatus, batteries, motor-generators, &c., as well as the tower clock, has been ordered, and most of it has been received. In the appended list is given the total number of electric dials now in place or projected.

Parliament Building..	44
Eastern Block..	35
Western Block..	60
Langevin Block..	48
Thistle Block..	2
Observatory (including tower clock)..	27
Post Office (including tower clock)..	21
Printing Bureau..	29
Mint..	29
Archives Building..	7
Total (including 2 tower clocks)..	302

ARMATURE-TIMES OF REPEATERS AND RELAYS.

In the telegraphic comparisons of time which form a part in determinations of longitude, it is usually assumed that the time of transmission of the telegraphic signals is the same in either direction. Where the highest accuracy is not required, as in stations used only for cartographical purposes, this assumption is undoubtedly

* Zeitübertragung durch das Telephon, by Dr. S. Riefler, in Zeitschrift für Instrumentenkunde, Feby., 1906.

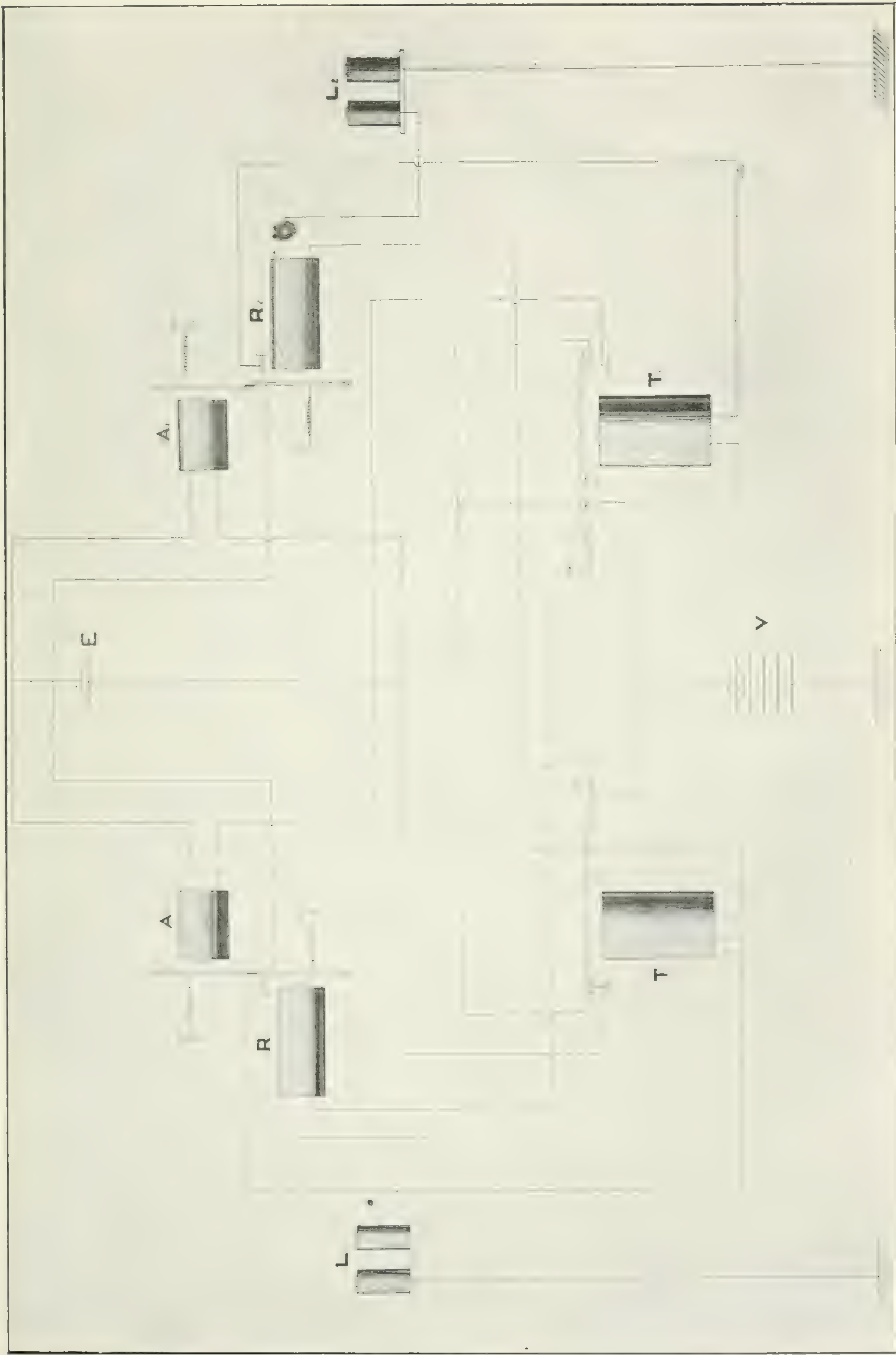


FIG. 5.—Connections of Milliken Hicks Repeaters.

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other type; consequently these were the only ones experimented with. The first experiment consisted in measuring the time of transmission in both directions under different conditions of adjustment, due care being taken that all the adjustments were perfectly normal, and such as might easily occur in actual work. The relays L_1 and L_2 were made to record one on each half of a double chronograph; to save time in scaling, as well as to eliminate errors in the determination of parallax, a clock was made to record on the chronograph simultaneously; the clock times of the signals sent and received could thus be scaled directly, their difference giving the time of transmission; in order that no individual peculiarities of the two coils of the chronograph might enter into the result, the connections were interchanged during the progress of each measurement. The times of transmission east and west are given below for each adjustment:—

	E.	W.
First adjustment..015 sec.	.054 sec.
Second adjustment..016 “	.069 “
Third adjustment..014 “	.034 “
Fourth adjustment..039 “	.008 “

Each of these values is from the mean of twenty signals; the probable error of each value works out about .002 sec. or .003 sec. The values of $\frac{t_{ew} - t_{we}}{2}$, i.e., the effects on a longitude determination (see equation 16), range from .026 sec. to – .015 sec., an amount by no means desirable in a primary longitude.

This appeared to be conclusive proof of the variations in transmission time, due to adjustment, which are liable to occur in actual practice; though the matter might have been left here, another experiment was performed, designed to test the effects of variations in current strength in line and local circuits; in addition to the direct information obtained, this would furnish an independent test of the reliability of transmission times measured in this way, since according to theory the latter should vary regularly, if at all, with the changes in current strength. An attempt was also made to separate the armature-times of the line relays from the time of transmission through the repeaters; though this attempt failed in the first instance, the results are given, as affording a practical example of the errors liable to be introduced into the measurement of short intervals of time, unless due precautions are taken. The first column in Table I. gives the line current, the second the voltage of the local circuit; under E. and W. are the transmission times east and west, measured as before. The last column gives the values obtained for the *difference* in the armature-times of the line relays, in the sense $\delta t_e - \delta t_w$; they were obtained by breaking the common circuit of L_1 and L_2 at V by means of a key (see Fig. 5), and measuring the difference of the clock-times of the signals recorded by L_1 and L_2 as before; apparently, at first sight, this should give the quantity required. Now evidently, if t_{we} and t_{ew} represent the actual times of transmission through the repeaters, $t_{we} = E - (\delta t_e - \delta t_w)$ and $t_{ew} = W + (\delta t_e - \delta t_w)$. On attempting, however to apply this correction, we are confronted with impossible negative values of t_{we} .

TABLE I. TIME OF TRANSMISSION OF SIGNALS THROUGH REPEATERS.

Line Current.	Local Voltage.	E.	W.	$\delta t_e - \delta t_w$.
.017 amp.	3.6	.031 sec.	.026 sec.	.016 sec.
.032 “	3.6	.012 “	.029 “	.041 “
.048 “	3.6	.044 “	.030 “	.064 “
.064 “	3.6	.013 “	.036 “	.073 “
.064 “	5.6	.050 “	.039 “	
.064 “	7.6	.050 “	.041 “	
.064 “	9.6	.050 “	.041 “	

For some time this was very puzzling and discouraging, as tending to throw doubt on the whole series of experiments, but the phenomenon is capable of a simple explanation as follows. The line relays L_1 and L_2 were not identical; L_1 (the western one) was of the type used as a signal relay in longitude operations, with split tubular cores; L_2 had the ordinary solid cores; consequently, L_2 had much the higher coefficient of self-induction. Now the two relays were still connected in parallel even after the opening of the circuit at V ; the higher self-induction of L_2 (as might indeed have been foreseen) simply tended to establish a circuit through itself and L_1 , thus quickening the action of L_1 and retarding its own, *i.e.*, making the measured value of $\delta t_e - \delta t_w$ too large. That such action is possible is proved conclusively by a piece of independent evidence which presented itself in actual work in connection with the time service. Three identical differentially wound polar relays were connected in parallel; in the case of two of them the current passed through both windings in series, in that of the third through only one; thus the value of the self-induction in the third was lower than in either of the others; this third relay was neutrally adjusted, *i.e.*, the armature would stay indifferently in either position when no current flowed, a *reversal* of current being necessary to operate it. Yet it was consistently operated by the simple breaking of the circuit, thus proving conclusively that the current passing through it not only died down more rapidly than it otherwise would, but was actually reversed.

These facts are of importance as serving to emphasize the care necessary in the use of divided circuits when used for the exact measurement of time; in particular, they indicate that chronographs should not be operated in parallel unless their windings are identical; rather they should be worked by separate relays (a separate battery is not, however, necessary); moreover, the relays, if operated by the same clock, should be identical.

The experiment with the repeaters was repeated in a slightly different form, to avoid the above error; in this case the armature-times of the line relays were eliminated during the measurement by interchanging them; half the signals were sent with L_1 connected to R_1 and L_2 to R_2 , the other half with L_2 connected to R_1 and L_1 to R_2 ; the range of current-strengths was also different. The results are shown in Table II.; the columns headed E and W are in this case the transmission times through the repeaters, freed from effect of the line relays. While of course not perfectly regular, they are sufficiently so to indicate the general law; with the exception of the values in the fourth line, which appear to be somewhat too small, they form an unbroken series increasing and decreasing again with the line current, while the times with the heavier local current are in each case, with the above

TABLE II. TIME OF TRANSMISSION OF SIGNALS THROUGH REPEATERS.

Line Current.	Local Voltage.	E.	W.
·017 amp.	5·0	·032 sec.	·022 sec.
·043 "	5·0	·049 "	·037 "
·086 "	5·0	·055 "	·040 "
·086 "	10·8	·051 "	·044 "
·044 "	10·8	·056 "	·044 "
·017 "	10·8	·043 "	·025 "

exception, slightly greater than the corresponding ones where it had the lower value; it may be noted in addition, that the values under E are consistently greater than those under W. The same general tendencies may be observed in Table I., though not in as regular a degree; this is due probably to the fact that each value in Table I. is derived from only twenty signals, as against forty in Table II.; the inclusion of

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the armature-times of the line relays in Table I. should make no difference in the *general* tendency, as the quantities follow similar laws.

The conclusion to be drawn from these experiments appears to be that the time of transmission through repeaters in opposite directions may vary within comparatively wide limits, being affected both by conditions of adjustment and by current strength. The effect of the first of these causes on longitude measurements may of course be eliminated by reversal of the repeaters without change of adjustment during the progress of the exchange, but with the differences of current strength in the two sections of the line this is not the case, the latter being dependent on length and condition of line, &c. In fact it is conceivable that under certain conditions—*e.g.*, a large difference of current strength in the two sections of the line, with a *corresponding* perfect adjustment of repeaters—the reversal of the repeaters without readjustment might only aggravate matters—might indeed introduce an error where otherwise none would have occurred. One thing is certain; repeaters should never be used in primary longitude work except in case of absolute necessity. In that case it would probably be best to increase the number of exchanges, even having several in immediate succession, and to insist on a complete and independent readjustment of all repeaters between exchanges, trusting to the principle of compensation of errors in the final result; under such conditions it would at least be reasonably certain that the result would not be affected by systematic error.

There still remains to be considered the possible difference in armature-times of the signal relays at the two stations, the quantity $\delta t_w - \delta t_e$ in equation (16), which enters for its full value into the longitude. Most of the signal relays on our longitude switch-boards are of the split-core type described above, having a resistance of about 330 ohms; one, however, is a polar relay, resistance 400 ohms in each winding. The armature-times of both these types were measured under different conditions of current and adjustment; the method of measurement was as follows. A relay with two pairs of points was controlled by the signal key; one pair of points recorded directly on one side of the chronograph, the other pair worked the relay whose armature-time was to be measured; the points of the latter recorded on the other side of the chronograph. To eliminate the individuality of the separate pairs of points of the first relay, their connections were interchanged during each measurement, as well as those of the two coils of the chronograph. Each measurement consisted of ten signals with each system of connection, or forty in all.

Table III. shows the effect on the split-core relay of independent variations of the three variable quantities, current, armature-tension, and distance of coils from armature; in only one case, one which can easily be guarded against in actual work, was the armature-time greater than .005 sec., while it did not reach even that value

TABLE III. ARMATURE-TIME OF SPLIT-CORE RELAY.

Current.	ADJUSTMENT.		Arm.-time.
	Coils.	Tension.	
.058 amp.	Close.	Loose.	.012 sec.
.058 "	Medium.	"	.001 "
.058 "	Distant.	"	.005 "
.058 "	Medium.	"	.005 "
.058 "	"	Medium.	.003 "
.058 "	"	Tight.	.0005 "
.029 "	"	Medium.	.002 "
.058 "	"	"	.0025 "
.117 "	"	"	.003 "

except when the tension was loose. This indicates that the relay with split tubular cores is reasonably efficient for longitude work, and may be rendered very highly so by care in keeping the adjustment keyed up to the highest point at which the relay will work. This fact renders practically unnecessary the complex balancing system often employed in longitude exchanges in Europe.

TABLE IV. ARMATURE-TIME OF POLAR RELAY.

Current.	ADJUSTMENT.		Arm.-time.	Remarks.
	Pole-piece.	Tension.		
·005 amp.	Close.	Loose.	·029 sec.	One winding.
·005 "	"	Tight.	·002 "	"
·005 "	Distant.	Loose.	·027 "	"
·005 "	"	Tight.	·006 "	"
·025 "			·025 "	Both windings.
·050 "			·056 "	One winding, other short-circuited.
·050 "			·021 "	One winding.
·025 "			·012 "	"
·012 "			·009 "	"
·005 "			·002 "	"
·005 "			·008 "	" other short-circuited.
·0025 "			·003 "	Both windings.

Table IV. shows the armature-times for the polar relay under similar conditions. In these relays the armature moves between two pole-pieces which form the permanent field; the distance between the pole-pieces is variable; this is the adjustment referred to in the second column; the 'tension,' in this case the effect of the permanent magnetic field, depends on the position of the armature relative to the pole-pieces. As will be seen from the table, the adjustment of the pole-pieces makes little difference, while the effect of variation of tension is very considerable; variations of current also produce a considerable effect. If both windings are used in series the armature-time appears to be slightly greater (for the same adjustment) than if only one winding is used, with the same number of ampere-turns; on the other hand, if the current be passed through one winding, and the other short-circuited, the armature-time is more than doubled. On the whole, though under the most favourable conditions the armature-time is fairly small, this type of relay is decidedly unsuitable for the measurement of short intervals of time, and should never be used in longitude operations or for working chronographs except in extremity.

ERRORS OF TRANSIT OBSERVATIONS.

The experiments described below were undertaken in the first instance merely as a test of the relative merits of the transit key and the travelling wire micrometer as methods of observing transits; the results which developed, however, served to call attention to some other phases of the question which seemed to call for investigation. That work is as yet by no means complete; still, some results have been arrived at which serve to show in a general way the causes underlying some of the larger errors in transit work, and to indicate the lines along which further investigation may most profitably be carried on. It should be premised that what follows refers only to observations with a portable instrument, that is, to the case in which the azimuth and collimation errors of the instrument must be determined from the observations themselves, and not by means of a fixed azimuth mark and collimating

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telescopes. For a more complete understanding of the conditions, it may be well to explain in a few words the general method of making the observations and reductions which has been customary here. It is to be noted also that the purpose of the regular observations is the determination of clock-error.

The instruments used are the ordinary reversible type of Cooke transit, with object-glass of about three inches aperture and three feet focal length. These instruments, though portable, are heavy and massive enough to be fairly stable during an evening's work. A complete time determination consists usually of the transits of twelve stars, six in each position of the instrument; of these six, one is a slow-moving north star for the determination of azimuth, while the remaining five are south of the zenith. After applying corrections for level error and for diurnal aberration, we have from each star an equation of the form $Cc + Aa + \Delta T = l$, where c and a are the collimation and azimuth errors respectively, $C = \sec \delta$, $A = \sin (\phi - \delta) \sec \delta$, and ΔT is the clock correction. The twelve equations are combined by least squares, giving equal weights to the separate observations, and hence are deduced the values of c , a , and ΔT . The probable error of the resulting ΔT is found in the usual way from the residuals given by the separate stars. It has been the usual custom to so select the stars as to 'balance' the set, that is, to have ΣA and ΣC each as nearly zero as convenient.

Every one who has had extended experience with such observations has noticed certain discrepancies which frequently show themselves in the results. In an extended series of observations for personal equation, the different values obtained, even from two determinations on the same night, may sometimes, if not frequently, differ by a tenth of a second or even more, while the probable errors of the individual sets may not in any case exceed one one-hundredth of a second. Nor is it only in personal equation observations that these discrepancies show themselves; in longitude determinations, a fair average of the extreme difference obtained during a few night's work would probably be about a tenth of a second or more; and the differences in clock-error obtained by the same observer on the same night from successive determinations, even when using a reliable clock, are often of about the same order of magnitude.

From a comparison of the magnitude of these frequent discrepancies with that of the corresponding probable errors, and from the fact that they do not follow the same law, and seem to have no connection, it is at once evident that the discrepancies are not the result of truly accidental errors, but are systematic in their nature; that is to say, that of two sets taken on the same night, one may be affected by a certain systematic error, the other by a different error also systematic. The simplest explanation of this fact, and the one which has been generally accepted,* is that these discrepancies are the result of real variations in the observer's personal equation. Indeed it seems *a priori* quite probable that a quantity so purely a personal one would depend on the observer's physical and mental state, and would consequently vary from night to night, and even during the same night. Such a supposition would fully and absolutely explain the observed facts; for even though the change were a gradual and regular one during any single night it would not be evident from a consideration of the separate observations, for the reason that the grouping into sets and the separate reduction of these would tend to mask its progressive character. And further, if the discrepancies were due to this cause, we should naturally expect that in observations with the travelling wire micrometer, which are comparatively free from personal error, they would disappear or at least be considerably reduced. It remained, then, only to test this hypothesis by actual experiment.

* See "Test of a Transit Micrometer", U.S. Coast and Geodetic Survey report, 1904; also "Die Beobachtungsmethode mittelst des Repsold'shen Registrirmikrometer in ihre Anwendung auf Längenbestimmungen," by Prof. Th. Albrecht, in *Astronomische Nachrichten*, No. 3699, Band 155.

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Two of the instruments belonging to the Observatory have been equipped with the micrometer attachment for a couple of seasons; but most of the observations taken with them were in the field, and the variations in the rates of the chronometers used made it impossible to place any dependence on conclusions drawn from these data. There remain some observations for personal equation taken here by Mr. McDiarmid, using the Riefler Sidereal clock, which might have been compared with observations taken by myself at the same time with the key. But any conclusions based on these data could scarcely have been final, for whichever way the balance went, the possibility would remain of explaining it by the superior accuracy of one or the other observer.* Consequently it was decided to make a test by conducting a series of observations with two instruments, one fitted with a transit micrometer, the other with a fixed field and a key. The method followed was to take as many time sets as convenient, four if possible, on the same night, with the key and micrometer alternately, repeating the programme on a sufficient number of nights. The accurate running of the Riefler clock made it possible to compare rigorously the results of all the observations on any night, and even, with some slight reservation, on succeeding nights.

Up to that time I had never used the transit micrometer, so that in the first place it was necessary to gain some experience in its use. It was decided, however, to so arrange the observations made for that purpose that they could be made use of to obtain some preliminary results. The observations were begun about the first of November last; only six stars were observed in each set instead of twelve, and four sets were taken on each clear night for six nights, making in all twelve sets with each instrument. The continuity of the series was broken at this point by a period of cloudy weather, and the later observations were confined to the gaining of experience with the micrometer. The indications given by these six nights' observations were not very conclusive; while the average inter-agreement of the observations for the whole period was about the same for key and micrometer, the result given by discarding one night's observations was quite decidedly in favour of the micrometer; considering the fact of my inexperience with the latter, it was perhaps natural that I should be confirmed in my former belief that the discrepancies in key observations were explainable on the assumption of variations in personal equation, and would tend to disappear in micrometer work. At the same time, it was of course realized that time-sets of only six stars were rather unreliable, and that the data were not extensive enough to warrant definite conclusions. It was decided, therefore, after having gained some experience in micrometer observations, to proceed with a new and more extended test, and to observe full sets of twelve stars in each case.

This series of observations was begun about the middle of December, and concluded early in February; during that time some forty-five sets had been observed on sixteen different nights, nearly an equal number with each instrument. It was not possible, on account of weather conditions, to adhere throughout to the full program of four sets on every night; sometimes only three or two were obtained, and on a few occasions only one. It was hoped, however, that even these might be made some use of, if the clock-rate proved constant enough from day to day. The results are shown in Table V. The second and third columns show respectively the clock corrections (in seconds) and their probable errors as deduced in the usual way; the fourth and fifth columns contain the discordances between the different observations on the same night with the same instrument. These range in the case of the key up to .129 sec., with an average of .049 sec.; in that of the micrometer to .103 sec., with an average of .045 sec.; the average probable error is .011 sec. for each instrument. It is worthy of note that on December 17th, when the largest discrepancy with the micrometer occurred, the probable errors of the two sets were only .009 sec. and .011 sec., while the discrepancy of .129 sec. with the key occurred between two sets whose probable errors were .007 sec. and .011 sec.

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TABLE V.

I	II	III	IV	V	VI	VII	VIII	IX	X	XI
Date	ΔT	r	Discordances.		T	Mean for night.	ΔT at mean rate.	Resid. for night	Residuals.	
			Key	Microm.					Key	Microm.
Dec. 17....	-2.704	.009			2.704					-.059
" 17....	-3.070*	.011	.024	.103	-2.625*	2.645	-2.645	.000	.020	
" 17....	-3.094*	.012			-2.649*				-.004	
" 17....	-2.601	.011			-2.601					.044
" 19....	2.612	.013			-2.612					.047
" 19....	-3.182*	.011	.129	.068	-2.737*	-2.659	-2.659	.000	-.078	
" 19....	-3.053*	.007			-2.608*				.051	
" 19....	-2.680	.011			-2.680					-.021
" 27....	-3.165	.012			2.720				-.001	-.001
Jan. 2....	-3.211*	.010			2.766	-2.766	-2.763	-.003	-.003	
" 6....	-2.746	.009			-2.746	-2.791	2.793	.002		.047
" 6....	-3.281*	.004			-2.836*				-.043	
" 9....	-2.803	.015			-2.803				.039	.012
" 9....	-3.221*	.015	.015	.017	-2.776*				.024	
" 9....	-3.236*	.015			2.791*	-2.797	-2.815	.018		-.005
" 9....	-2.820	.014			-2.820					
" 11....	-2.856	.007			-2.856					-.026
" 11....	-3.308*	.007	.046		-2.863*	-2.845	-2.830	-.015	-.033	
" 11....	-3.262*	.012			2.817*				.013	
" 15....	-2.721	.008		-2.721					-.037	
" 15....	-3.107*	.009		-2.662*	-2.691	-2.684	-.007	.022		
" 16....	-2.617	.013	-2.617	-2.617				-2.647	.030	.030
" 18....	2.562	.013	-2.562	-2.562				-2.573	.011	.011
" 21....	2.508	.010	-2.568							-.046
" 21....	-2.899*	.009	.063		-2.454*	-2.493	-2.462	-.031	.008	
" 21....	-2.962*	.009			-2.517*				-.055	
" 23....	-2.778*	.013			-2.333*				.055	
" 23....	2.409	.012		.034	.007	-2.409	-2.378	-2.388	.010	
" 23....	-2.402	.015	-2.402							-.014
" 23....	-2.812*	.012	-2.367*			.021				
" 28....	2.214	.020					-2.214	-2.205	-2.204	-.001
" 28....	2.641*	.013			-2.196*	.008				
" 30....	-2.452*	.008			-2.007*					
" 30....	-2.044	.007	.040	.066	-2.044	-2.019				
" 30....	1.978	.011			-1.978					
" 30....	-2.492	.010			2.017*					
Feb. 4....	-1.515	.011								1.515
" 4....	-1.967*	.013	.064		-1.522*	-1.541				
" 4....	-2.051*	.011			-1.586*					
" 6....	-1.670*	.008			-1.225*					
" 6....	-1.169	.007		.008	.011	1.169	-1.198			
" 6....	-1.180	.012	-1.180							
" 6....	-1.662*	.014	-1.217*							
Means.....		.011*								
		.011	.049	.045					.028	.029

*Key observations.

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We may examine this series of observations in another way, which will give perhaps a better comparison of the performances of the two instruments. It has been mentioned that the standard clock of the Observatory has a very steady rate; if it should so happen that the series of observations with one instrument showed a pronounced tendency to agree more closely with the supposition of a regular rate-curve, that would indicate, other things being equal, that that series of observations was less affected by fluctuating errors. From the values in column II. we can obtain the mean systematic difference between the key and micrometer observations, a quantity which of course corresponds to personal equation; it amounts to $\cdot445$ sec. The application of this correction to the key observations makes all the observations on any single night strictly comparable; the derived values are given in column VI., and the means for each night in column VII.

From a consideration of these it is apparent that as regards clock-rate the whole period falls naturally into three parts, during the first two of which at least the rate was very nearly constant. The irregularity during the last few days is no doubt due to the fact that the outside case of the clock was then being installed, which, in addition to a certain amount of direct disturbance, probably gave rise to considerable irregular changes of temperature. The decided change in rate at January 11th is somewhat puzzling; it may be partially accounted for by changes in local temperature, as about that time, on account of the extreme coldness of the weather, the electric heater proved just insufficient to keep the temperature up to its former value, and several incandescent lamps were left burning continually in the clock room to help turn the scale; a slight increase in temperature (though only about $\cdot2^{\circ}$ C) was indicated by the thermometer in the upper part of the clock, and it is possible that the change in distribution of the heating may have induced a larger difference in the temperature of the pendulum, though this seems doubtful. On the other hand, there was exactly at that date a small but very definite change in the amplitude of swing of the pendulum, which was read daily; for the month immediately preceding, the amplitude varied between $91\cdot6'$ and $92\cdot1'$, the average being $91\cdot9'$; from January 11th till the end of the month it varied between $91\cdot2'$ and $91\cdot6'$, the average being $91\cdot3'$; this can have no connection with temperature, as the monthly averages for the three preceding months had been respectively $91\cdot8'$, $91\cdot7'$ and $91\cdot7'$, while the average temperatures were $25\cdot8^{\circ}$, $25\cdot1^{\circ}$ and $23\cdot5^{\circ}$. Whatever the cause of the simultaneous change in amplitude and rate, there is little doubt that they were closely connected.

However that may be, it is sufficient for the purpose of our comparison that during each of the first two periods the evidence points to a practically constant rate, so that up to January 28th we are enabled to make a comparison of all the observations without regard to date. During the first period (December 17th to January 11th) the mean rate was $- \cdot0074$ sec. per day; during the second (January 11th to January 28th), $+ \cdot0359$ sec. per day. The theoretical clock corrections at these constant rates are given in column VIII., while column IX. exhibits the differences between columns VII. and VIII. The smallness of these differences at once gives colour to the supposition that the rate was practically uniform during each period, and allows us without fear of error to make use of the test above referred to. This consists in a comparison of the residuals of each separate observation from the assumed rate-curve; these are tabulated for the two instruments in columns X. and XI. The mean value for the key is $\cdot028$ sec., and for the micrometer $\cdot029$ sec., the largest values being $\cdot078$ sec. and $\cdot059$ sec. respectively. Or if we treat these quantities as real residuals, and determine from them in the usual way the probable error of a single observation, we get $\cdot025$ sec. in the case of the key, and $\cdot024$ sec. in that of the micrometer.

This series of observations, then, in whatever way we examine it, points to the conclusion that so far as irregular variations are concerned there is very little to choose between the key and the micrometer. True, the values are on the whole per-

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haps slightly smaller in the case of the micrometer, but the difference is so slight that we would not be justified in drawing any conclusions from it.

In order to check the conclusions arrived at, an investigation was made of all the available micrometer observations made at the Observatory; those taken in the field were useless for the purpose, as stated before, on account of the inaccuracies of chronometer rate. The entire data were found to consist of thirteen nights' work, on each of which two time sets had been taken. These nights were distributed over several months; the largest discrepancy which occurred was $\cdot 108$ sec., the average value being $\cdot 035$ sec., while the average of the probable errors of all the sets was $\cdot 010$ sec. From fifteen nights' key observations taken during the same period the largest discrepancy was $\cdot 101$ sec., the average being $\cdot 034$ sec., and the average probable error $\cdot 010$ sec. In this case the discrepancies are somewhat smaller than those obtained in December and January, as was of course to be expected from the difference in temperature and the consequent difference of comfort in making the observations. As in the former case, however, the values are practically the same for the two methods of observing.

The conclusion, then, would appear to be forced upon us, that if irregular fluctuations of personal error do occur in key observations, their influence is effectually masked by some other source or sources of error which are common to observations with the transit micrometer. This is, so far as I am aware, the first comparative test of the kind to be made under identical conditions for both instruments, and it is perhaps not surprising that what seems to have been an erroneous assumption should have received general acceptance; indeed, the present investigation was undertaken in the first place with the firm expectation of merely confirming the generally accepted opinion. These conclusions, however, it must be remembered, do not invalidate the claim for the micrometer of the practical elimination of personal equation and all the advantages thereby entailed.

Taking it for granted, then, that these effects are practically independent of the instrument used, and therefore not due to variations of personal equation, it becomes legitimate, in order to obtain more extensive data on the real magnitude of the variations, to take into account any additional key observations available. These consist of about fifty nights' work, on each of which at least two time determinations were taken; the average value of the discordances amounts to $\cdot 039$ sec., the average of the probable errors being $\cdot 012$ sec. Combining these with the observations already considered, both with key and micrometer, we obtain, as the average discordance from nearly a hundred nights' work, the quantity $\cdot 039$ sec., the average probable error corresponding being $\cdot 011$ sec. An idea of the magnitude of the real probable error to which such an average discordance corresponds may be obtained from the values derived from columns X. and XI. in Table V., and also from the personal equation observations made by Mr. McDiarmid and myself. These last consist of between twenty and thirty separate determinations; the probable error of a single determination, computed by the residuals from the mean, amounts to $\cdot 038$ sec. Now this is evidently the probable error of the *difference* of two time sets; that of a single one would therefore be $\cdot 027$ sec. The average discordance on the same night during these observations was $\cdot 035$ sec. Combining this with the probable error of $\cdot 024$ sec. or $\cdot 025$ sec. from Table V. and the corresponding discordances, we may assume as a rough average that the final mean discordance of $\cdot 039$ sec. corresponds to a real 'probable error' of about $\cdot 025$ sec. That is to say, the real liability to error exceeds more than two-fold the nominal value obtained in the usual way from the residuals of the separate stars.

It must be admitted, of course, that to such a limited number of observations as the twelve stars of a time set, the theory of the probability of errors does not rigorously apply; for this reason it is to be expected that the probable error obtained from the application of this principle should be systematically too small; and it might easily happen that in some cases it would be vastly so. But in the average of a large

number of observations, such as is here considered, it seems impossible to account for effects of the observed magnitude by any such hypothesis. To account for at least a part of the discrepancy, we must rather look for some source of error which from its nature would not show itself in the residuals, which might systematically affect the result of a complete set by a considerable quantity, and which might vary from one set to another.

In considering the question *à priori*, the explanation seemed most likely to lie in defective determinations of azimuth caused by errors in the observations of polar stars; this view was strengthened by the reflection that when the azimuths of the two 'clamps' of a time set are reduced separately, they frequently differ by a considerable amount. A simple test of the validity of this explanation can be obtained by observing in the course of a set as large a number of polar stars as possible, and examining the effect of a choice of different ones in reduction. Table VI. shows the

TABLE VI.

Date	T	<i>a</i>	<i>r</i>	Difference
Dec. 19.....	- 2·654	- ·060	·016	·073
	- 2·581	- ·196	·013	
Dec. 19*.....	- 3·182	·485	·012	·069
	- 3·113	·360	·016	
Feb. 21.....	·122	- ·244	·008	·013
	·135	- ·264	·008	
Mar. 6.. ..	·698	·317	·010	·046
	·744	·244	·014	
Mar. 25.....	- ·655	- ·349	·019	·018
	·637	- ·398	·017	

* Key observations.

result of such observations and reductions on several nights. In most cases four polars and ten time stars were observed in each set; the polars were in every case ones that might have been used in the course of ordinary observations—*i.e.*, ranging from about 75° to 85° declination. The reduction was made first with one pair of polars, omitting the others, and afterwards with another pair and the *same* time stars. The second and third columns give the clock corrections and azimuths obtained in this way from each set, the fourth column containing the probable errors of the corresponding clock corrections. In the last column is given the difference between the two time determinations from each set. It will be seen that in three cases out of the five the two values of the azimuth differ by a considerable amount, while in the same three cases the resulting effect on the time determination is about ·05 sec. or over. In some cases almost as large a change would have resulted by replacing only one of the polars, though in the majority of cases each separate polar gives a quite distinct value of the azimuth.

There can be no doubt that we have here the principal source of error in such observations as we have been considering. While in itself it may not be of sufficient magnitude to completely account for all the discrepancies observed, it would seem to do so fully when taken in conjunction with ordinary accidental errors of observation and with the errors arising from defective level readings, which, however, it greatly

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overshadows in magnitude. The fault does not appear to lie, at least in the main, in catalogue errors or in magnitude equation, but simply in accidental errors of observations of the polar stars, which of course become systematic with respect to any one set. It remains to be seen whether any method of observing can be devised to overcome the difficulty. Some tentative experiments have been made to this end, but the investigation is not yet complete. An account of these experiments, together with a discussion of the theoretical conditions, is reserved for a future report.

RISING AND SETTING OF THE SUN AT OTTAWA.

The tables given below for the rising and setting of the sun at Ottawa for 1907 were computed by W. M. Tobey, and have been checked over carefully by both Mr. Tobey and myself. The exact formulae for rising and setting (in standard civil time) are respectively:—

$T_R = 12^h\ 02^m\ 52^s + e_1 - t_1$
 $T_S = 0^h\ 02^m\ 52^s + e_2 + t_2$

where e = equation of time for corresponding epoch

$$\sin \frac{1}{2} t = \sqrt{\frac{\sin \frac{1}{2} (\zeta + \phi - \delta) \sin \frac{1}{2} (\zeta - \phi + \delta)}{\cos \phi \cos \delta}}$$

 $\zeta = 90^\circ + \text{horizontal refraction} + \text{sun's semi-diameter.}$
 $\phi = \text{latitude.}$
 $\delta = \text{sun's apparent declination for corresponding epoch.}$

The values adopted for the horizontal refraction and semi-diameter were 33' and 16' respectively, giving $\zeta = 90^\circ\ 49'$; the latitude was taken to the nearest minute ($45^\circ\ 24'$). For convenience in computation, the values used for e and δ were those corresponding to 6 a.m. and 6 p.m., respectively; this introduces a small variable error, whose maximum value, however, does not exceed four or five seconds, and is negligible in comparison with the uncertainties of refraction.

The object of carrying the computation to seconds was that the tables might be of permanent value, since tables for any ensuing year can be formed from them by interpolation. For since the length of the tropical year is 365.24221 days, and the length of the civil year 365 days or 366 days, 1908, 1912, &c., being leap years, it follows that if T_{1907} be the time of rising or setting on any particular day, and ΔT the difference between that day and the next (or preceding), we will have

$T_{1907+n} = T_{1907} + K \Delta T$

where $K = - .24221\ n + \text{the greatest integer in } \frac{n+2}{4}.$

It is to be noted, however, that in a leap year this value of K applies in January and February only, and must be increased by unity for the remaining months. The values of K are given for the next few years:—

1908..	— .242
	.758
1909..516
1910..273
1911..031
1912..	— .211
	.789
1913..547
1914..305
1915..062

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Date.	JANUARY.						FEBRUARY.						MARCH.						APRIL.						Date.
	Rising.			Setting.			Rising.			Setting.			Rising.			Setting.			Rising.			Setting.			
	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	
1907	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	1907
1	7	42	59	4	29	46	7	25	18	5	08	27	6	43	18	5	48	32	5	45	23	6	29	28	1
2	7	43	03	4	30	40	7	24	08	5	09	54	6	41	32	5	49	54	5	43	30	6	30	45	2
3	7	43	04	4	31	36	7	22	55	5	11	21	6	39	45	5	51	17	5	41	37	6	32	02	3
4	7	43	02	4	32	34	7	21	42	5	12	48	6	37	58	5	52	39	5	39	44	6	33	19	4
5	7	42	57	4	33	35	7	20	25	5	14	15	6	36	10	5	54	01	5	37	53	6	34	36	5
6	7	42	51	4	34	36	7	19	09	5	15	42	6	34	21	5	55	22	5	36	00	6	35	53	6
7	7	42	41	4	35	39	7	17	50	5	17	09	6	32	32	5	56	43	5	34	08	6	37	10	7
8	7	42	29	4	36	45	7	16	30	5	18	37	6	30	42	5	58	03	5	32	17	6	38	25	8
9	7	42	14	4	37	52	7	15	07	5	20	04	6	28	51	5	59	25	5	30	26	6	39	43	9
10	7	41	56	4	39	01	7	13	44	5	21	31	6	27	01	6	00	45	5	28	36	6	41	01	10
11	7	41	36	4	40	11	7	12	19	5	22	59	6	25	09	6	02	06	5	26	46	6	42	17	11
12	7	41	13	4	41	23	7	10	53	5	24	26	6	23	18	6	03	26	5	24	57	6	43	34	12
13	7	40	48	4	42	35	7	09	25	5	25	53	6	21	26	6	04	45	5	23	08	6	44	51	13
14	7	40	20	4	43	48	7	07	56	5	27	20	6	19	33	6	06	05	5	21	20	6	46	08	14
15	7	39	50	4	45	04	7	06	25	5	28	46	6	17	41	6	07	25	5	19	33	6	47	26	15
16	7	39	17	4	46	21	7	04	53	5	30	12	6	15	48	6	08	44	5	17	46	6	48	42	16
17	7	38	41	4	47	38	7	03	20	5	31	38	6	13	54	6	10	03	5	16	00	6	49	59	17
18	7	38	04	4	48	56	7	01	45	5	33	05	6	12	01	6	11	21	5	14	15	6	51	16	18
19	7	37	24	4	50	16	7	00	10	5	34	30	6	10	07	6	12	40	5	12	31	6	52	32	19
20	7	36	41	4	51	36	6	58	33	5	35	56	6	08	13	6	13	58	5	10	47	6	53	49	20
21	7	35	56	4	52	58	6	56	55	5	37	21	6	06	19	6	15	16	5	09	04	6	55	06	21
22	7	35	09	4	54	19	6	55	17	5	38	46	6	04	25	6	16	35	5	07	22	6	56	23	22
23	7	34	20	4	55	42	6	53	37	5	40	11	6	02	30	6	17	53	5	05	41	6	57	40	23
24	7	33	28	4	57	05	6	51	57	5	41	35	6	00	36	6	19	10	5	04	01	6	58	55	24
25	7	32	35	4	58	29	6	50	14	5	42	59	5	58	41	6	20	28	5	02	22	7	00	13	25
26	7	31	37	4	59	53	6	48	31	5	44	22	5	56	47	6	21	45	5	00	44	7	01	28	26
27	7	30	39	5	01	18	6	46	47	5	45	46	5	54	53	6	23	03	4	59	07	7	02	44	27
28	7	29	39	5	02	43	6	45	03	5	47	09	5	52	58	6	24	20	4	57	31	7	04	00	28
29	7	28	35	5	04	09	5	51	05	6	25	37	4	55	57	7	05	18	29
30	7	27	31	5	05	34	5	49	11	6	26	54	4	54	23	7	06	32	30
31	7	26	26	5	07	00	5	47	17	6	28	11	31

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Date.	MAY.						JUNE.						JULY.						AUGUST.						Date.
	Rising.			Setting.			Rising.			Setting.			Rising.			Setting.			Rising.			Setting.			
	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	
1907	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	1907
1	4	52	50	7	07	47	4	18	24	7	42	39	4	17	39	7	54	47	4	45	36	7	31	52	1
2	4	51	19	7	09	03	4	17	49	7	43	31	4	18	11	7	54	36	4	46	46	7	30	36	2
3	4	49	49	7	10	19	4	17	16	7	44	22	4	18	45	7	54	23	4	47	55	7	29	18	3
4	4	48	21	7	11	33	4	16	45	7	45	11	4	19	22	7	54	08	4	49	05	7	27	58	4
5	4	46	54	7	12	50	4	16	17	7	45	59	4	20	09	7	53	51	4	50	15	7	26	37	5
6	4	45	29	7	14	05	4	15	51	7	46	44	4	20	40	7	53	31	4	51	27	7	25	14	6
7	4	44	03	7	15	17	4	15	27	7	47	28	4	21	22	7	53	09	4	52	37	7	23	50	7
8	4	42	41	7	16	31	4	15	06	7	48	10	4	22	05	7	52	45	4	53	50	7	22	25	8
9	4	41	20	7	17	45	4	14	48	7	48	50	4	22	50	7	52	17	4	55	01	7	20	57	9
10	4	40	09	7	18	58	4	14	32	7	49	28	4	23	37	7	51	48	4	56	12	7	19	29	10
11	4	38	41	7	20	11	4	14	18	7	50	05	4	24	25	7	51	16	4	57	25	7	17	59	11
12	4	37	25	7	21	22	4	14	06	7	50	41	4	25	15	7	50	41	4	58	38	7	16	27	12
13	4	36	10	7	22	35	4	13	56	7	51	14	4	26	06	7	50	05	4	59	50	7	14	55	13
14	4	34	57	7	23	46	4	13	49	7	51	44	4	26	58	7	49	27	5	01	02	7	13	23	14
15	4	33	45	7	24	56	4	13	46	7	52	13	4	27	51	7	48	46	5	02	14	7	11	46	15
16	4	32	36	7	26	06	4	13	42	7	52	39	4	28	46	7	48	03	5	03	28	7	10	11	16
17	4	31	27	7	27	15	4	13	41	7	53	05	4	29	42	7	47	18	5	04	40	7	08	33	17
18	4	30	21	7	28	24	4	13	45	7	53	28	4	30	40	7	46	29	5	05	53	7	06	55	18
19	4	29	16	7	29	33	4	13	50	7	53	47	4	31	39	7	45	40	5	07	06	7	05	15	19
20	4	28	15	7	30	38	4	13	55	7	54	05	4	32	38	7	44	49	5	08	19	7	03	35	20
21	4	27	14	7	31	45	4	14	06	7	54	20	4	33	38	7	43	55	5	09	31	7	01	53	21
22	4	26	15	7	32	50	4	14	17	7	54	33	4	34	39	7	42	59	5	10	45	7	00	11	22
23	4	25	18	7	33	54	4	14	31	7	54	44	4	35	42	7	42	00	5	11	58	6	58	27	23
24	4	24	24	7	34	57	4	14	47	7	54	53	4	36	45	7	41	00	5	13	11	6	56	43	24
25	4	23	31	7	35	59	4	15	05	7	54	59	4	37	49	7	39	58	5	14	24	6	54	58	25
26	4	22	41	7	36	59	4	15	25	7	55	03	4	38	53	7	38	55	5	15	37	6	53	12	26
27	4	21	53	7	37	59	4	15	48	7	55	05	4	39	59	7	37	49	5	16	50	6	51	26	27
28	4	21	06	7	38	58	4	16	12	7	55	04	4	41	06	7	36	41	5	18	03	6	49	38	28
29	4	20	22	7	39	56	4	16	39	7	55	00	4	42	13	7	35	31	5	19	16	6	47	50	29
30	4	19	40	7	40	52	4	17	08	7	54	55	4	43	20	7	34	20	5	20	29	6	46	01	30
31	4	19	01	7	41	46	4	44	27	7	33	07	5	21	42	6	14	12	6	14	12	31

Date.	SEPTEMBER.						OCTOBER.						NOVEMBER.						DECEMBER.						Date.
	Rising.			Setting.			Rising.			Setting.			Rising.			Setting.			Rising.			Setting.			
	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	
1907	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	1907
1	5	22	55	6	42	21	5	59	41	5	45	06	6	40	49	4	51	34	7	21	15	4	21	48	1
2	5	24	08	6	40	32	6	00	56	5	43	12	6	42	14	4	50	07	7	22	25	4	21	24	2
3	5	25	21	6	38	40	6	02	12	5	41	19	6	43	37	4	48	42	7	23	34	4	21	02	3
4	5	26	34	6	36	49	6	03	28	5	39	26	6	45	01	4	47	18	7	24	42	4	20	43	4
5	5	27	47	6	34	56	6	04	44	5	37	33	6	46	26	4	45	56	7	25	48	4	20	26	5
6	5	29	00	6	33	04	6	06	01	5	35	41	6	47	50	4	44	36	7	26	53	4	20	12	6
7	5	30	13	6	31	11	6	07	18	5	33	49	6	49	14	4	43	17	7	27	56	4	20	01	7
8	5	31	26	6	29	17	6	08	35	5	31	57	6	50	39	4	42	00	7	28	58	4	19	53	8
9	5	32	39	6	27	23	6	09	52	5	30	07	6	52	03	4	40	45	7	29	58	4	19	47	9
10	5	33	52	6	25	29	6	11	11	5	28	17	6	53	28	4	39	31	7	30	56	4	19	43	10
11	5	35	05	6	23	35	6	12	29	5	26	27	6	54	51	4	38	19	7	31	51	4	19	44	11
12	5	36	18	6	21	40	6	13	46	5	24	38	6	56	15	4	37	09	7	32	45	4	19	47	12
13	5	37	31	6	19	45	6	15	04	5	22	50	6	57	39	4	36	02	7	33	38	4	19	51	13
14	5	38	44	6	17	50	6	16	24	5	21	03	6	59	01	4	34	55	7	34	28	4	19	58	14
15	5	39	58	6	15	54	6	17	42	5	19	16	7	00	25	4	33	51	7	35	17	4	20	08	15
16	5	41	11	6	13	59	6	19	02	5	17	30	7	01	48	4	32	49	7	36	03	4	20	21	16
17	5	42	24	6	12	03	6	20	21	5	15	44	7	03	10	4	31	49	7	36	47	4	20	36	17
18	5	43	37	6	10	07	6	21	41	5	14	01	7	04	32	4	30	51	7	37	29	4	20	55	18
19	5	44	50	6	08	11	6	23	01	5	12	17	7	05	53	4	29	55	7	38	08	4	21	15	19
20	5	46	03	6	06	15	6	24	22	5	10	35	7	07	15	4	29	02	7	38	45	4	21	39	20
21	5	47	17	6	04	19	6	25	43	5	08	53	7	08	35	4	28	11	7	39	20	4	22	05	21
22	5	48	31	6	02	23	6	27	03	5	07	13	7	09	54	4	27	22	7	39	52	4	22	33	22
23	5	49	44	6	00	27	6	28	25	5	05	34	7	11	12	4	26	35	7	40	22	4	23	04	23
24	5	50	58	5	58	31	6	29	47	5	03	55	7	12	32	4	25	50	7	40	51	4	23	38	24
25	5	52	12	5	56	36	6	31	09	5	02	18	7	13	50	4	25	08	7	41	15	4	24	14	25
26	5	53	26	5	54	40	6	32	31	5	00	42	7	15	06	4	24	29	7	41	37	4	24	52	26
27	5	54	41	5	52	45	6	33	53	4	59	08	7	16	22	4	23	52	7	41	57	4	25	32	27
28	5	55	55	5	50	50	6	35	16	4	57	34	7	17	37	4	23	17	7	42	14	4	26	16	28
29	5	57	10	5	48	55	6	36	39	4	56	02	7	18	51	4	22	45	7	42	29	4	27	02	29
30	5	58	25	5	47	00	6	38	03	4	54	31	7	20	04	4	22	15	7	42	41	4	27	49	30
31	6	39	26	4	53	02	7	42	51	4	28	39	31

I have the honour to be, sir,
Your obedient servant,
R. M. STEWART.

APPENDIX No. 5.

REPORT OF THE CHIEF ASTRONOMER, 1907.

TABULAR STATEMENT OF LONGITUDE AND
LATITUDE OBSERVATIONS, 1906

BY

J. Macara.

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SESSIONAL PAPER No. 25a

APPENDIX No. 5.

TABULAR STATEMENT OF LONGITUDE AND LATITUDE OBSERVATIONS.

OTTAWA, ONT., March 30, 1907.

W. F. KING, Esq., B.A., LL.D.,
Chief Astronomer,
Department of the Interior,
Ottawa.

SIR,—I have the honour to transmit herewith a tabular statement of the differences of longitude and the latitude results of stations observed in 1906. Annexed thereto is, also, a description of the stations occupied.

A synopsis of the statement giving the longitude and latitude of the various stations will be found on page 256.

I have the honour to be, sir,

Your obedient servant,

J. MACARA.

DIFFERENCE OF LONGITUDE BETWEEN NEW LISKEARD AND DOMINION OBSERVATORY, OTTAWA.

SESSIONAL PAPER No. 25a

Date.	DIFFERENCE OF CHRONOGRAPH.			CLOCK CORRECTION.				DIFFERENCE OF LONGITUDE.				Time of Trans- mission.		
	Western Signals.		Eastern Signals.	Western Station.	Probable Error.	Eastern Station.	Probable Error.	Western Signals.	Eastern Signals.	Mean.	Probable Error.		<i>v.</i>	
	h. m.	s.	h. m.	s.				m.	s.	m.	s.			
1906.														
June 2...	0	15	44.134	0	15	44.004	-14.779	+ .010	-05.914	+ .014	m.	s.	s.	.096
" 3...		15	43.495		15	43.375	-15.374	+ .018	-05.882	+ .020				.089
" 6...		15	44.165		15	44.052	-14.171	+ .011	-05.544	+ .008				.103
" 8...		15	45.487		15	45.367	-12.549	+ .010	-05.135	+ .015				.003
" 9...		15	47.916		15	47.761	-09.820	+ .010	-04.820	+ .016				.000
" 10...		15	45.941		15	45.812	-11.595	+ .014	-04.615	+ .011				.018
														.065

Observers—W.—F. A. McDIARMID.	h. m. s.	s.
E.—R. M. STEWART.	15 52.838	+ .007
Weighted mean ..	.364	+ .004
Personal Equation....	51.797	+ .052
λDominion Observatory....	5 18 44.999	+ .052
λNew Liskeard		

DIFFERENCE OF LONGITUDE BETWEEN RIVIÈRE OUELLE AND DOMINION OBSERVATORY, OTTAWA.

Date.	DIFFERENCE OF CHRONOGRAPH.				CLOCK CORRECTION.				DIFFERENCE OF LONGITUDE.				Time of Trans- mission.	
	Western Signals.		Eastern Signals.		Western Station.		Eastern Station.		Probable Error.		Probable Error.			v.
	h. m.	s.	h. m.	s.	h. m.	s.	h. m.	s.	h. m.	s.	h. m.	s.		
1906.														
June 21...	0 22	17.823	0 22	17.682	s. -29.099	s. +.013	s. -00.504	s. +.010	h. m. 0 22	s. 46.418	h. m. 0 22	s. 46.348	s. +.016	s. .054
" 25...		22.966		22.812	-30.095	+ .008	-06.588	+ .009		46.473		46.396	+ .012	.006
" 27...		53.114		52.982	+03.256	+ .006	-03.325	+ .010		46.533		46.467	+ .012	.079
" 29...		06.215		06.096	-39.970	+ .016	+00.266	+ .012		46.451		46.392	+ .020	.060

Observers--W.--R. M. STEWART.
E.--F. A. McDIARMID.

Weighted mean.....	h. m.	s.
Personal Equation.....	0 22	46.402
λDominion Observatory.....	5 02	51.797
λRivière Ouelle.....	4 40	05.759
		±.009
		±.004
		±.052
		±.052

DIFFERENCE OF LONGITUDE BETWEEN MANIWAKI AND DOMINION OBSERVATORY, OTTAWA.

SESSIONAL PAPER No. 25a

Date.	DIFFERENCE OF CHRONOGRAPH.		CLOCK CORRECTION.				DIFFERENCE OF LONGITUDE.				Time of Trans- mission.
	Western Signals.	Eastern Signals.	Western Station.	Probable Error.	Eastern Station.	Probable Error.	Western Signals	Eastern Signals.	Mean.	Probable Error.	
1906.	m. s.	m. s.	s.	s.	s.	s.	m. s.	m. s.	m. s.	s.	s.
July 6..	51.993	51.917	-10.700	+ .010	- .290	+ .011	1 02.327	1 02.365	1 02.365	± .015	.038
" 7..	54.987	54.889	-07.944	+ .012	- .410	+ .011	1 02.423	1 02.472	1 02.472	± .016	.049
" 9..	1 02.727	1 02.654	- .234	+ .018	- .497	+ .008	1 02.391	1 02.427	1 02.427	± .020	.037

Observers—W.—F. A. McDIARMID.
E.—R. M. STEWART.

Weighted mean.....	h. m. s.	1 02.418	± .009
Personal Equation.....		.364.....	
λDominion Observatory....	5 02	51.797	± .052
λManiwaki	5 03	54.579	± .052

DIFFERENCE OF LONGITUDE BETWEEN BOUNDARY AND VANCOUVER.

Date.	DIFFERENCE OF CHRONOGRAPH.				CLOCK CORRECTION.				DIFFERENCE OF LONGITUDE.				Time of Trans- mission.										
	Western Signals.		Eastern Signals.		Western Station.		Eastern Station.		Probable Error.		Eastern Station.			Probable Error.									
	h.	m.	s.	h.	m.	s.	m.	s.	s.	h.	m.	s.		h.	m.	s.							
1906.																s.							
Aug. 22..	1	11	50.247	1	11	49.790	1	29.856	1	11	413	±.028	1	11	31.804	1	11	31.347	1	11	31.576	±.036	..219
" 25..	11	30	723	11	30	284	21	426	22	472	±.013	±.013	769	330	549	±.016	..219						
" 27..	11	14	084	11	13	635	12	309	30	087	±.010	±.010	862	413	637	±.014	..224						
" 29..	10	57	959	10	57	516	00	385	34	271	±.020	±.012	845	402	623	±.023	..221						
" 31..	11	08	610	11	08	203	13	281	36	537	±.010	±.010	866	459	662	±.014	..203						
Sept. 2..	11	05	078	11	04	657	12	846	39	483	±.013	±.010	715	294	505	±.016	..211						

Observers—W.—F. A. McDIARMID.
E.—OTTO KLOTZ.

Weighted mean... .. h. m. s. 1 11 31.596
λVancouver..... .. 8 12 28.461
λBoundary 9 24 00.057

SESSIONAL PAPER No. 25a

DIFFERENCE OF LONGITUDE BETWEEN BOUNDARY AND FORT EGBERT.

Date.	DIFFERENCE OF CHRONOGRAPH.		CLOCK CORRECTION.				DIFFERENCE OF LONGITUDE.				Time of Trans- mission.
	Western Signals.	Eastern Signals.	Western Station.	Probable Error.	Eastern Station.	Probable Error.	Western Signals.	Eastern Signals.	Mean.	Probable Error.	
1906.											
Aug. 19.	s. 48.145	s. 48.145	s. -3.446		m. 1 s. 34.744	s. +.010	s. 50.045	s. 50.045	s. 50.045		
" 22.	41.291	41.291	-1.837		29.434	+ .014	49.980	49.980	49.980		
" 23.	37.182	37.182	-0.316		26.837	+ .019	49.971	49.971	49.971		
" 25.	27.758	27.762	+3.654		21.408	+ .010	49.996	49.992	49.994		
" 28.	10.077	10.077	+6.949		06.941	+ .009	49.915	49.915	49.915		
" 29.	02.057	02.065	+8.575		00.712	+ .020	50.080	50.072	50.076		

Observers—W.—EDWIN SMITH.
E.—F. A. McDIARMID.

h. m. s.

Mean..... 49.998
λFort Egbert.. 9 24 50.058
λBoundary..... 9 24 00.060

LONGITUDE AND LATITUDE OF STATIONS OBSERVED IN 1906.

Place	Difference of Longitude	To	Longitude		Longitude		Latitude	
	h. m. s.		h. m. s.		° ' "		° ' "	
Dominion Observatory.....	1 775	Cliff St. transit house. . .	5 02 51.797		75 42 56.96		47 30 33.58	
New Liskeard.....	15 53.202	Dominion Observatory.....	5 18 44.999		79 41 14.99		47 29 04.86	+ 12
Rivière Ouelle	22 46.038	" "	4 40 05.759		70 01 26.39		46 22 28.40	+ 11
Maniwaki.	1 02.782	" "	5 03 54.579		75 58 38.69		64 40 51.42	+ 13
Boundary	1 11 31.596	Vancouver (1900).. . . .	*9 23 59.970		140 59 59.55			+ 16
"	1 11 31.596	" (1905).....	*9 24 00.057		141 00 00.86			
"	49.998	Fort Egbert.....	*9 24 00.060		141 00 00.90			
Vancouver.							49 17 46.07	+ 13

* Adjusted Longitude, 9h. 24m. 00.027s.

LOCAL POSITIONS OF ASTRONOMICAL STATIONS.

Dominion Observatory.—The reference point of the longitudes observed in 1906 is a temporary transit house, the meridian of which is $0^s.12$ east of the centre of the dome of the observatory.

New Liskeard.—The observatory pier is 25.5 feet south and 836.6 feet west of an iron post which is 145 feet S. $5^{\circ} 20'$ W. of the southwest corner of the Temiskaming and Northern Ontario Railway station house.

Rivière Ouelle.—The observatory pier is 18.7 feet south, and 180.3 feet east of the first mooring post on the east side of the wharf. It is also about 70 feet from the Intercolonial Railway crossing at the end of the wharf.

Maniwaki.—The observatory pier is 112.8 feet south and 69.8 feet west of the southwest corner of the Canadian Pacific Railway station house.

Boundary.—The observatory is on the south bank of the Yukon river and is 352 feet east of the 'Ogilvie Line' and about 20 feet south from the shore of the Yukon river.

Vancouver.—The observatory is at Brockton Point in Stanley Park.

